SYLLABUS

Module -1

Basic Concepts: Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh.

Module -2

Network Theorems: Superposition, Reciprocity, Millman‘s theorems, Thevinin‘s and Norton‘s theorems and Maximum Power transfer theorem.

Module -3

Transient behavior and initial conditions: Behavior of circuit elements under switching condition and their Representation, evaluation of initial and final conditions in RL, RC and RLC circuits for AC and DC excitations.

Laplace Transformation & Applications: Solution of networks, step, ramp and impulse responses, waveform Synthesis.

Module -4

Resonant Circuits: Series and parallel resonance, frequency- response of series and Parallel circuits, Q–Factor, Bandwidth.

Module -5

Two port network parameters: Definition of z, y, h and transmission parameters, modeling with these parameters, relationship between parameters sets.

Text Books:

1. M.E. Van Valkenberg (2000), “Network analysis”, Prentice Hall of India, 3rd edition, 2000, ISBN: 9780136110958.

2. Roy Choudhury, “Networks and systems”, 2nd edition, New Age International Publications, 2006, ISBN: 9788122427677.

Reference Books:

1. Hayt, Kemmerly and Durbin “Engineering Circuit Analysis”, TMH 7th Edition, 2010.

2. J. David Irwin /R. Mark Nelms, “Basic Engineering Circuit Analysis”, John Wiley, 8th edition, 2006.

3. Charles K Alexander and Mathew N O Sadiku, “Fundamentals of Electric Circuits”, Tata McGraw-Hill, 3rdEd, 2009.

Module 1: Basic Circuit Concepts

Circuit Elements:

Any two terminal circuit components are called circuit elements.

Types:

1) Active elements: Deliver the energy to the network

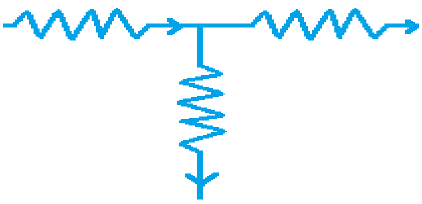
Examples: Voltage Source, Current Source

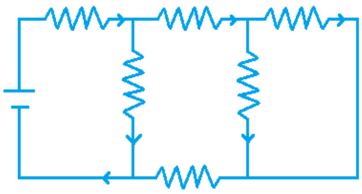
2) Passive elements: Absorb the energy from the network

Examples: Resistors, Capacitors, Inductors

Network:

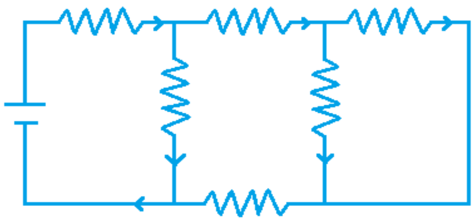
Interconnection of two are more circuit elements is called network





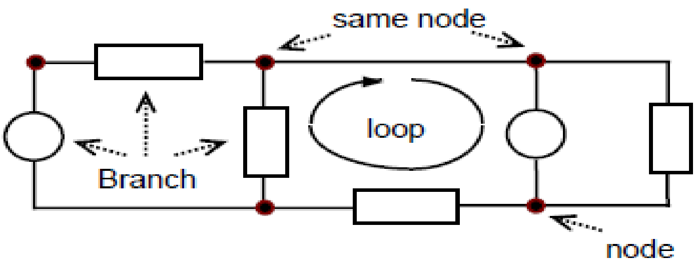
Circuit:

Network with at least one closed path is called circuit



Note: Every circuit is a network but all networks are not circuits

Network Terminology



- **Branch**

  A branch represents a single element, such as a resistor or a battery

- **Node**

  A node is the point or junction in a circuit connecting two or more branches or circuit elements. The node is usually indicated by a dot (.) in a circuit

- **Loop**

  A loop is any closed path in a circuit

- **Mesh**

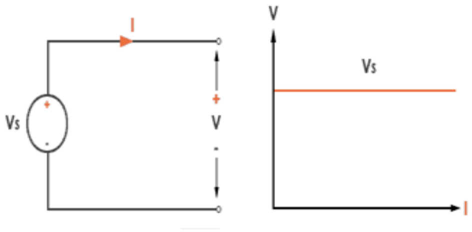
  It is a loop that contains no other loop within it.

Energy sources:

- Energy sources
  - Voltage source
    - Ideal VS
    - Practical VS
  - Current source
    - Ideal CS
    - Practical CS

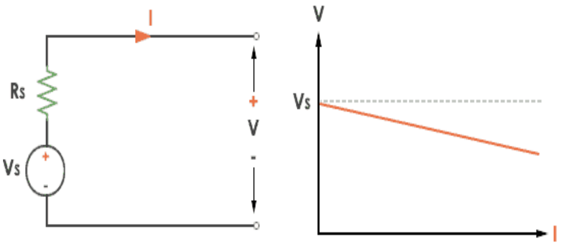
Ideal VS:

- Whose internal resistance is zero
- Irrespective of the load current, terminal voltage is constant



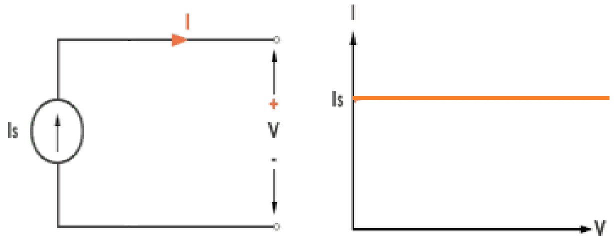
Practical VS:

- Which has finite internal resistance and connected in series with the source
- Terminal voltage decreases with increase in load current



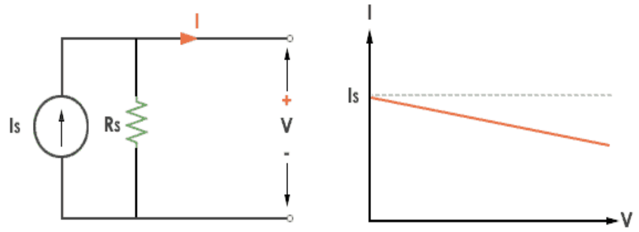
Ideal CS:

- Has infinite internal resistance
- Irrespective of the load voltage, terminal current is constant



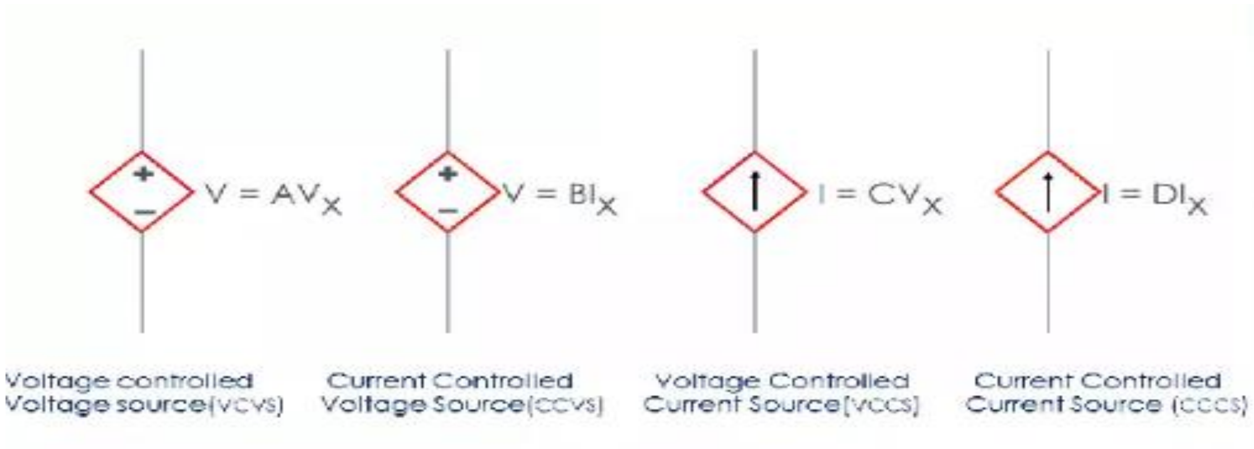
Practical CS:

- Has finite internal resistance
- Terminal current decreases with increase in load current



Dependent sources/ Controlled sources:

- Sources whose voltage/current depends on voltage/current that appears at some other location of the network.
- Represented by diamond symbol
- 4 types



Classification of Networks:

1) Linear and Non linear networks

A ***Linear circuit*** is one whose parameter are constant i.e., they do not change with voltage or current.

Examples: Network consisting of R, L and C

A ***Non linear circuit*** is one whose parameters change with voltage or current.

Examples: Network consisting of diode and transistor

2) Unilateral and Bilateral networks

The circuit whose properties or characteristics change with the direction of its operation is said to be ***Unilateral***.

Examples: A diode rectifier is a unilateral, because it cannot perform rectification in both directions.

A ***Bilateral circuit*** is one whose properties or characteristics are the same in either direction.

Examples: R, L & C.

3) Active and Passive network

Network consisting of only passive elements is called **Passive** network

Examples: Network consisting of R, L and C

Network consisting of at least one active element is called **Active** network

Examples: Network consisting VS and CS

4) Lumped and Distributed network

Network in which elements are physically separable is called **Lumped** network.

Examples: R, L and C

Network in which elements cannot be physically separable is called **Distributed** network.

Examples: Transmission lines having R, L, C all along their length.

Source Transformation

Source Transformation involves the transformation of voltage source to its equivalent current source and vice-versa.

Consider a voltage source with series resistance R and a current source with same resistance R in parallel as shown below.

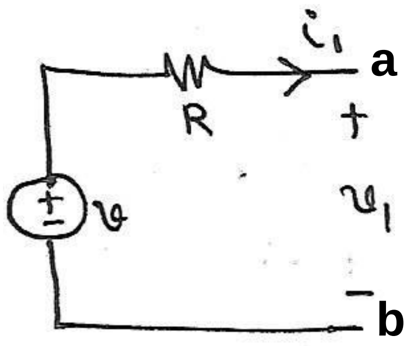


Fig: Voltage source

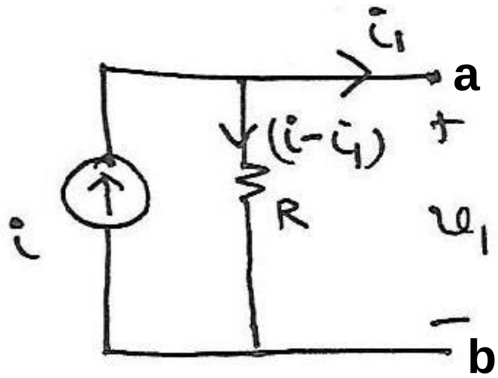


Fig: Current source

The terminal voltage and current relationship in the case of voltage source is;

$v_1 = v - i_1 R$ …… (1)

The terminal voltage and current relationship in the case of current source is;

$i_1 = i - v_1 / R$

$v_1 = i R - i_1 R$ …… (2)

If the voltage source above has to be equivalently transformed to or represented by a current source then the terminal voltages and currents have to be same in both cases.

This means eqn. (1) should be equal to eqn. (2).

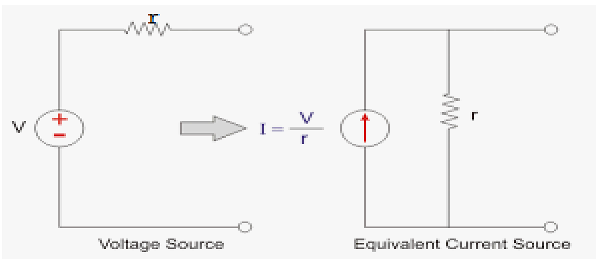
This implies,

$v = i R$

or

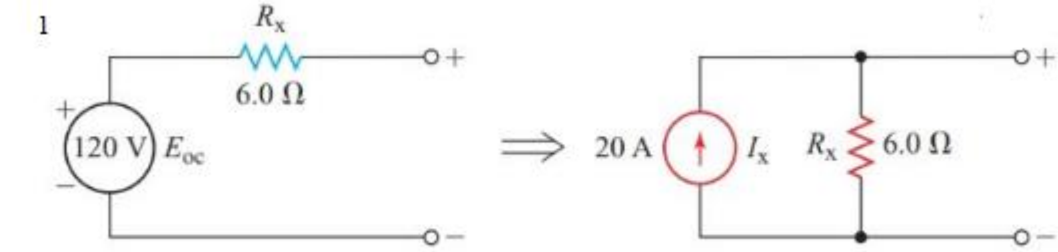
$i = v / R$…(3)

If eqn.(3) holds good, then the voltage source above can be equivalently transformed to or represented by, the current source shown above and vice-versa.



Examples:

1



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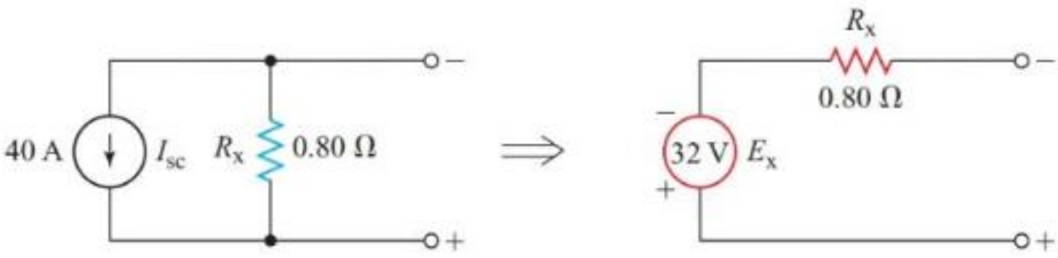
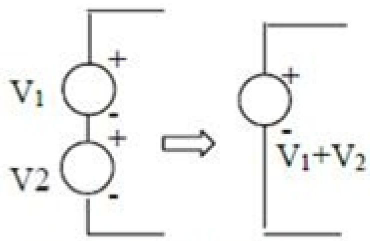
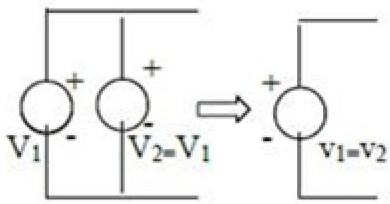


Combination of sources:

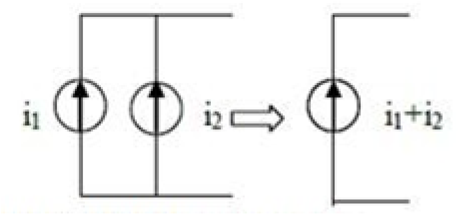
1. Two ideal voltage sources in series



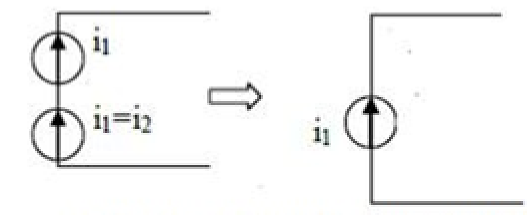
2. Two ideal voltage sources in parallel



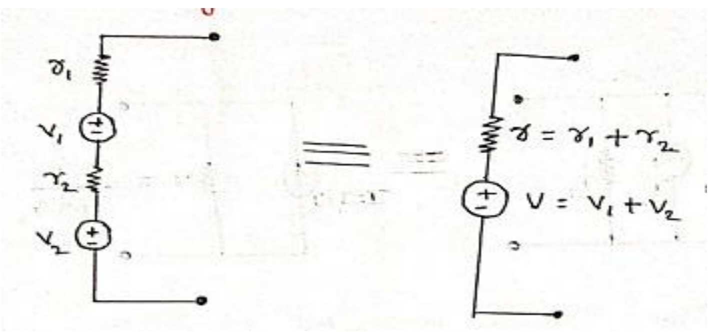
3. Two ideal current sources in parallel



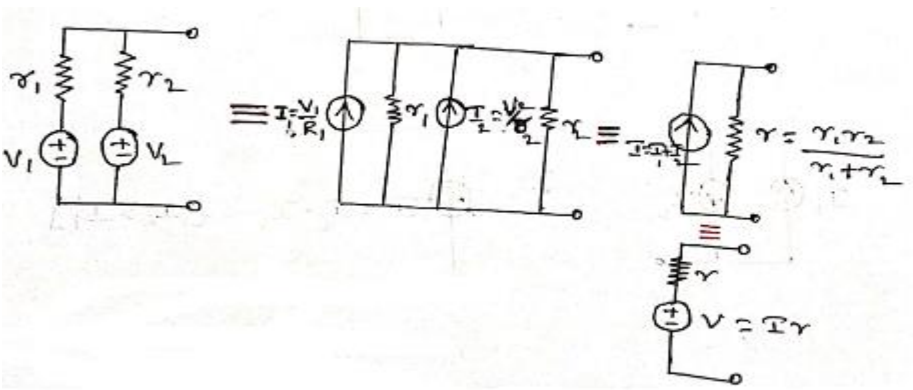
4. Two ideal current sources in series



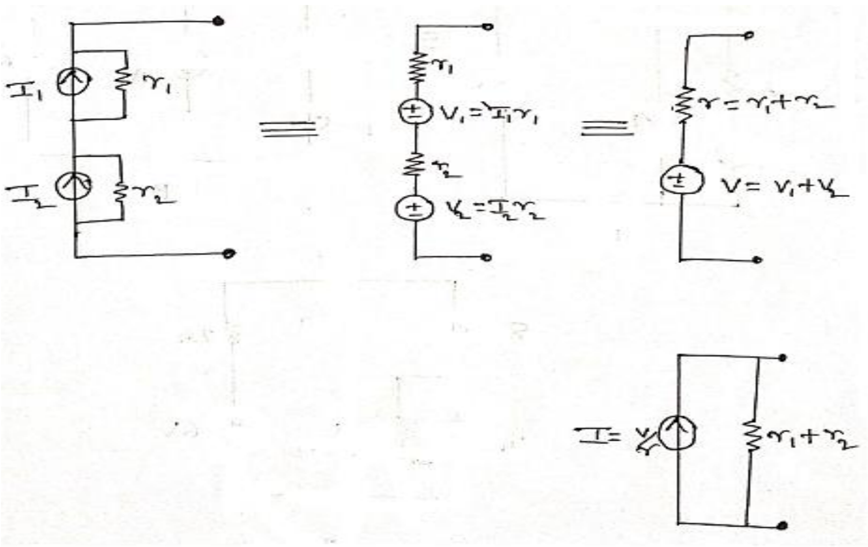
5. Two practical voltage sources in series



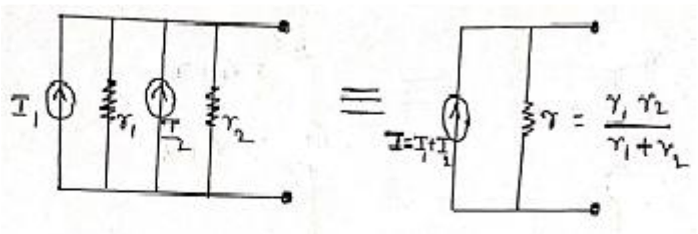
6. Two practical voltage sources in parallel



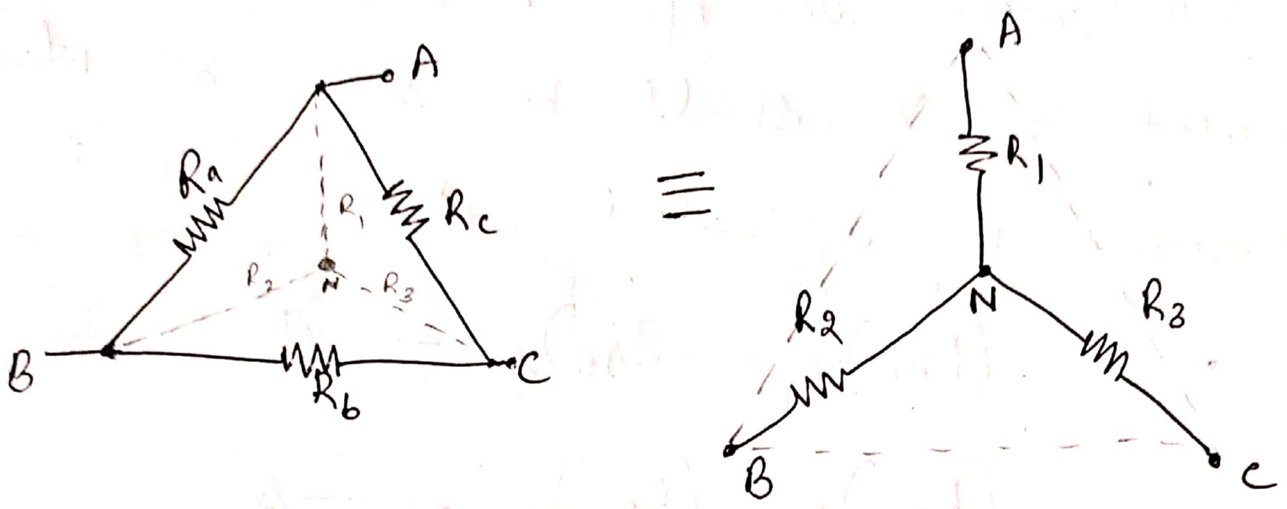
7. Two practical current sources in series



8. Two practical current sources in parallel



Star-Delta transformation ($\Delta - Y$) :-



Let R_a, R_b & R_c be the elements connected in Δ n/w b/n the terminals A, B & C. Let the n/w consisting of R_1, R_2 & R_3 be the equivalent Y n/w across the same terminals A, B & C.

For converting the given Δ n/w into equivalent Y n/w, it is necessary to derive the relations for R_1, R_2 & R_3 in-terms of R_a, R_b & R_c.

Ily to convert the known Y into equivalent Δ n/w, it is neccessary to derive the relation for R_a, R_b & R_c in terms of R_1, R_2 & R_3.

i) Delta to Star transformation :-

The resistance b/n A & B when connected in Y should be same as when connected in equivalent Δ.

i.e, $(R_{AB})_Y = (R_{AB})_\Delta$ —— ①

$(R_{BC})_Y = (R_{BC})_\Delta$ —— ②

$(R_{CA})_Y = (R_{CA})_\Delta$ —— ③

$$(R_{AB})_Y = R_1 + R_2$$

$$(R_{AB})_\Delta = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c}$$

from ①

$$R_1 + R_2 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \quad \text{—— ④}$$

lly $$R_2 + R_3 = \frac{R_b(R_c+R_a)}{R_a+R_b+R_c} \quad \text{—— ⑤}$$

$$R_3 + R_1 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad \text{——— (6)}$$

$\rightarrow$ (4) − (5) gives.

$$R_1 + \cancel{R_2} - \cancel{R_2} - R_3 = \frac{R_a(R_b + R_c) - R_b(R_c + R_a)}{R_a + R_b + R_c}$$

$$R_1 - R_3 = \frac{R_a \cancel{R_b} + R_a R_c - R_b R_c - \cancel{R_b} R_a}{R_a + R_b + R_c}$$

$$R_1 - R_3 = \frac{R_a R_c - R_b R_c}{R_a + R_b + R_c} \quad \text{——— (7)}$$

$\rightarrow$ (6) + (7) gives.

$$\cancel{R_3} + R_1 + R_1 - \cancel{R_3} = \frac{R_c R_a + \cancel{R_c R_b} + R_a R_c - \cancel{R_b R_c}}{R_a + R_b + R_c}$$

$$\cancel{2} R_1 = \frac{\cancel{2} R_a R_c}{R_a + R_b + R_c}$$

$$\boxed{R_1 = \frac{R_a R_c}{R_a + R_b + R_c}} \quad \text{——— (8)}$$

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$$\text{Illy} \quad R_2 = \frac{R_b R_a}{R_a + R_b + R_c} \quad \text{—} (9) \quad \& \quad R_3 = \frac{R_c R_b}{R_a + R_b + R_c} \quad \text{—} (10)$$

ii) Star to delta transformation :-

To get the expressions for R_a, R_b & R_c in terms of R_1, R_2 & R_3, eqns (8), (9) & (10) are used.

(8) × (9) gives
$$R_1 \times R_2 = \frac{R_a^2 R_b R_c}{(R_a + R_b + R_c)^2} \quad \text{—} (11)$$

(9) × (10) gives
$$R_2 \times R_3 = \frac{R_a R_b^2 R_c}{(R_a + R_b + R_c)^2} \quad \text{—} (12)$$

(10) × (8) gives
$$R_3 \times R_1 = \frac{R_a R_b R_c^2}{(R_a + R_b + R_c)^2} \quad \text{—} (13)$$

→ (11) + (12) + (13) gives

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_a^2 R_b R_c + R_a R_b^2 R_c + R_a R_b R_c^2}{(R_a + R_b + R_c)^2}$$

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$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c\left[R_a+\cancel{R_b}+R_c\right]}{\left(R_a+\cancel{R_b}+R_c\right)^2}$$

$$= \frac{R_aR_bR_c}{(R_a+R_b+R_c)}$$

R_3

from eqn (10) $R_3 = \dfrac{R_bR_c}{R_a+R_b+R_c}$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = R_aR_3$$

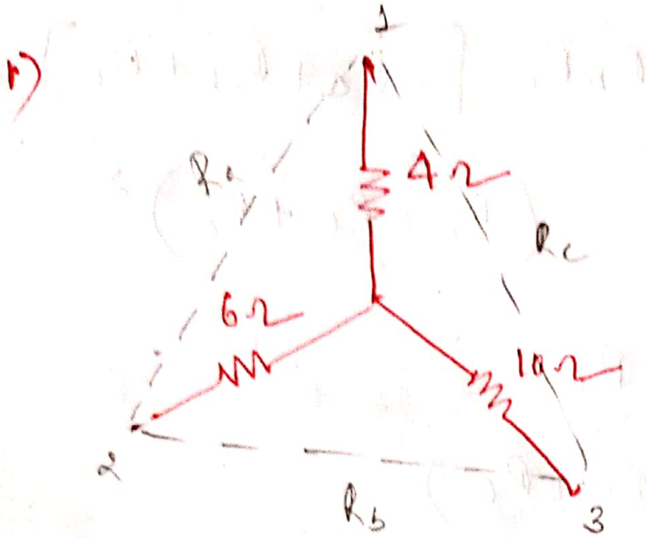
$$\Rightarrow \boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}.$$

llly

$$\boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}}$$

$$\boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}}$$

1)



$$R_a = \frac{4\times 6 + 6\times 10 + 10\times 4}{10}$$

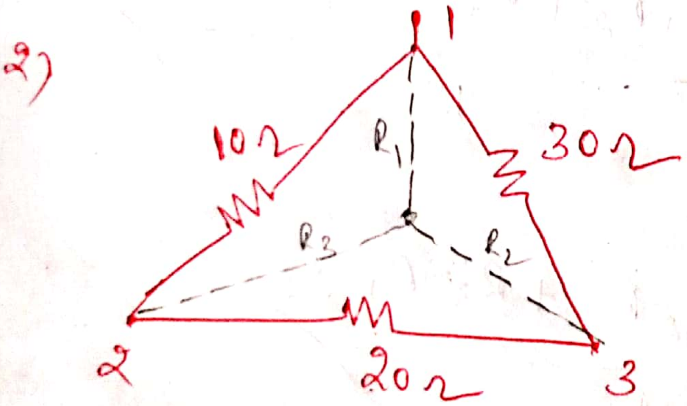
$$= 12.4\Omega$$

$$R_b = \frac{4\times 6 + 6\times 10 + 10\times 4}{4}$$

$$= 31\Omega$$

$$R_c = \frac{124}{6} = 20.66\Omega.$$

2)



$$R_1 = \frac{10\times 30}{10+30+20}$$

$$R_1 = 5\Omega$$

$$R_2 = \frac{30\times 20}{10+30+20}$$

$$R_2 = 10\Omega$$

$$R_3 = \frac{10\times 20}{10+20+30} = 3.33\Omega$$

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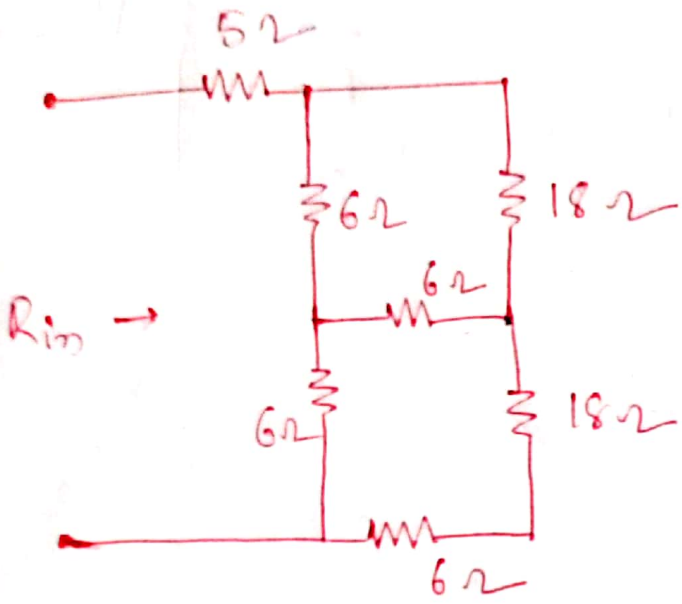
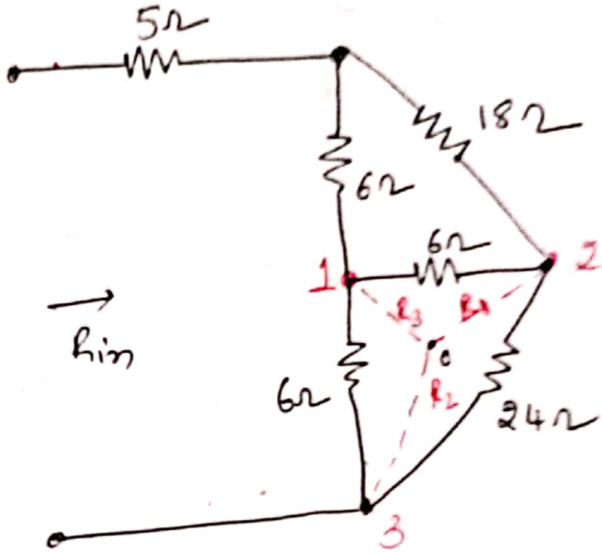
1) Determine R_{in} using Star Delta transformation in the n/w shown below:

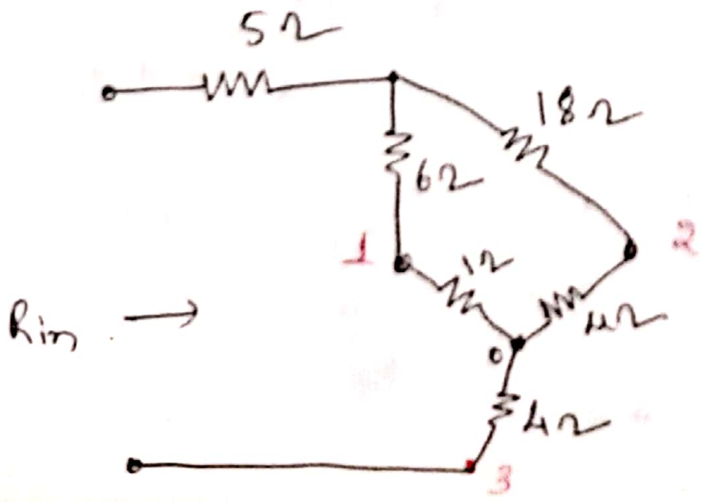


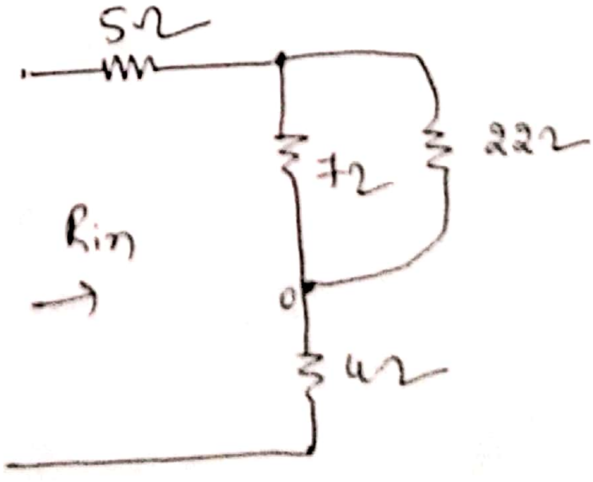


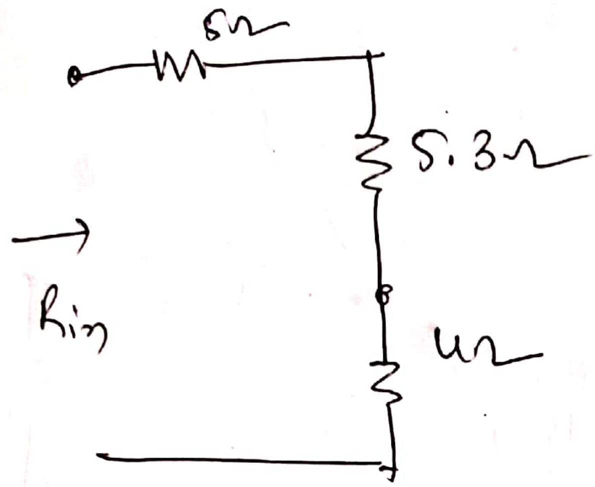
$$R_1 = \frac{24 \times 6}{6+6+24} = 4\Omega$$

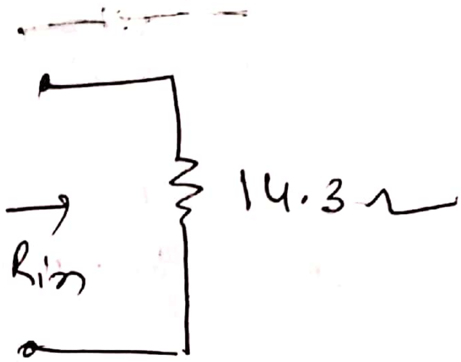
$$R_2 = \frac{24 \times 6}{6+6+24} = 4\Omega$$

$$R_3 = \frac{6 \times 6}{6+6+24} = 1\Omega$$



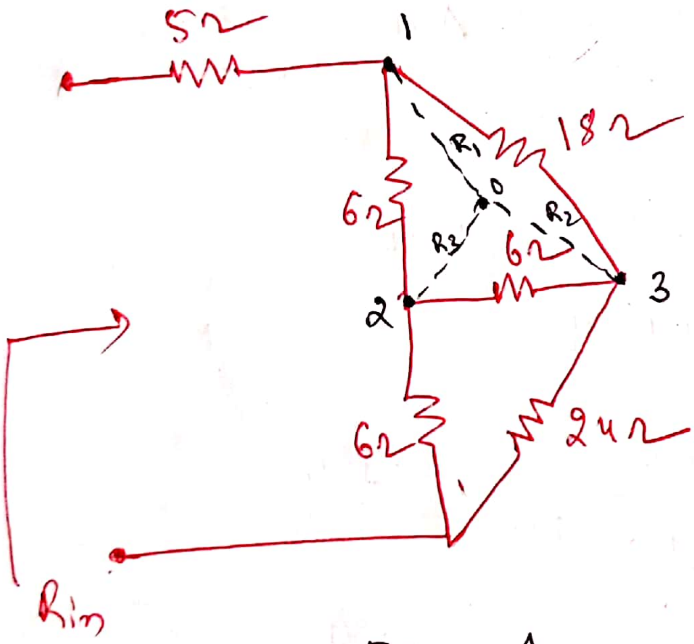




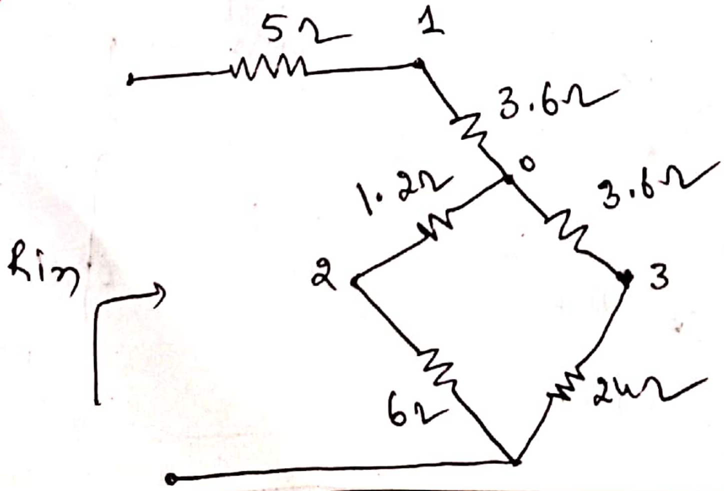


$$\boxed{R_{in} = 14.3\,\Omega}$$

(or)



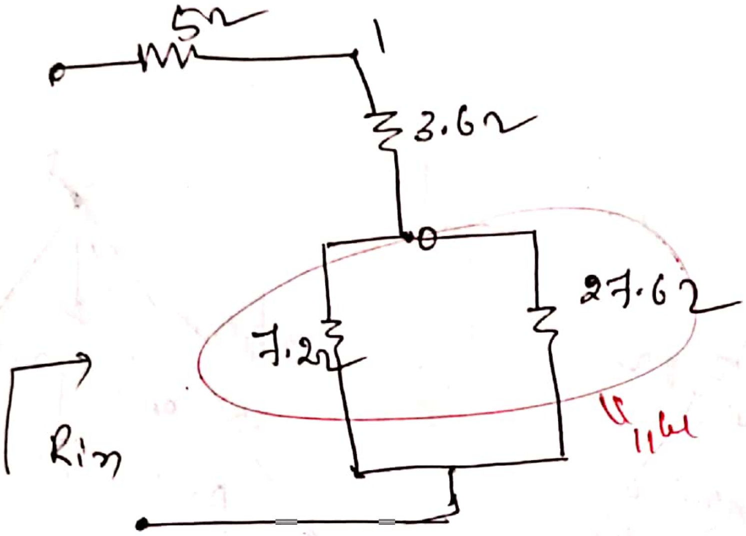
$$R_1 = \frac{6 \times 18}{6 + 18 + 6}$$

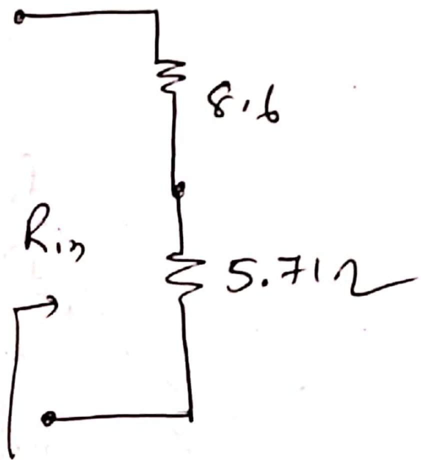


$$R_1 = 3.6\,\Omega$$

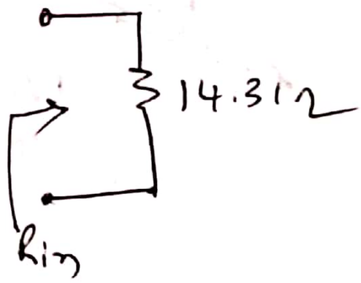
$$R_2 = \frac{18 \times 6}{6+18+6} = 3.6\Omega$$

$$R_3 = \frac{6 \times 6}{6+18+6} = 1.2\Omega$$

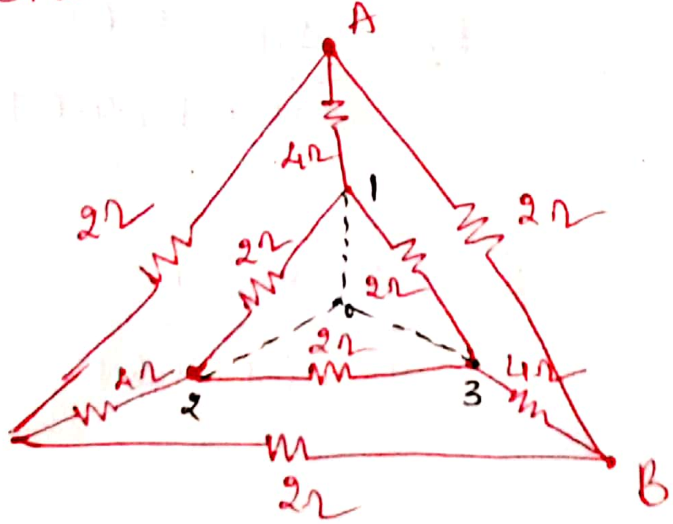


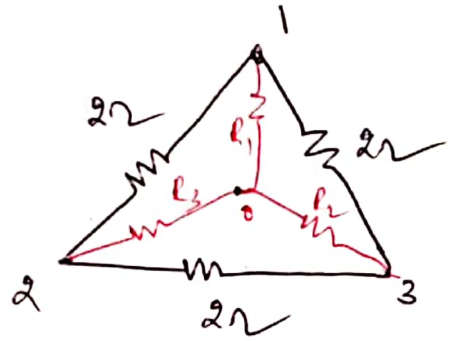


$$\boxed{R_{in} = 14.31\Omega}$$



2) Find the equivalent resistance b/n A & B for the ckt below.

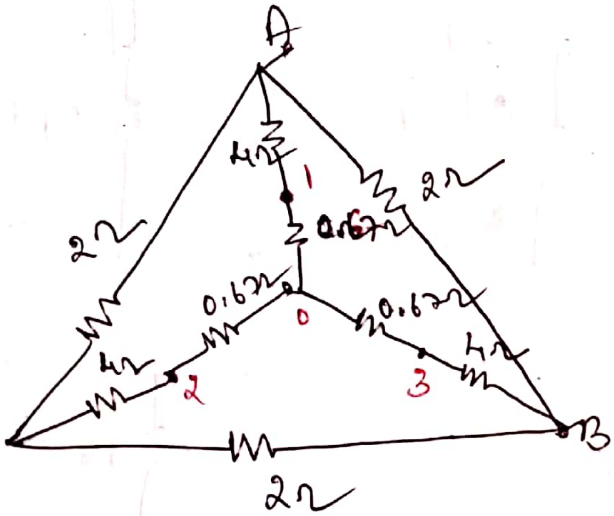


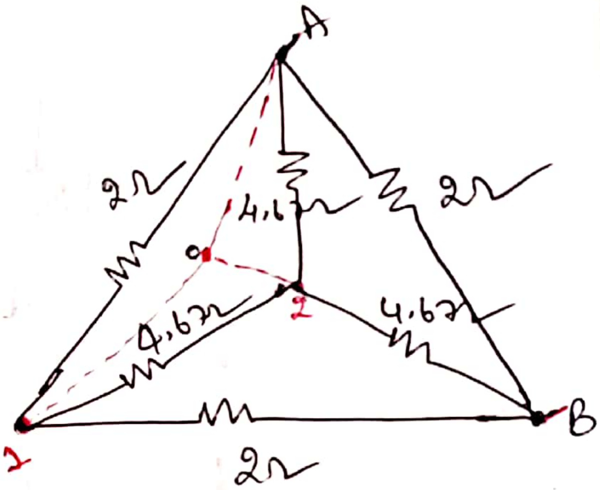


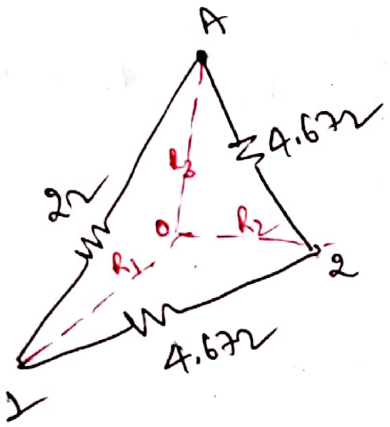
Inner delta Into Star

$$R_1 = \frac{2 \times 2}{2+2+2} = 0.67\Omega$$

$$\text{||ly} \quad R_2 = R_3 = 0.67\Omega$$



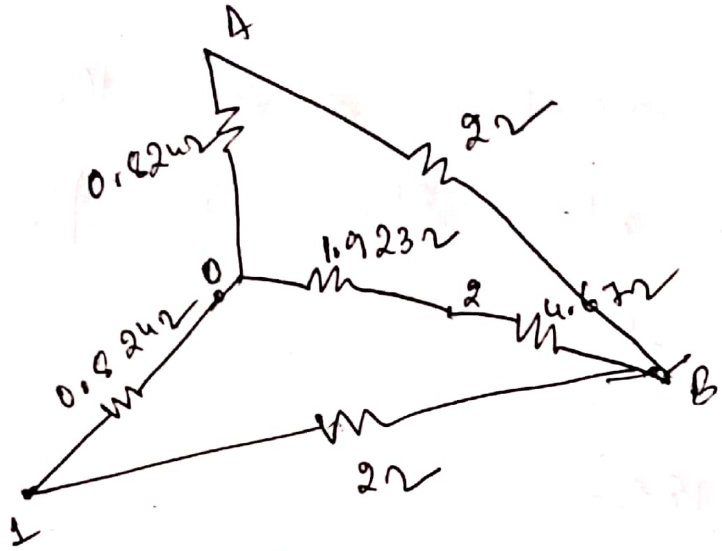


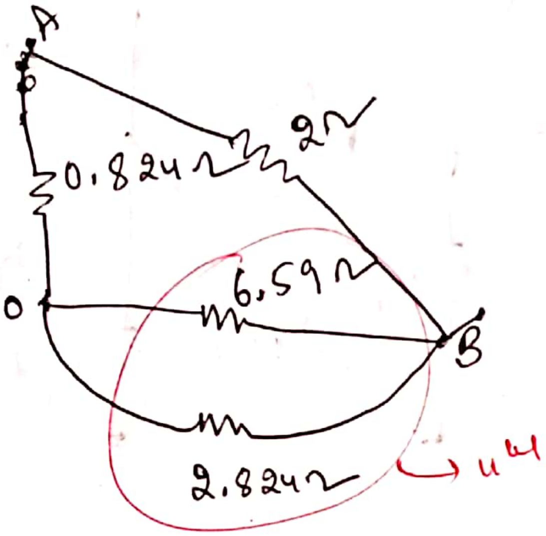


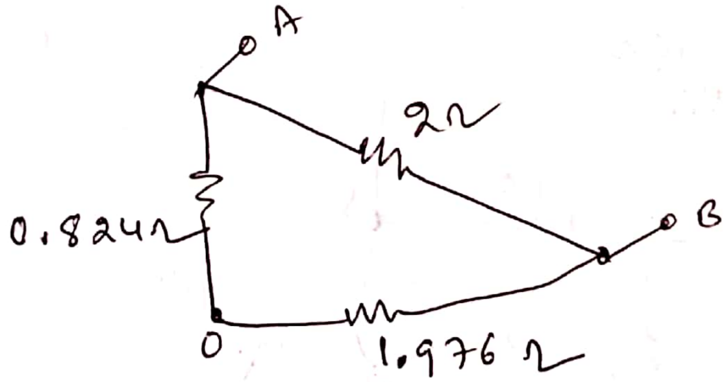
$$R_1 = \frac{2 \times 4.67}{2+4.67+4.67} = 0.824\Omega$$

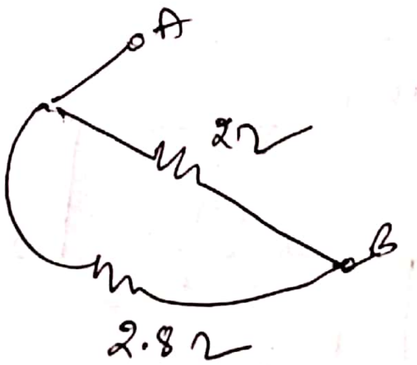
$$R_2 = \frac{4.67 \times 4.67}{2+4.67+4.67} = 1.923\Omega$$

$$R_3 = \frac{2 \times 4.67}{2+4.67+4.67} = 0.824\Omega$$



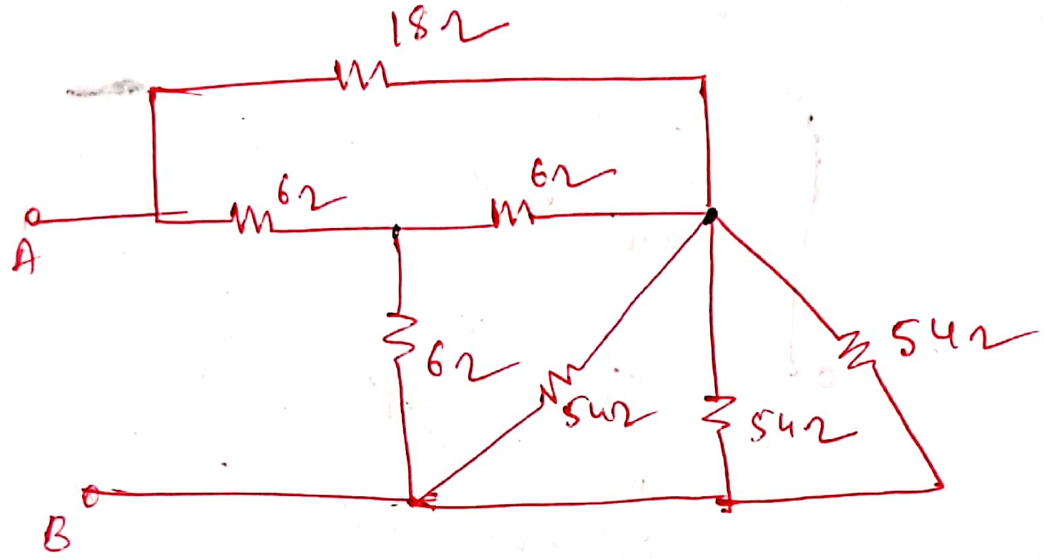


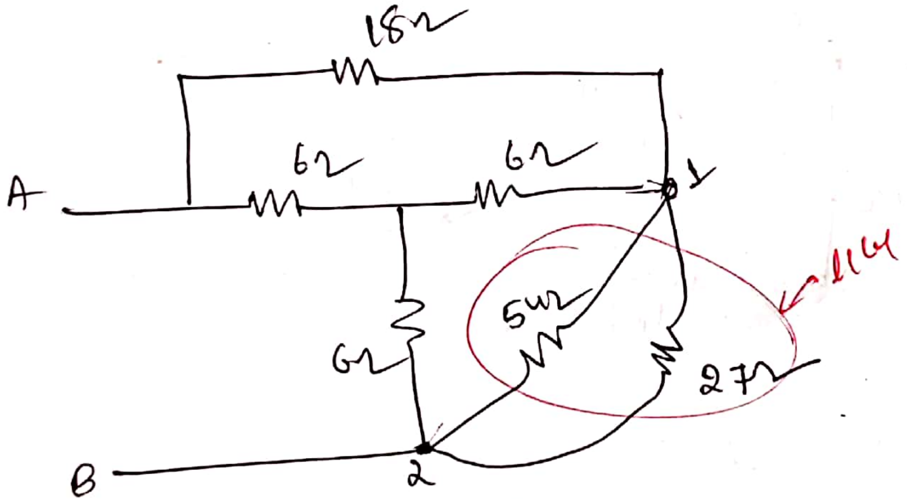


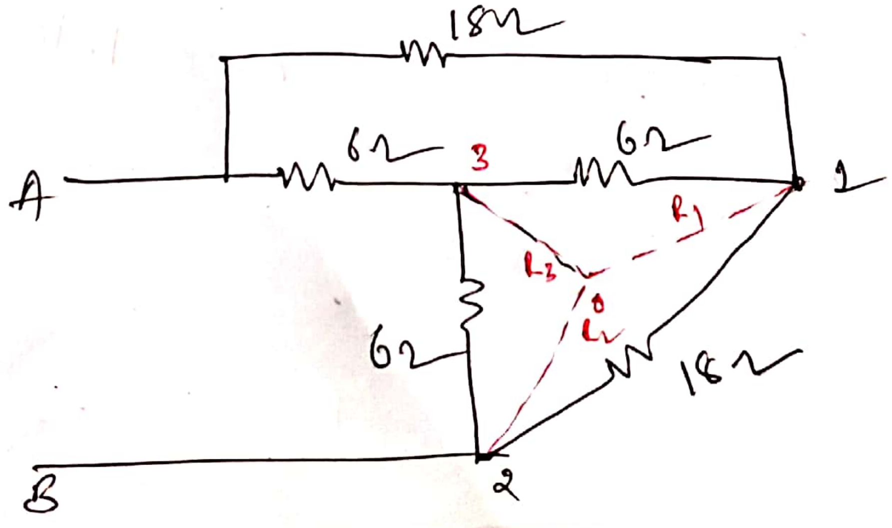


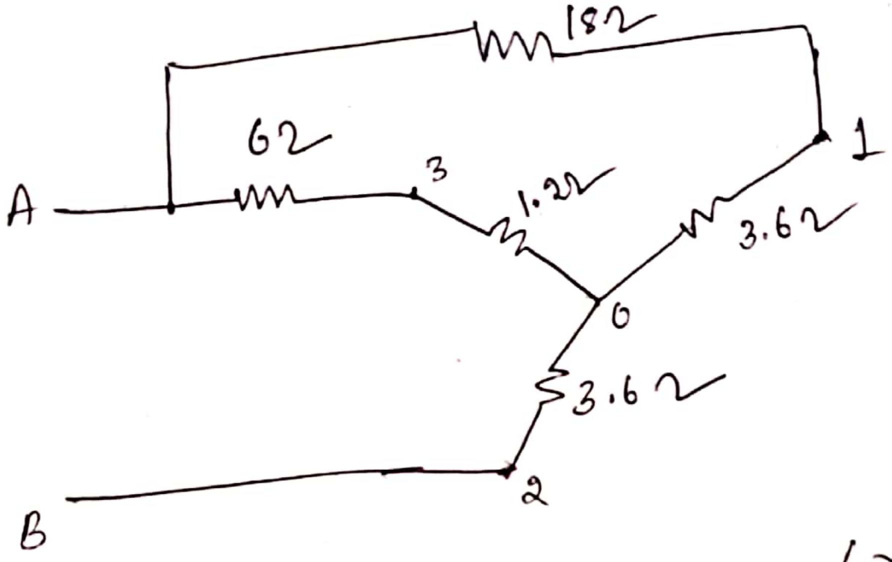
$$\boxed{R_{AB} = 1.167\,\Omega}$$

3) Compute the resistance across the terminals A & B of the n/w shown in fig using Δ-Y transformation





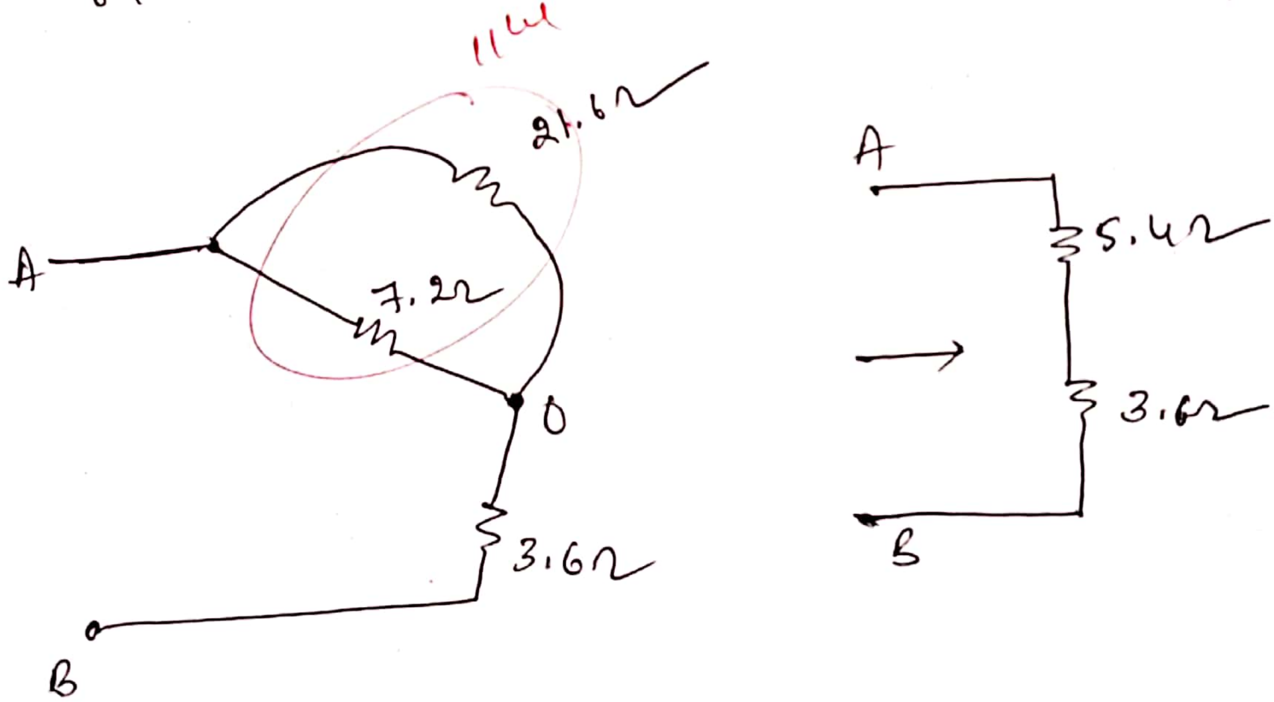




$$R_1 = \frac{6 \times 18}{6 + 18 + 6} = 3.6\,\Omega$$

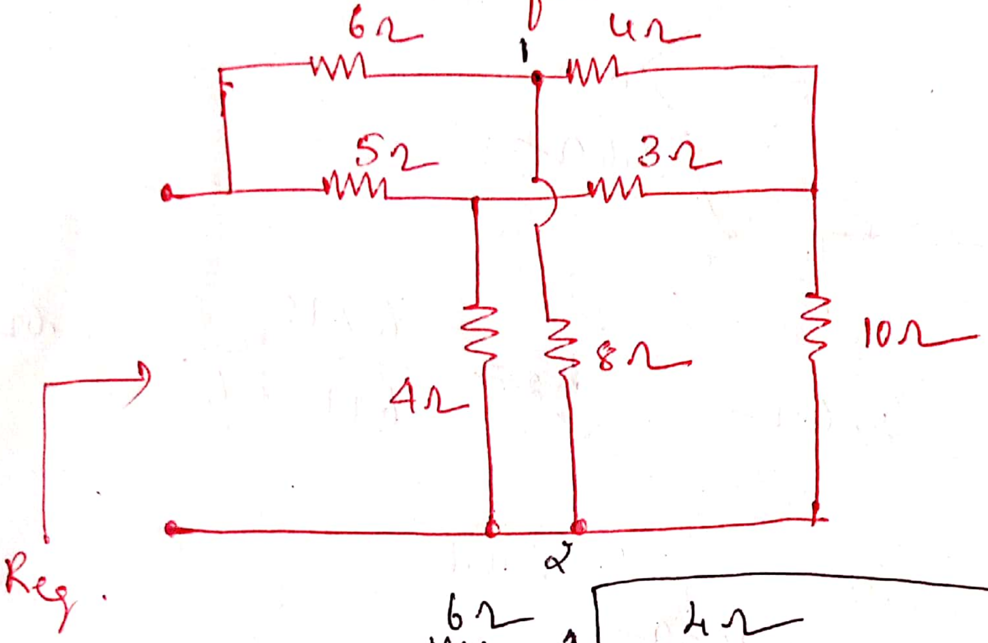
$$R_2 = \frac{6 \times 18}{6 + 18 + 6} = 3.6\,\Omega$$

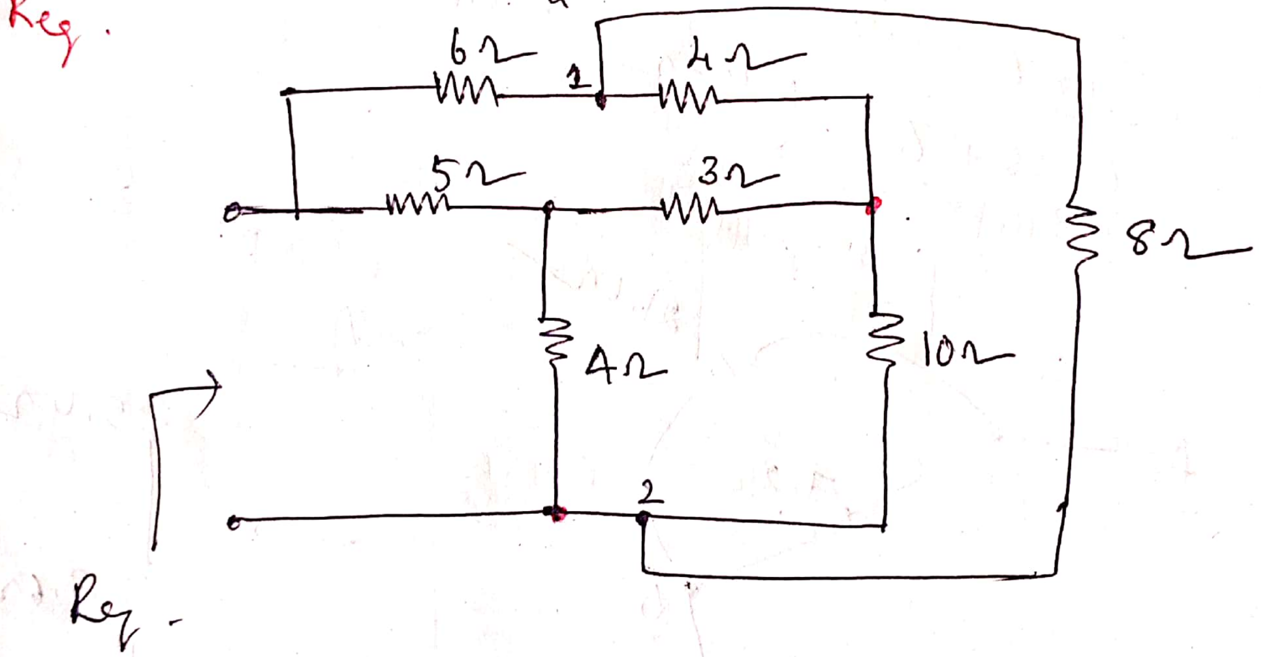
$$R_3 = \frac{6 \times 6}{6 + 18 + 6} = 1.2\,\Omega$$

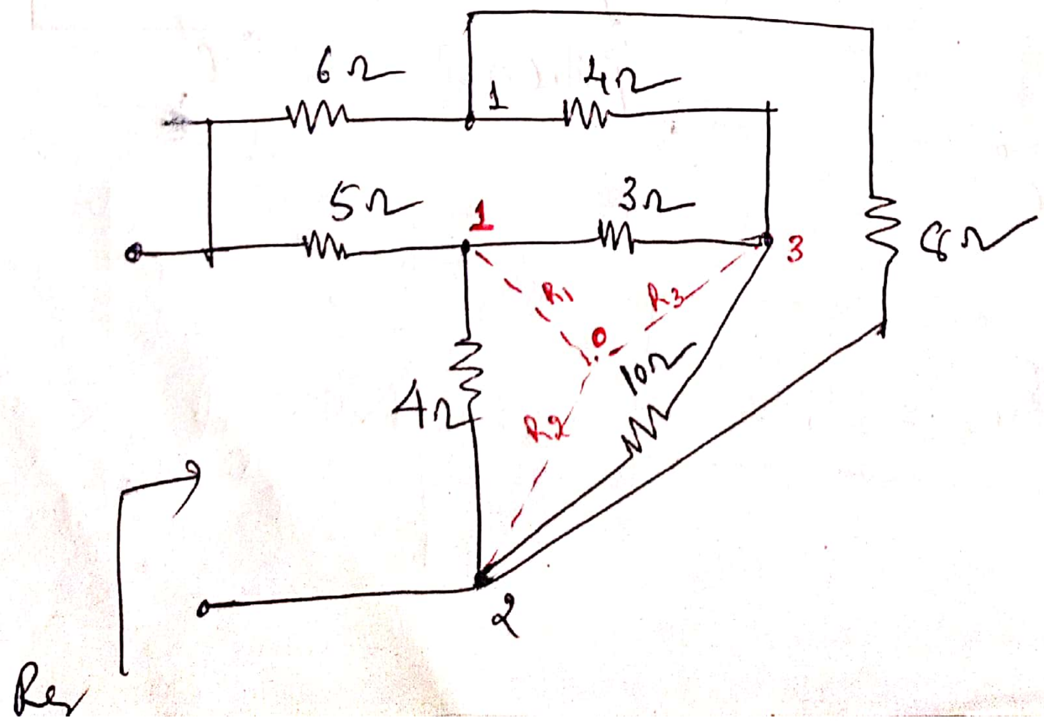


$$\boxed{R_{AB} = 9\,\Omega}$$

4) In the n/w shown below, find Req using Star delta transformation.



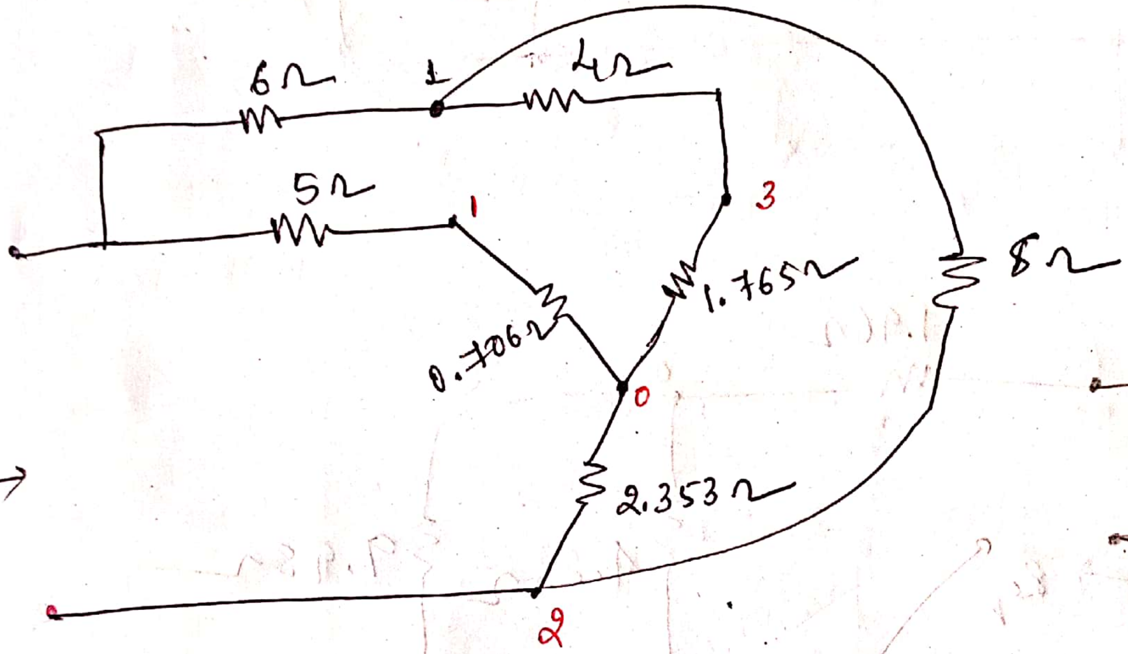


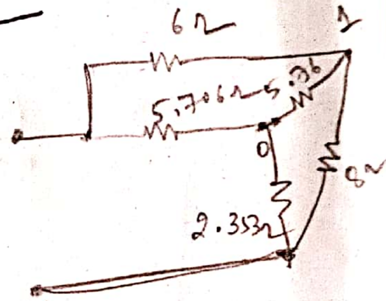


$$R_1 = \frac{4 \times 3}{4+3+10} = 0.706\,\Omega \qquad R_2 = \frac{4 \times 10}{4+10+3}$$

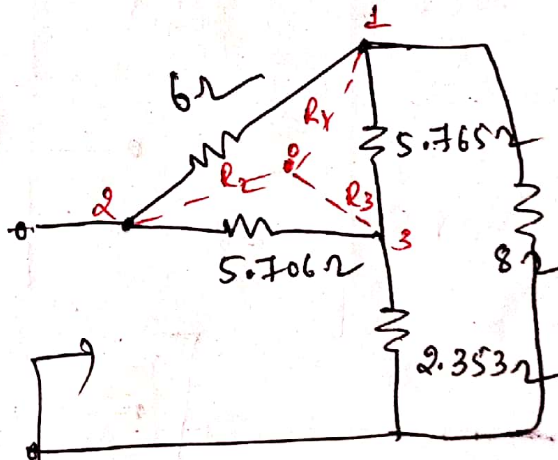
$$R_2 = 2.353$$

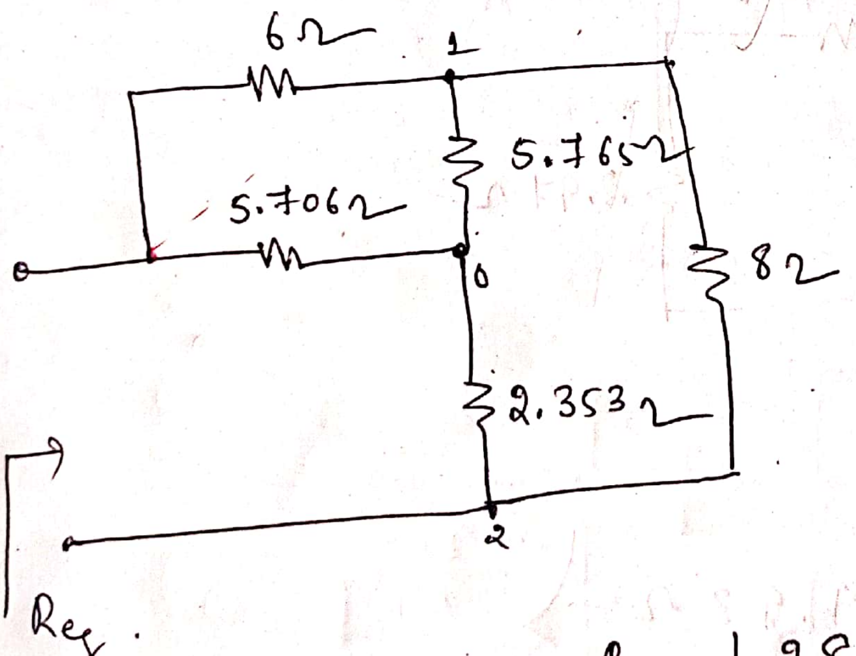
$$R_3 = \frac{3 \times 10}{3+4+10} = 1.765\,\Omega$$





Req.





Req

Req.

$R_1 = 1.98\,\Omega$ $\qquad R_3 = 1.88\,\Omega$

$R_2 = 1.96\,\Omega$

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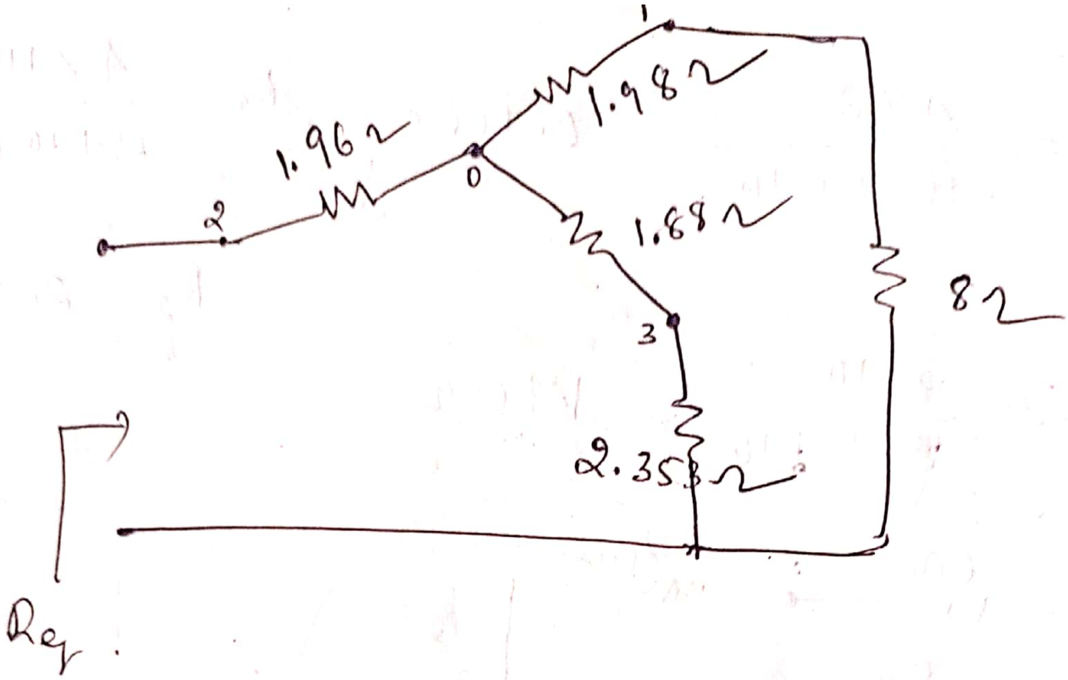
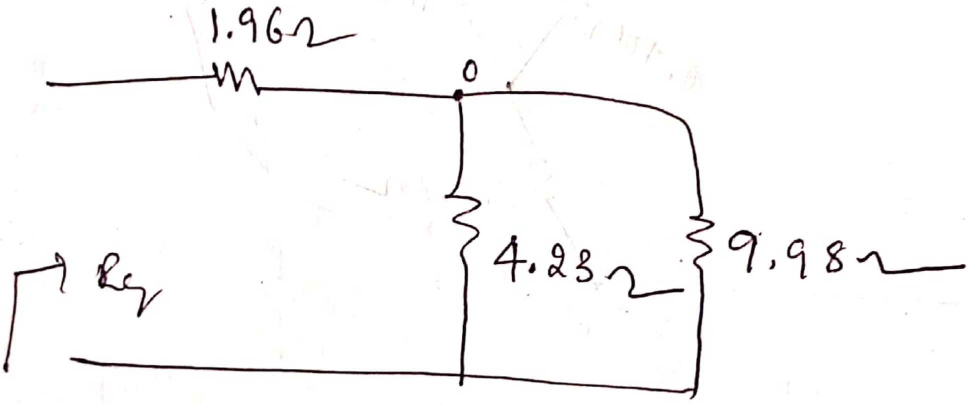
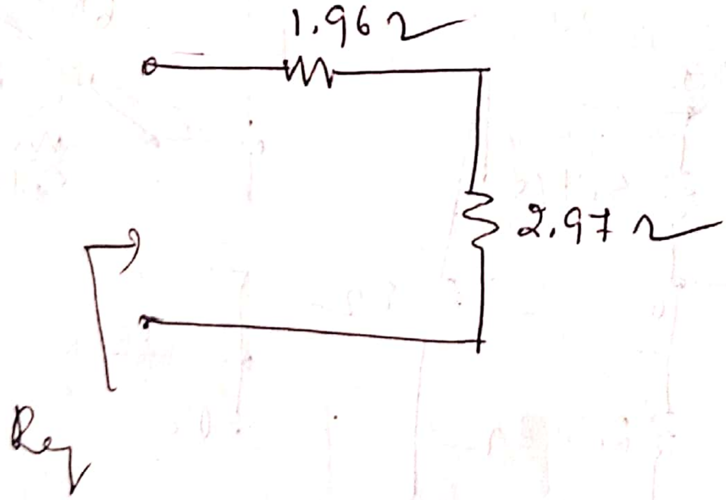
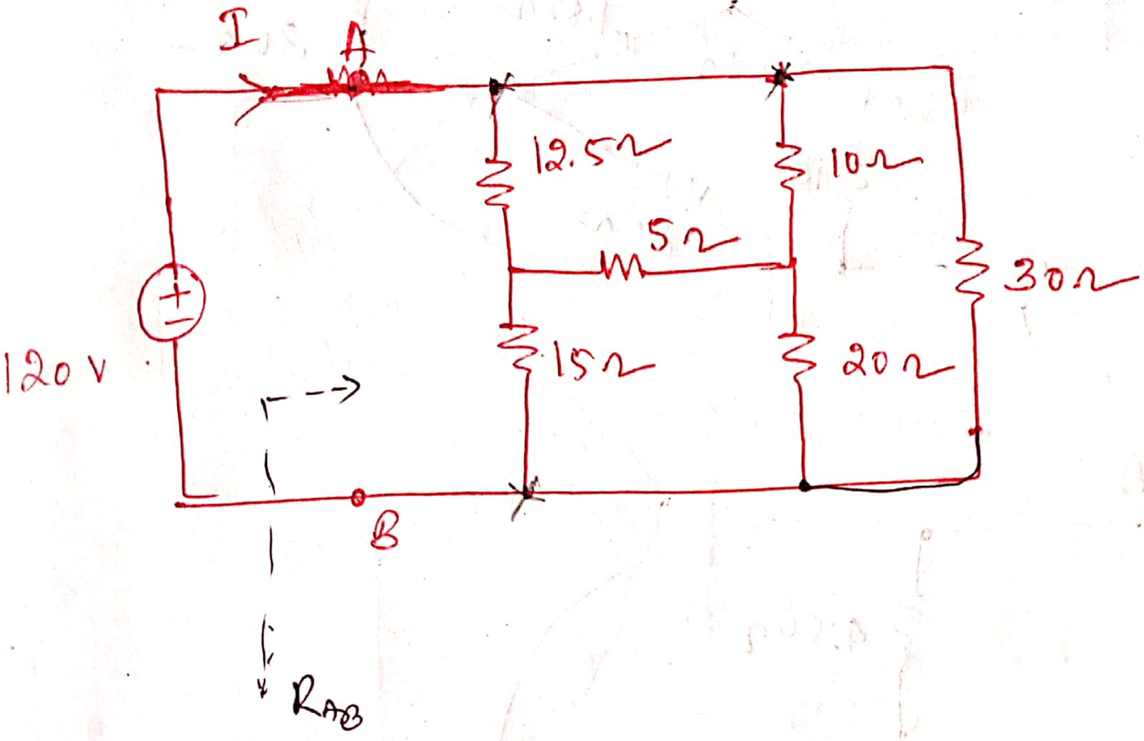


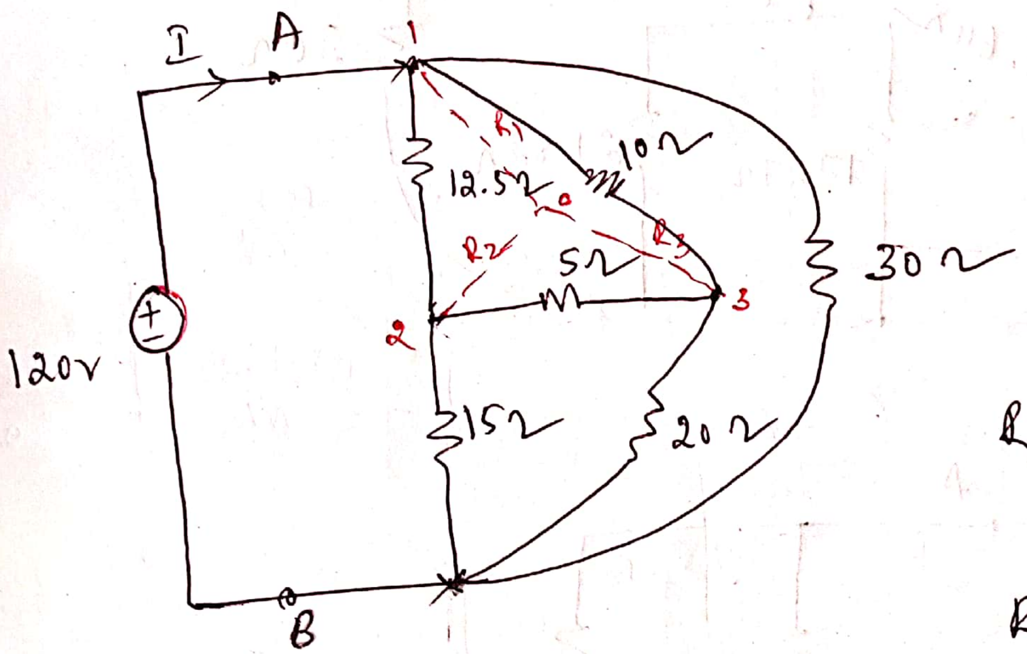




$R_{eq} = 4.93\,\Omega$

5) Obtain equivalent resistance R_{AB} for the given ckt & find I.





$$R_1 = \frac{12.5 \times 10}{12.5 + 10 + 5}$$

$$R_1 = 4.54\,\Omega$$

$$R_2 = \frac{12.5 \times 5}{12.5 + 10 + 5}$$

$$R_2 = 2.273\,\Omega$$

$$R_3 = \frac{5 \times 10}{12.5 + 10 + 5}$$

$$R_3 = 1.82\,\Omega$$

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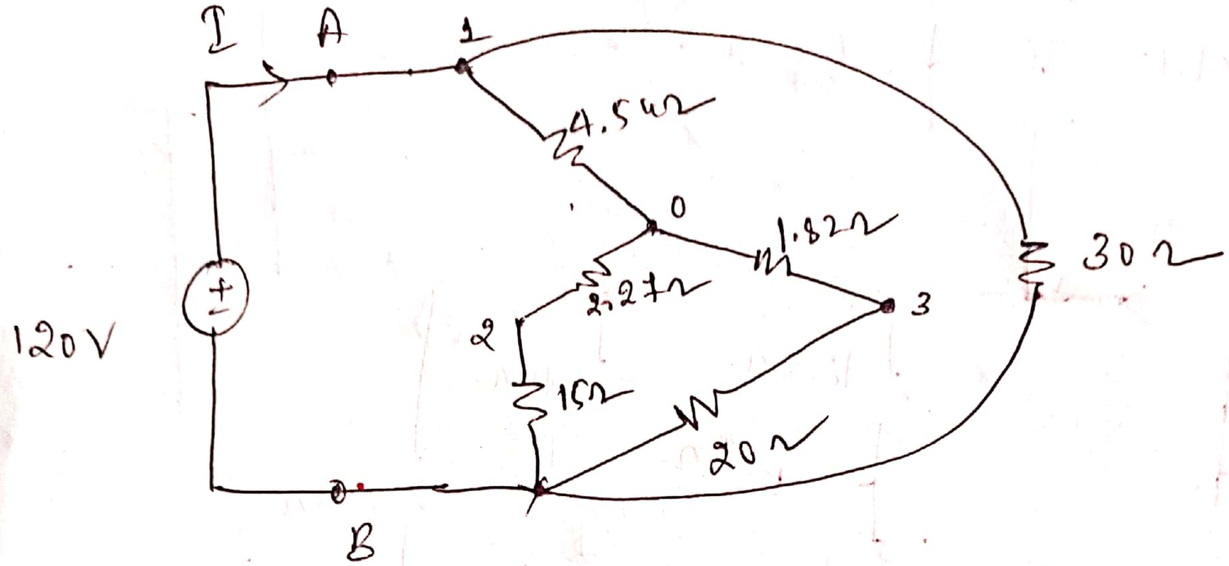
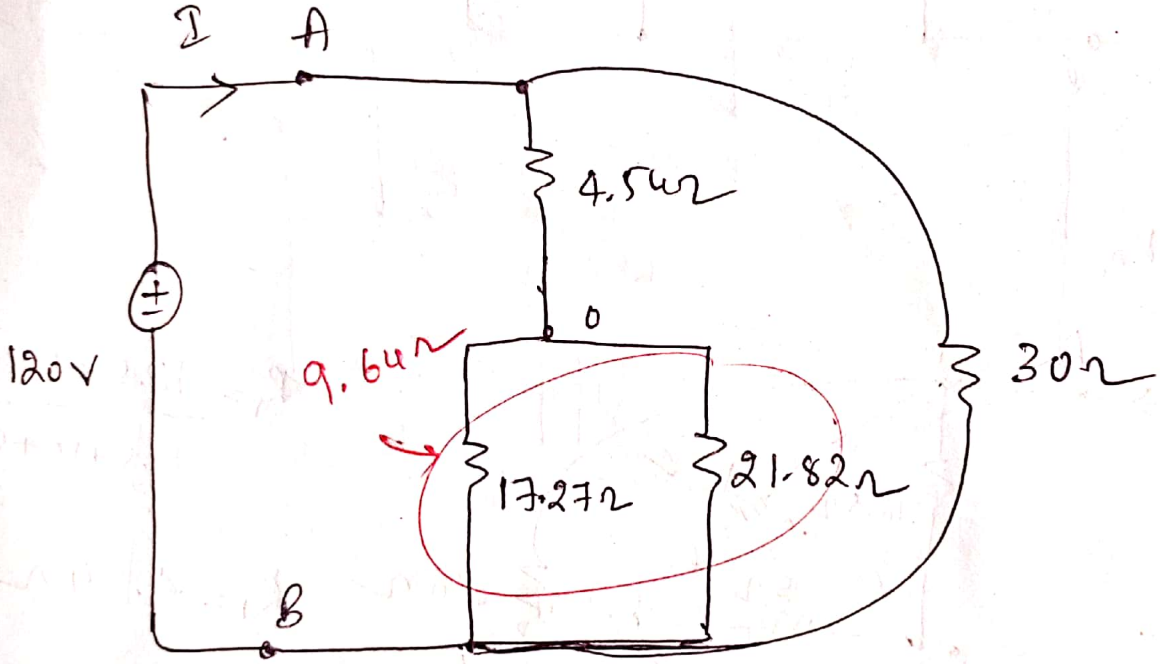
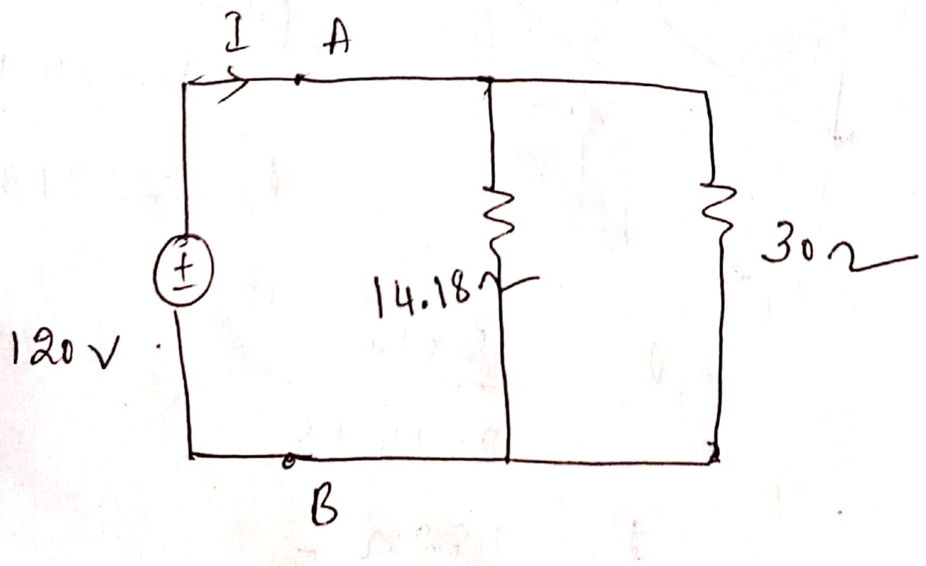
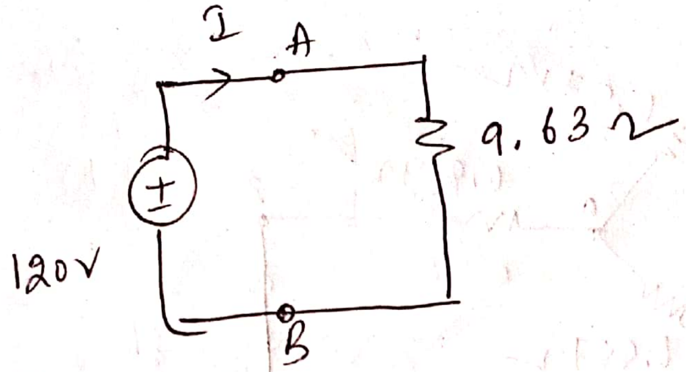

I
A
1
4.54Ω
0
1.82Ω
2.27Ω
3
2
15Ω
20Ω
30Ω
120V
B


I
A
4.54Ω
0
9.64Ω
17.27Ω
21.82Ω
30Ω
120V
B

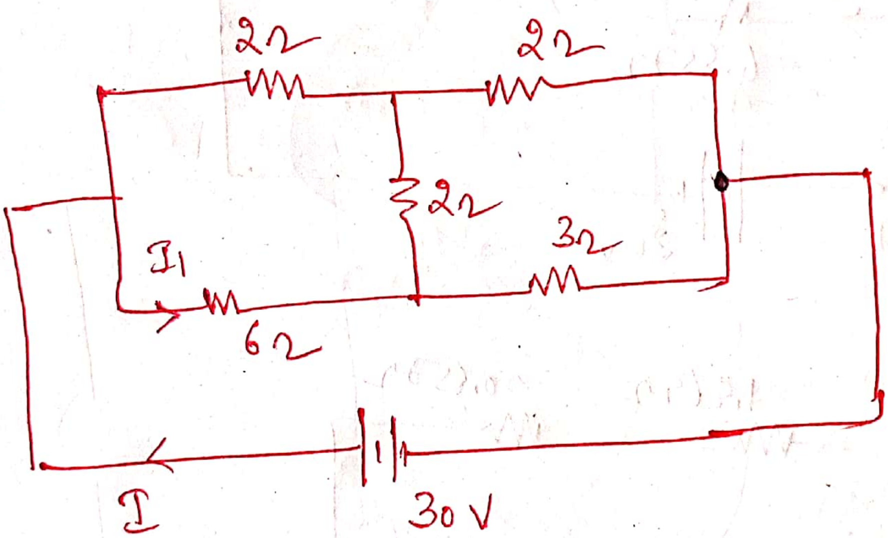

I
A
14.18Ω
30Ω
120V
B

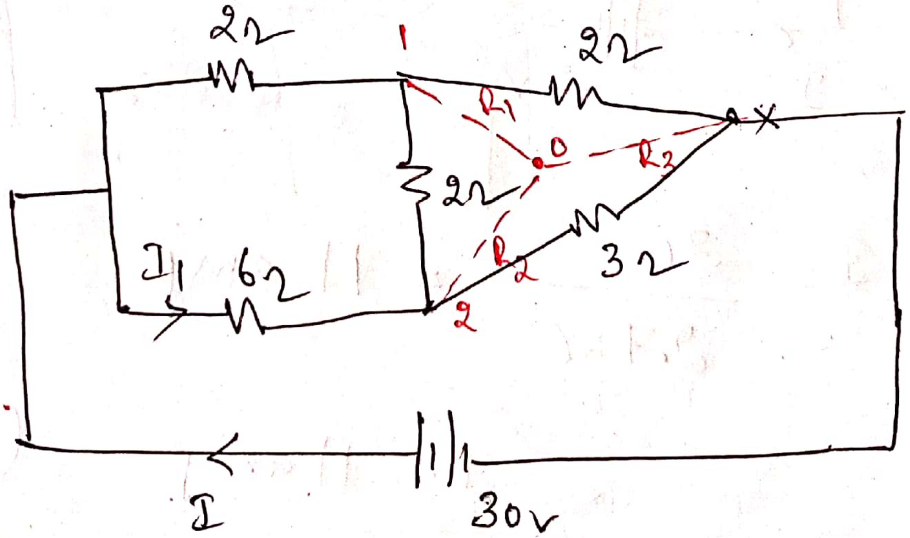


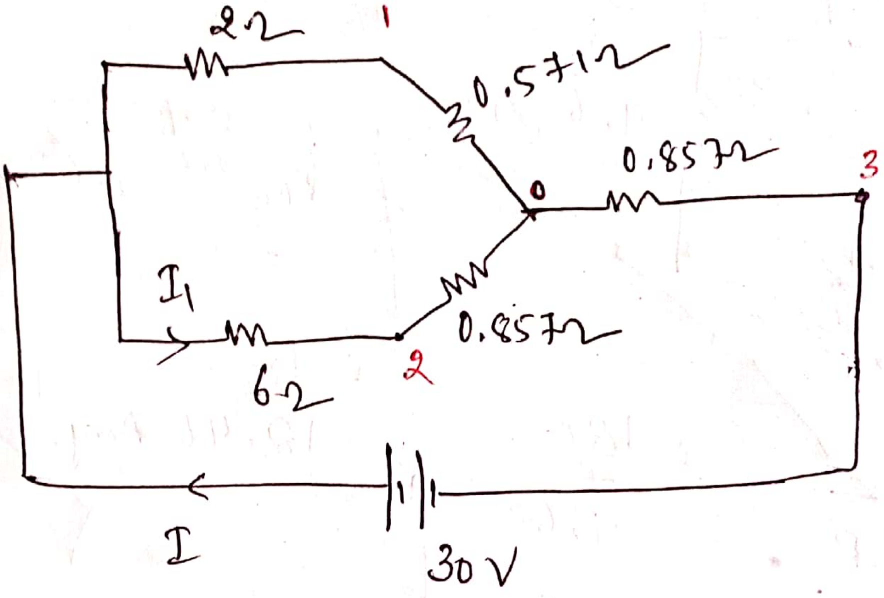
$R_{AB} = 9.63\,\Omega$

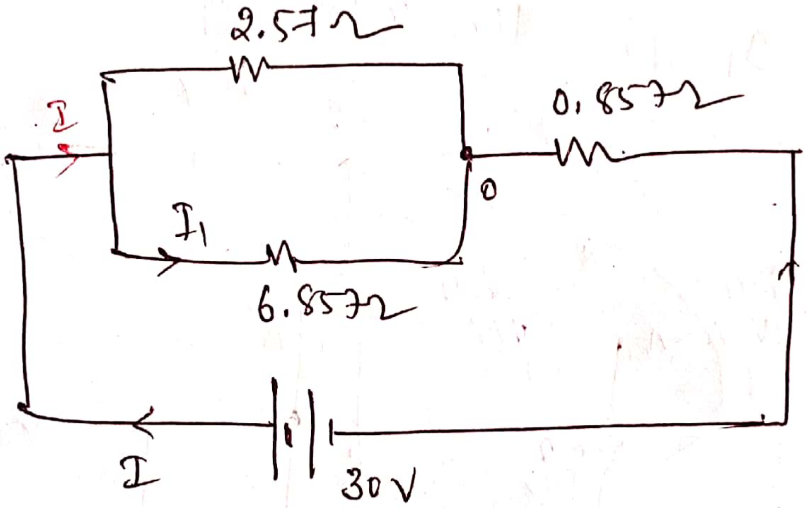
$$I = \frac{V}{R} = \frac{120}{9.63} = 12.46 \text{ Amp.}$$

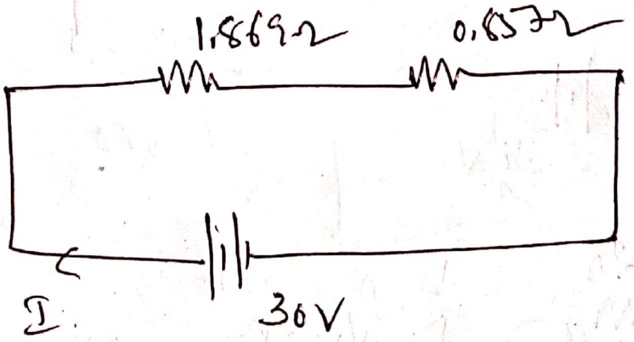
6) Find I & I_1 in the n/w shown using Δ-Y transformation.











$$I = \frac{30}{2.726} = 11 \text{ amp}$$

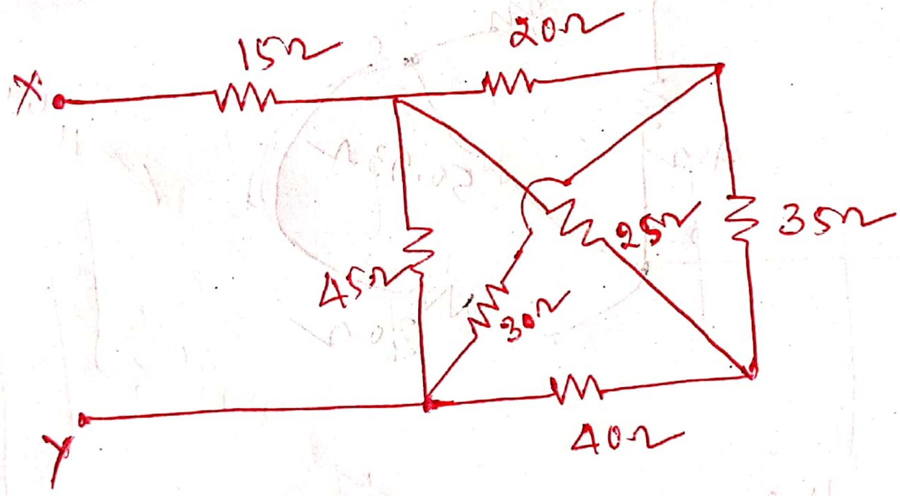
$$\boxed{I = 11 \text{ amp}}$$

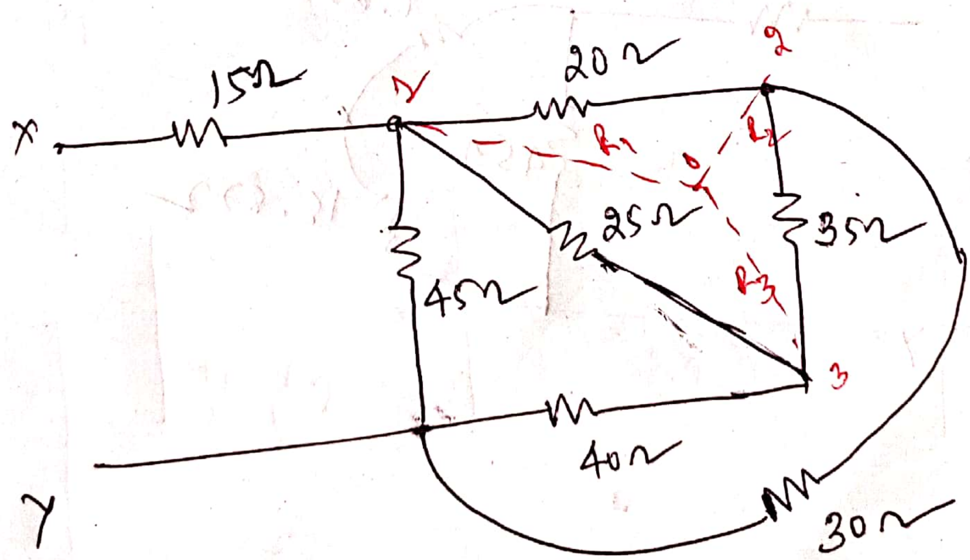
$$I_1 = \frac{I \times 2.57\Omega}{2.57 + 6.857\Omega}$$

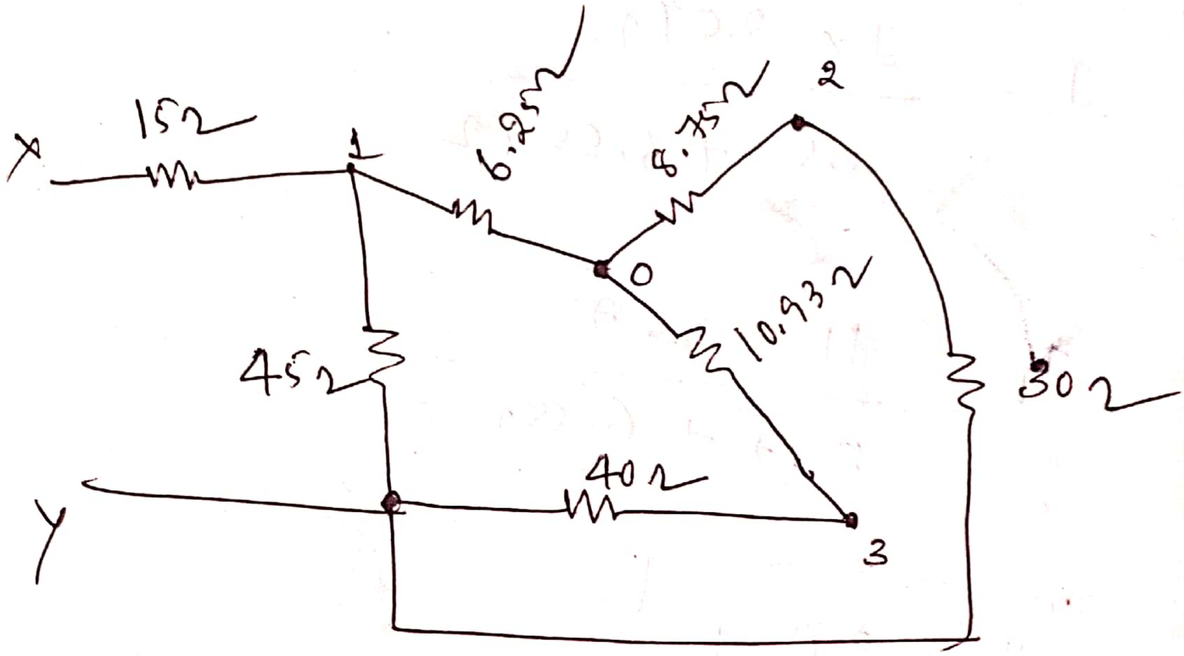
$$I_1 = \frac{11 \times 2.57}{2.57 + 6.857}$$

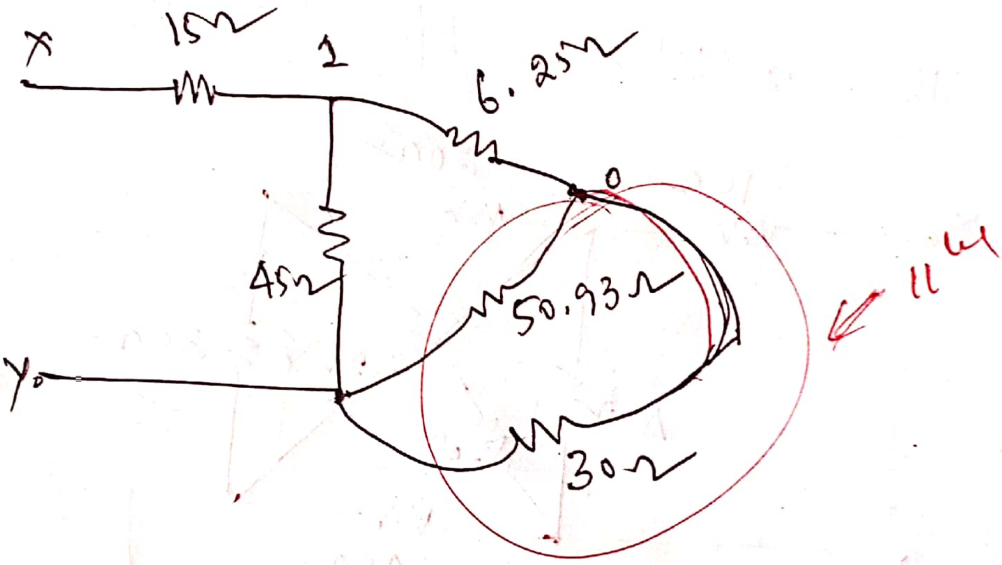
$$\boxed{I_1 = 3\ \text{Amp}}$$

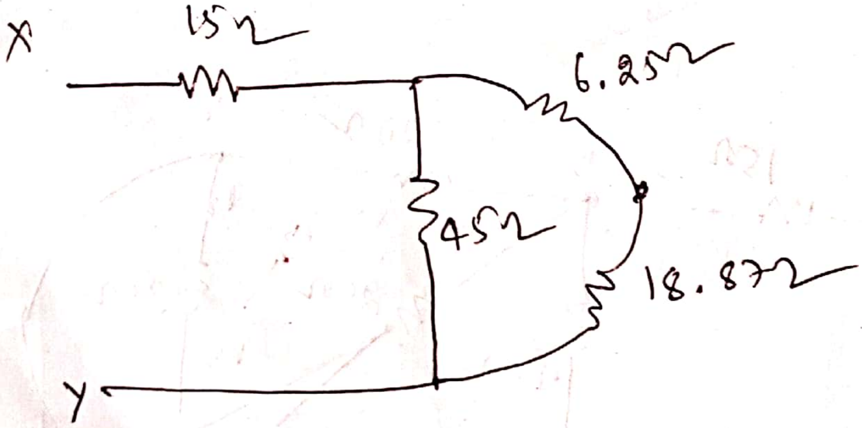
7) Find the resistence b/w the terminals x & y

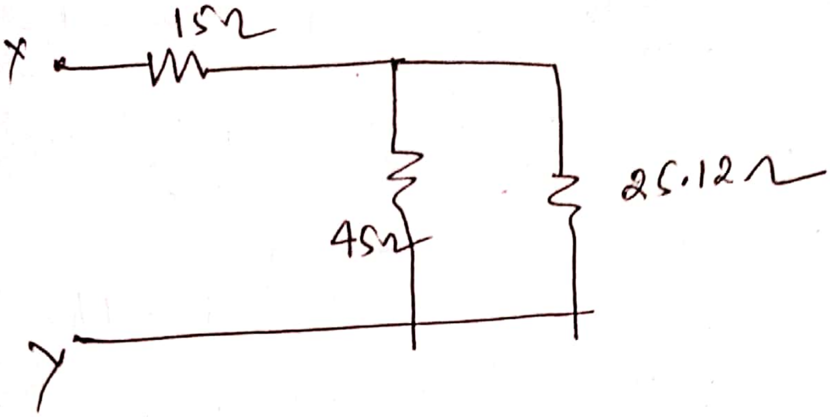


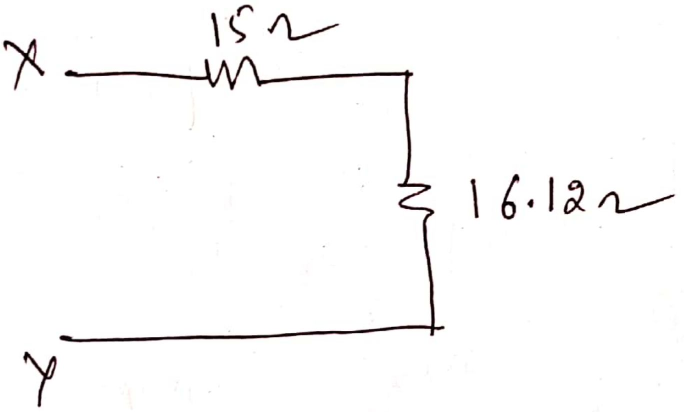



X
15Ω
1
6.25Ω
8.75Ω
2
0
10.93Ω
45Ω
30Ω
40Ω
Y
3


X
15Ω
1
6.25Ω
0
45Ω
50.93Ω
Y
30Ω

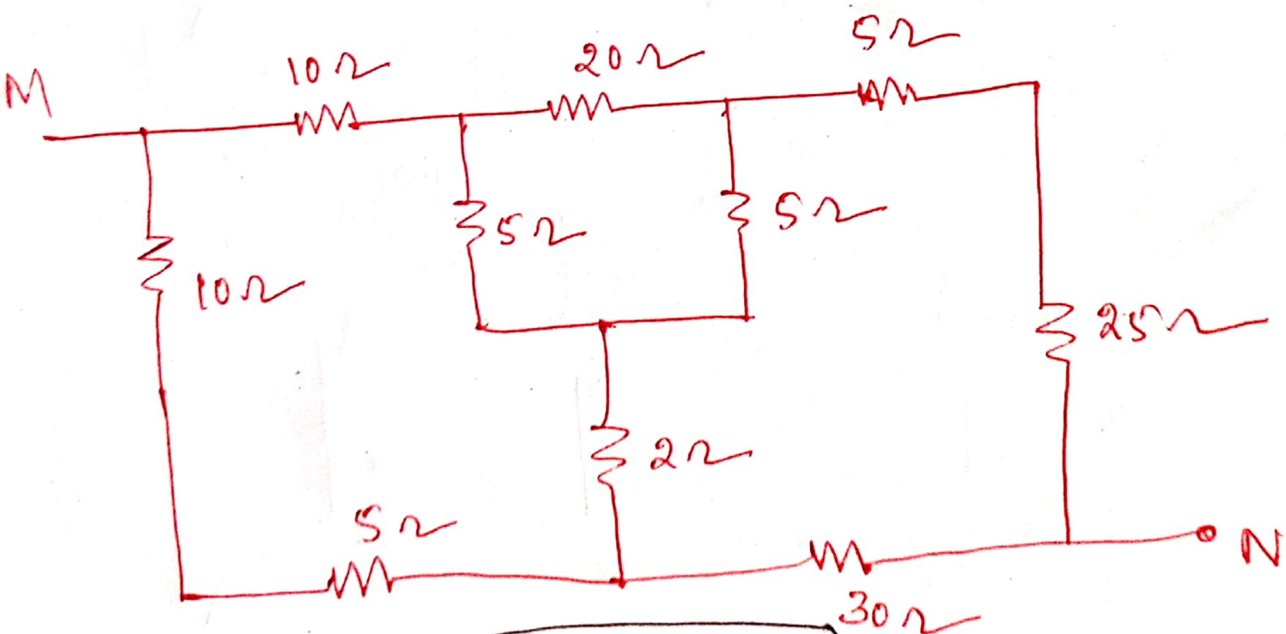

X
15Ω
6.25Ω
45Ω
18.87Ω
Y





$$R_{XY} = 31.12\ \Omega$$

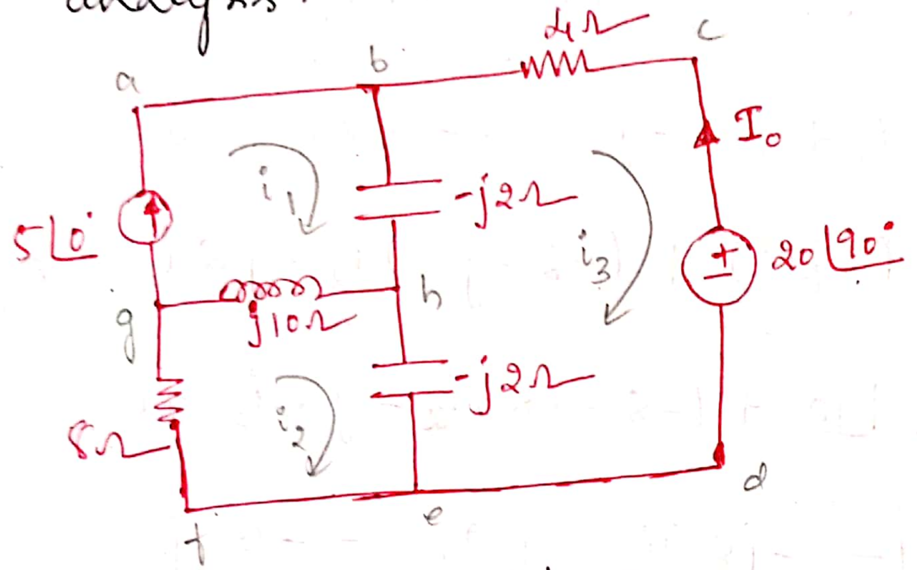
9)



$$R_{MN} = 22.84\ \Omega$$

Mesh Analysis :- (Loop or Current Analysis)

1) Determine the current I_o in the circuit using mesh analysis.



$j = 1\angle 90^\circ$

$-j = 1\angle -90^\circ$

$j2 = 2\angle 90^\circ$

From the mesh abhga

$$\boxed{i_1 = 5\angle 0^\circ} \quad \text{——— (1)}$$

→ KVL to the mesh bcdehb

$$-4i_3 - 20\angle 90^\circ + j2(i_3 - i_2) + j2(i_3 - i_1) = 0$$

$$-4i_3 - 20\angle 90^\circ + j2i_3 - j2i_2 + j2i_3 - j2i_1 = 0.$$

$$-j2i_2 + (-4 + j4i_3) - 20\angle 90^\circ - j2(5\angle 0^\circ) = 0$$

$$-j2i_2 + (-4 + j4i_3) - 20\angle 90^\circ - 10\angle 90^\circ = 0$$

$$-j2i_2 + (-4 + j4i_3) = 30\angle 90^\circ \quad \text{——— (2)}$$

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→ KVL to ghefg

$$-j10(i_2 - i_1) + j2(i_2 - i_3) - 8i_2 = 0$$

$$-j10\,i_2 + j10\,i_1 + j2\,i_2 - j2\,i_3 - 8\,i_2 = 0$$

$$j10\,i_1 + (-8 - j8)\,i_2 - j2\,i_3 = 0$$

$$j10(5\angle 0^\circ) + (-8 - j8)\,i_2 - j2\,i_3 = 0$$

$$50\angle 90 + (-8 - j8)\,i_2 - j2\,i_3 = 0$$

$50j$

$$(-8 - j8)\,i_2 - j2\,i_3 = -50j$$

$$(-8 - j8)\,i_2 - j2\,i_3 = 50\angle{-90^\circ} \quad \text{—— (3)}$$

matrix form

$$\begin{bmatrix} -j2 & -4+j4 \\ -8-j8 & -j2 \end{bmatrix} \begin{bmatrix} I_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30\angle 90^\circ \\ 50\angle{-90^\circ} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -j2 & -4+j4 \\ -8-j8 & -j2 \end{vmatrix}$$

$$\Delta = -4 - (-8 - j8)(-4 + j4)$$

$$\Delta = -4 - (+32 + \cancel{32}j - \cancel{32}j + 32)$$

$$\Delta = -4 - 64 = -68$$

$$\Delta = -68$$

$$\Delta_2 = \begin{vmatrix} -j2 & 30j \\ -8-j8 & -50j \end{vmatrix}$$

$$\Delta_2 = -100 - [(-8-j8)(30j)]$$

$$\Delta_2 = -100 - [-240j + 240]$$

$$\Delta_2 = -340 + j240$$

$$I_0 = 6.12 \angle 144.79° \, A$$

$$I_3 = \frac{\Delta_2}{\Delta} = \frac{-340 + j240}{-68}$$

$$I_3 = 5 - j3.53$$

$$I_3 = 6.12 \angle -35.21 \text{ Amp}$$

From the fig

$$I_0 = -I_3 = -6.12 \angle 35.21° \text{ Amp.}$$

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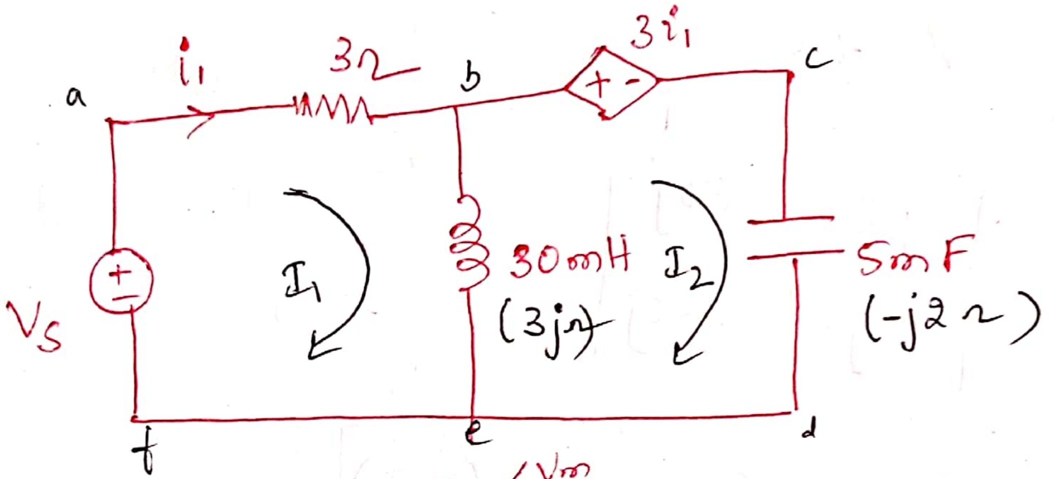
2) Find the Steady State Sinusoidal Current i_1 for the Circuit when $V_s = 10\sqrt{2}\cos(100t + 45^\circ)$



Given $V_s = \underbrace{10\sqrt{2}}_{V_m} \cos(\underbrace{100t}_{\omega t} + 45^\circ)$

$$V_s = \frac{V_m}{\sqrt{2}} \angle 45^\circ$$

$$= \frac{10\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} \angle 45^\circ = 10 \angle 45^\circ \text{ volts}$$

$\omega t = 100t$

$\omega = 100$

$X_L = 2\pi f L$

$= \omega L$

$= 100 \times 30\text{ mH}$

$= 3\Omega$ $\boxed{X_L = j3\Omega}$

$X_C = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{\omega C}$

$X_C = \dfrac{1}{100 \times 5 \times 10^{-3}}$

$\boxed{X_C = -j2\Omega}$

From the figure

$$\boxed{i_1 = I_1}$$

→ KVL to mesh abefa

$$-3I_1 - j3(I_1 - I_2) + 10\angle 45 = 0$$

$$-3I_1 - 3jI_1 + 3jI_2 = -10\angle 45^\circ$$

$$(-3-3j)I_1 + 3jI_2 = -10\angle 45^\circ] \quad \text{—— (1)}$$

→ KVL to mesh bcdeb.

$$-3i_1 + 2jI_2 - 3j(I_2 - I_1) = 0$$

$$-3I_1 + 2jI_2 - 3jI_2 + 3jI_1 = 0$$

$$(-3+3j)I_1 - jI_2 = 0 \quad \text{—— (2)}$$

WKT,

$$\begin{bmatrix} -3-3j & 3j \\ -3+3j & -j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -10\angle 45^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -3-3j & 3j \\ -3+3j & -j \end{vmatrix}$$

$$= 3j - 3 - (3j)(-3+3j)$$

$$= 3j - 3 + 9j + 9 = 6 + j12$$

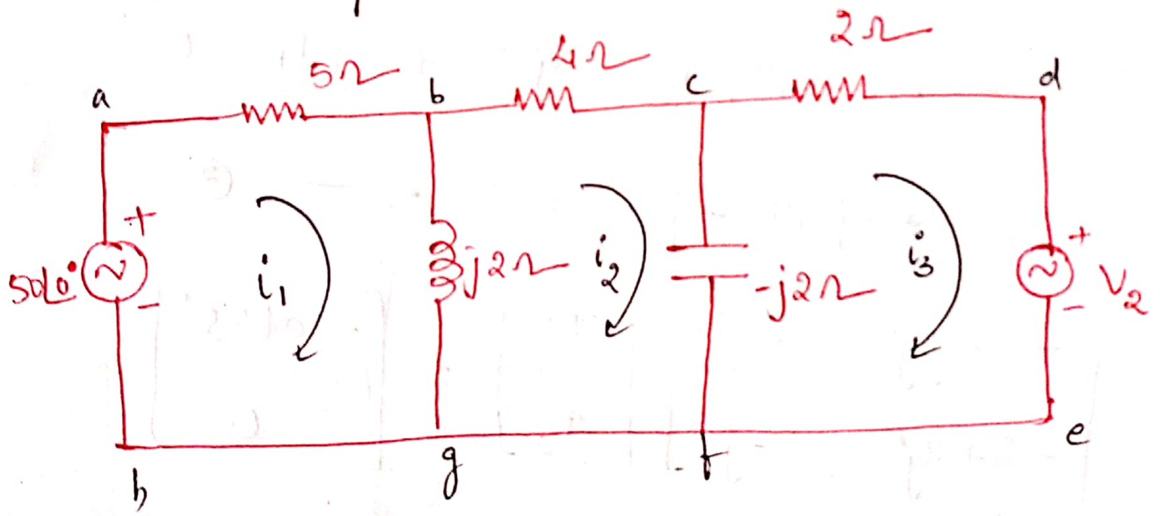
$$\Delta_1 = \begin{vmatrix} -10\angle 45 & 3j \\ 0 & 1\angle -90 \end{vmatrix}$$

$$\Delta_1 = -10\angle -45^\circ$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{-10\angle -45^\circ}{6+12j} = 0.745\angle 71.57^\circ$$

Imp July-6
July-07

3) In the circuit shown determine V_2 which results in '0' (Zero) current through 4Ω resistor
** use mesh analysis



Given, current through 4Ω is zero

$$\boxed{i_2 = 0}$$

→ KVL to the mesh abgha

$$-5i_1 - j2(i_1 - i_2) + 50\angle 0^\circ = 0$$

$$-5i_1 - j2i_1 = -50\angle 0^\circ$$

$$\cancel{-}(5+j2)i_1 = \cancel{-}50\angle 0^\circ$$

$$i_1 = \frac{50\angle 0^\circ}{5+j2} = \underline{\underline{9.28\angle -21.81^\circ \text{ amp}}}$$

→ KVL to mesh bcfgb

$$-4i_2 + j2(i_2 - i_3) - j2(i_2 - i_1) = 0$$

$$-\cancel{4i_2}^{\,0} - j2i_3 + j2i_1 = 0$$

$$j2i_1 = j2i_3$$

$$i_3 = i_1 = \underline{\underline{9.28\angle -21.81^\circ}} \text{ amp}$$

→ KVL to mesh cdefc.

$$-2i_3 - V_2 + j2(i_3 - i_2) = 0$$

$$-2i_3 - V_2 + j2i_3 = 0$$

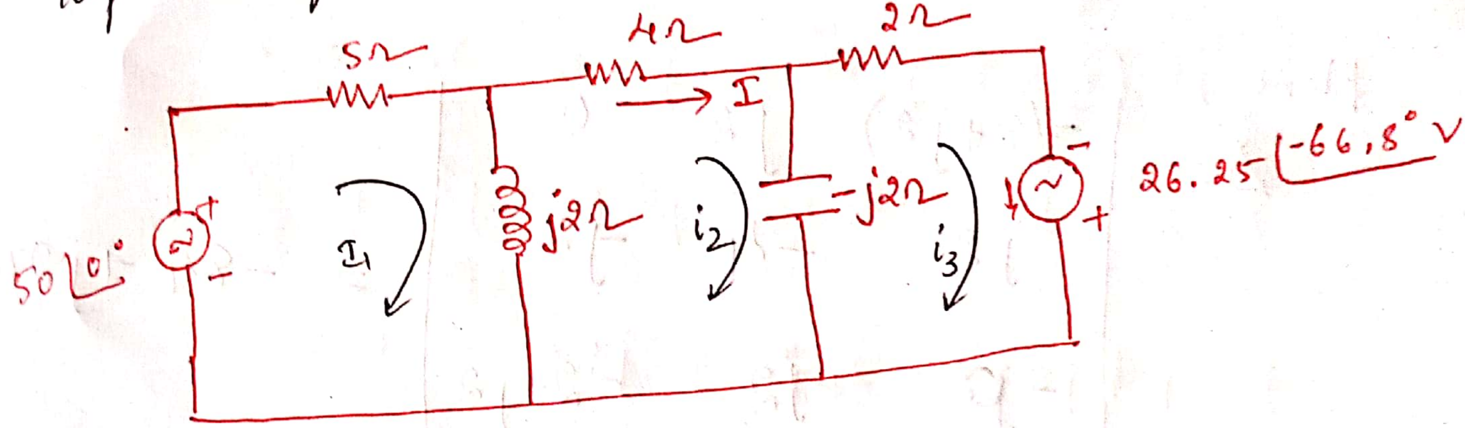
$$V_2 = (-2 + j2)i_3$$

$$V_2 = 2.82\angle 135^\circ \times 9.28\angle -21.81^\circ$$

$$\boxed{V_2 = 26.16\angle 113.19^\circ \text{ volts}}$$

mesh analysis

4) In the circuit shown below, find I through loop analysis.



→ KVL to loop 1

$$-5i_1 - j2(i_1 - i_2) + 50\angle 0^\circ = 0$$

$$-5i_1 - j2i_1 + j2i_2 = -50\angle 0^\circ$$

$$(-5 - j2)i_1 + j2i_2 = -50\angle 0^\circ \quad \text{——— (1)}$$

→ KVL to loop 2

$$-4i_2 + j2(i_2 - i_3) - j2(i_2 - i_1) = 0$$

$$-4i_2 + \cancel{j2i_2} - j2i_3 - \cancel{j2}i_2 + j2i_1 = 0$$

$$+j2i_1 - 4i_2 - j2i_3 = 0 \quad \text{——— (2)}$$

→ KVL to loop 3.

$$-2i_3 + 26.25\angle -66.8^\circ + j2(i_3 - i_2) = 0$$

$$-2i_3 + 26.25\angle -66.8^\circ + j2i_3 - j2i_2 = 0$$

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$$-j2i_2 + (-2+j2)\,i_3 = -26.25\angle -66.8^\circ \quad \text{——— (3)}$$

Here

$$\Delta = \begin{vmatrix} -5-j2 & j2 & 0 \\ j2 & -4 & -j2 \\ 0 & -j2 & -2+j2 \end{vmatrix} = -84 + j24$$

From the figure $I = I_2 = \dfrac{\Delta_2}{\Delta}$

$\downarrow$

$$\therefore \quad \Delta_2 = \begin{vmatrix} -5-j2 & -50\angle 0^\circ & 0 \\ j2 & 0 & -j2 \\ 0 & -26.25\angle -66.8^\circ & -2+j2 \end{vmatrix}$$

$2\angle -90^\circ$

$$\Delta_2 = (-5-j2)\left[0 + 52.5\angle -156.8\right] + 50\angle 0^\circ \left[j2(-2+j2)\right]$$

$$\Delta_2 = 5.38\angle -158.19 \times 52.5\angle -156.8 + 50\angle 0^\circ \left[-4j - 4\right]$$

$$\Delta_2 = 282.4\angle -315^\circ + 50\angle 0^\circ \times 5.65\angle -135^\circ$$

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$$\Delta_2 = 282.4 \angle -315^\circ + 282.5 \angle -135^\circ$$

$$\Delta_2 = \cancel{282.8}\ \cancel{199.6} + j\cancel{199.6} - \cancel{199.7} - j\cancel{199.7}$$

$$\boxed{\Delta_2 = 0}$$

$$\therefore \boxed{I = 0}$$

Node analysis or voltage analysis :-

Procedure :-

→ All the principle node of the n/w are identified & one of them is taken as reference node at zero potential. Usually the node at which maximum no of branches are connected is taken as reference node.

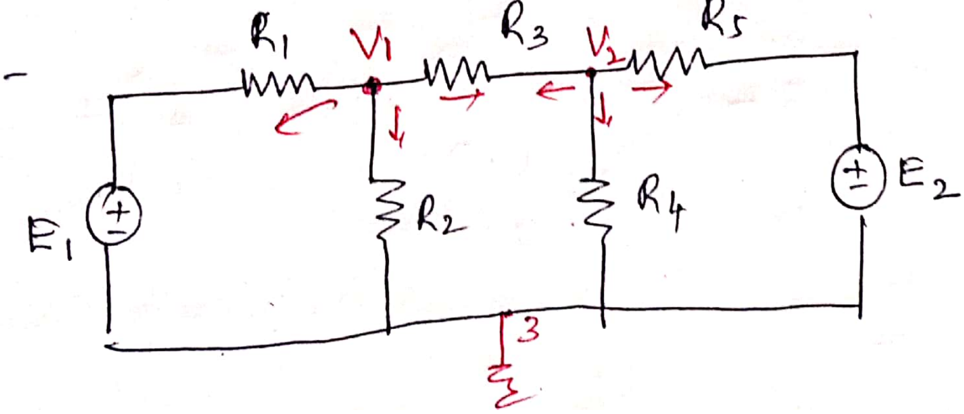
→ The remaining nodes are assigned with node voltages V_1, V_2, V_3 .... etc.

→ The node voltage equations are written using the KCL method.

→ The node voltage equations are solved using cramer's rule to get V_1, V_2, V_3 ... etc.

→ Once the node voltages are known the current in all the branches of the n/w can be found.

Ex:-



Applying KCL at node 1,

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0$$

$$\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] V_1 - \frac{V_2}{R_3} = \frac{E_1}{R_1} \quad \text{——— (1)}$$

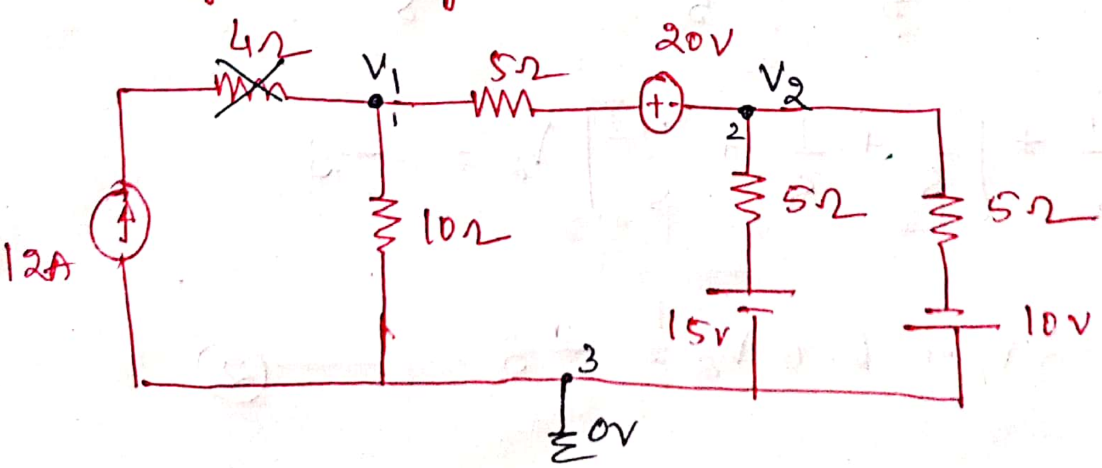
Applying KCL @ node 2.

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_2 - E_2}{R_5} = 0$$

$$\frac{V_2}{R_3} - \frac{V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_2}{R_5} - \frac{E_2}{R_5} = 0$$

$$-\frac{V_1}{R_3} + \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] V_2 = \frac{E_2}{R_5} \quad \text{——— (2)}$$

1) In the network shown, find V_1 & V_2 using node voltage analysis.



Apply KCL at node 1

$$-12 + \frac{V_1}{10} + \frac{V_1 - 20 - V_2}{5} = 0$$

$$-12 + \frac{V_1}{10} + \frac{V_1}{5} - \frac{20}{5} - \frac{V_2}{5} = 0$$

$$-12 + \left[\frac{1}{10} + \frac{1}{5}\right] V_1 - 4 - \frac{V_2}{5} = 0$$

$$\left[\frac{1}{10} + \frac{1}{5}\right] V_1 - \frac{V_2}{5} = 16$$

$$0.3V_1 - 0.2V_2 = 16 \quad \text{———} \; (1)$$

Apply KCL @ node 2.

$$\frac{V_2 + 20 - V_1}{5} + \frac{V_2 - 15}{5} + \frac{V_2 + 10}{5} = 0$$

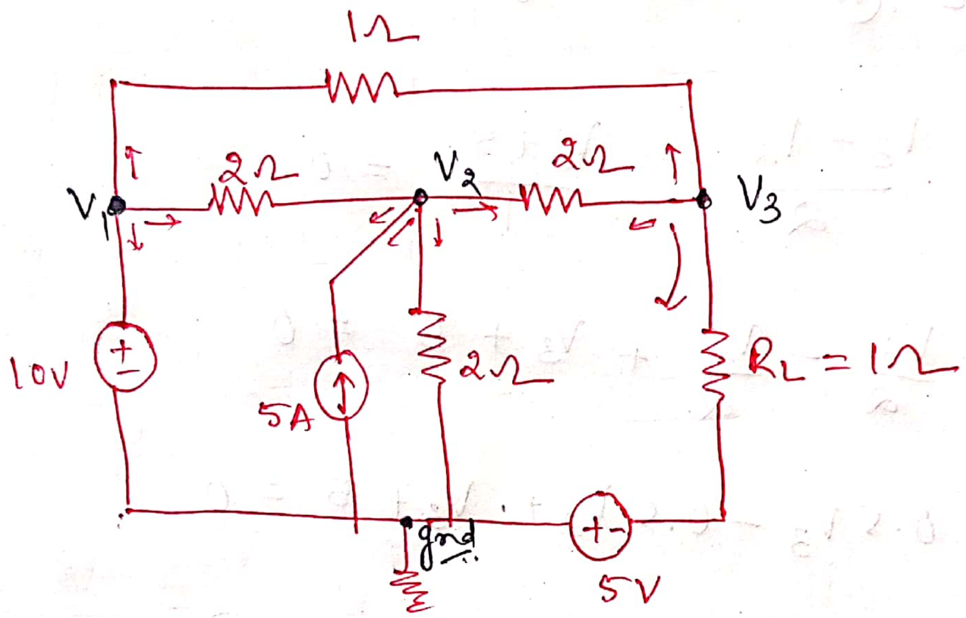
$$\frac{V_2}{5} + 4 - \frac{V_1}{5} + \frac{V_2}{5} - 3 + \frac{V_2}{5} + 2 = 0$$

$$-\frac{V_1}{5} + \left[\frac{1}{5} + \frac{1}{5} + \frac{1}{5}\right] V_2 = -3$$

$$-0.2V_1 + 0.6V_2 = -3 \quad \text{———} \; (2)$$

$$\boxed{V_1 = 64.28\,V} \quad \& \quad \boxed{V_2 = 16.42\,V}$$

2) Find 'I' through R_L in the n/w shown



From the figure

$$V_1 - 10 = 0$$

$$\boxed{V_1 = 10\,V}$$

Apply KCL @ node V_2.

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2} - 5 + \frac{V_2 - V_3}{2} = 0$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{2} - 5 + \frac{V_2}{2} - \frac{V_3}{2} = 0$$

$$0.5V_2 - 0.5V_1 + 0.5V_2 - 5 + 0.5V_2 - 0.5V_3 = 0$$

$$-0.5V_1 + 1.5V_2 - 0.5V_3 - 5 = 0$$

$$-0.5\times 10 + 1.5V_2 - 0.5V_3 - 5 = 0$$

$$1.5V_2 - 0.5V_3 = 10 \quad \text{—— ①}$$

Apply KCL @ node V_3.

$$\frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{2} + \frac{V_3 + 5}{1} = 0$$

$$V_3 - V_1 + \frac{V_3}{2} - \frac{V_2}{2} + V_3 + 5 = 0$$

$$V_3 - 10 + 0.5V_3 - 0.5V_2 + V_3 + 5 = 0$$

$$-0.5V_2 + 2.5V_3 = 5 \quad \text{—— ②}$$

Solving ① & ② we get

$$\boxed{V_3 = 3.57 \text{ volts}}$$

Current through R_L is

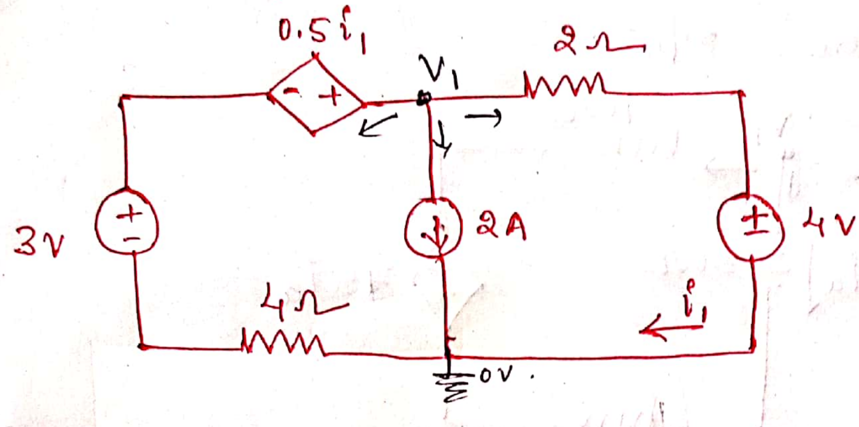
$$I = \frac{V}{R}$$

$$I = \frac{V_3 + 5}{R_L}$$

$$= \frac{3.57 + 5}{1}$$

$$\boxed{I = 8.57 \text{ amp}}$$

3) Find i_1 using nodal analysis.



From the figure $i_1 = \frac{V_1 - 4}{2}$

Apply KCL @ node V_1

$$\frac{V_1 - 0.5 i_1 - 3}{4} + 2 + \frac{V_1 - 4}{2} = 0$$

$$\frac{V_1}{4} - \frac{0.5 i_1}{4} - 0.75 + \cancel{2} + 0.5 V_1 - \cancel{2} = 0$$

$$0.25 V_1 - 0.125 \left[\frac{V_1 - 4}{2}\right] - 0.75 + 0.5 V_1 = 0$$

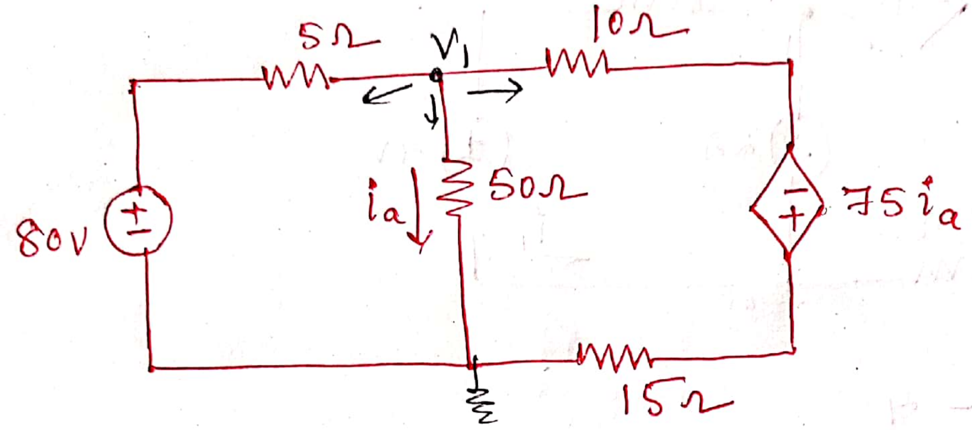
$$0.75 V_1 - 0.0625 V_1 + 0.25 - 0.75 = 0$$

$$0.6875 V_1 - 0.5 = 0$$

$$V_1 = \frac{0.5}{0.6875} = 0.727 \text{ volts}$$

$$\therefore i_1 = \frac{0.727 - 4}{2} = -1.636 \text{ Amp}$$

4) Find power delivered by the dependent voltage source in the n/w.



From the figure, $i_a = \frac{V_1}{50}$

Apply KCL @ node V_1 ..

$$\frac{V_1 - 80}{5} + \frac{V_1}{50} + \frac{V_1 + 75 i_a}{25} = 0$$

$$0.2V_1 - 16 + 0.02V_1 + 0.04V_1 + 3i_a = 0$$

$$0.26V_1 + 3i_a - 16 = 0$$

$$0.26V_1 + 3\left(\frac{V_1}{50}\right) = 16$$

$$0.32V_1 = 16$$

$$V_1 = 16/0.32 = 50 \text{ volts}$$

$$\therefore i_a = \frac{V_1}{50} = 50/50 = 1 \text{ Amp}$$

→ Current through dependent voltage source branch

$$= \frac{V_1 + 75 i_a}{25} = \frac{V_1 + 75\left(\frac{V_1}{50}\right)}{25}$$

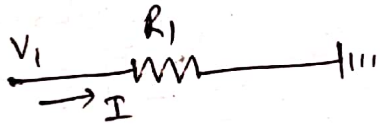
$$= \frac{50 + 75(1)}{25} = 5 \text{ Amp}$$

$\therefore$ power $= V \times I$

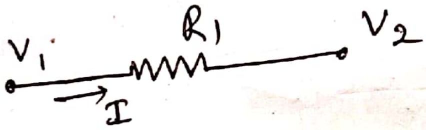
$= 75 i_a \times 5 = 75 \times 1 \times 5$

$= 375$ watts

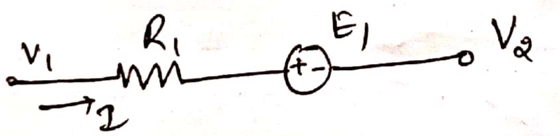
Note :-

1) 

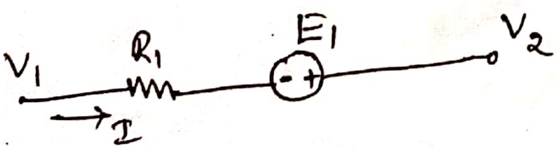
$$I = \frac{V_1}{R_1}$$

2) 

$$I = \frac{V_1 - V_2}{R_1}$$

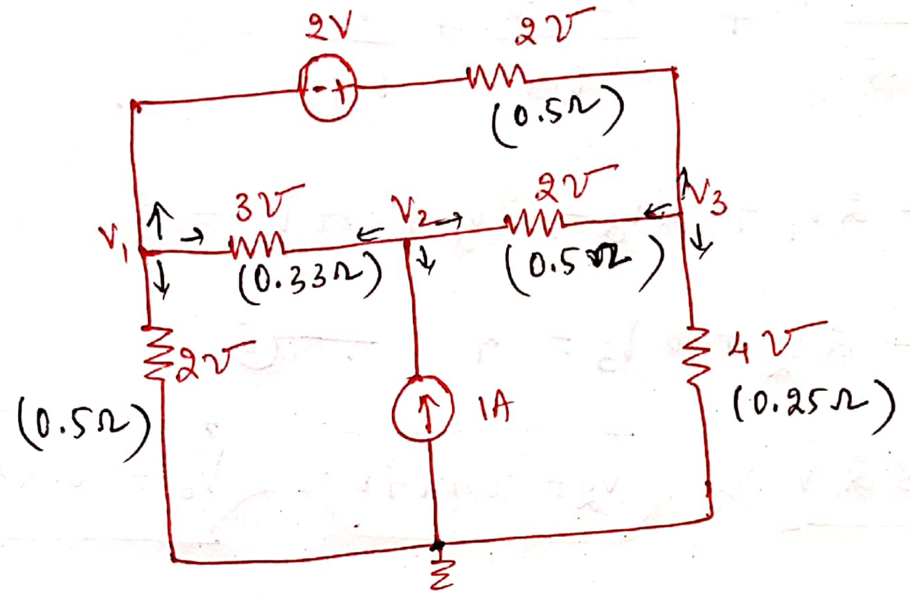
3) 

$$I = \frac{V_1 - E_1 - V_2}{R_1}$$

4) 

$$I = \frac{V_1 + E_1 - V_2}{R_1}$$

5) In the Circuit shown below, Use nodal analysis to determine the voltages V_2 & V_3.



Apply KCL @ node V_1

$$\frac{V_1}{0.5} + \frac{V_1 - V_2}{0.33} + \frac{V_1 + 2 - V_3}{0.5} = 0$$

$$2V_1 + 3V_1 - 3V_2 + 2V_1 - 2V_3 + 4 = 0$$

$$7V_1 - 3V_2 - 2V_3 = -4 \quad \text{——— (1)}$$

@ node 2

$$\frac{V_2 - V_1}{0.33} + \frac{V_2 - V_3}{0.5} - 1 = 0$$

$$-3V_1 + 3V_2 + 2V_2 - 2V_3 - 1 = 0$$

$$-3V_1 + 5V_2 - 2V_3 = 1 \quad \text{——— (2)}$$

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→ @ node 3

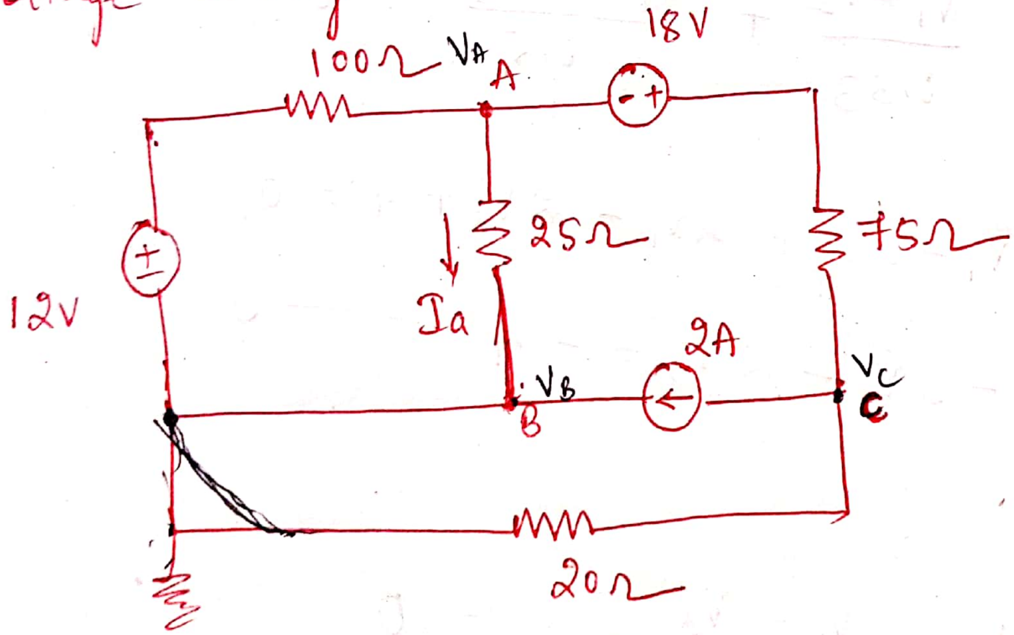
$$\frac{V_3 - 2 - V_1}{0.5} + \frac{V_3 - V_2}{0.5} + \frac{V_3}{0.25} = 0$$

$$2V_3 - 4 - 2V_1 + 2V_3 - 2V_2 + 4V_3 = 0$$

$$-2V_1 - 2V_2 + 8V_3 = 4 \quad \text{——} (3)$$

$V_1 = -0.382\,V$ $V_2 = 0.147\,V$ $V_3 = 0.441\,V$

6) Find I_a in the circuit shown using node voltage analysis.



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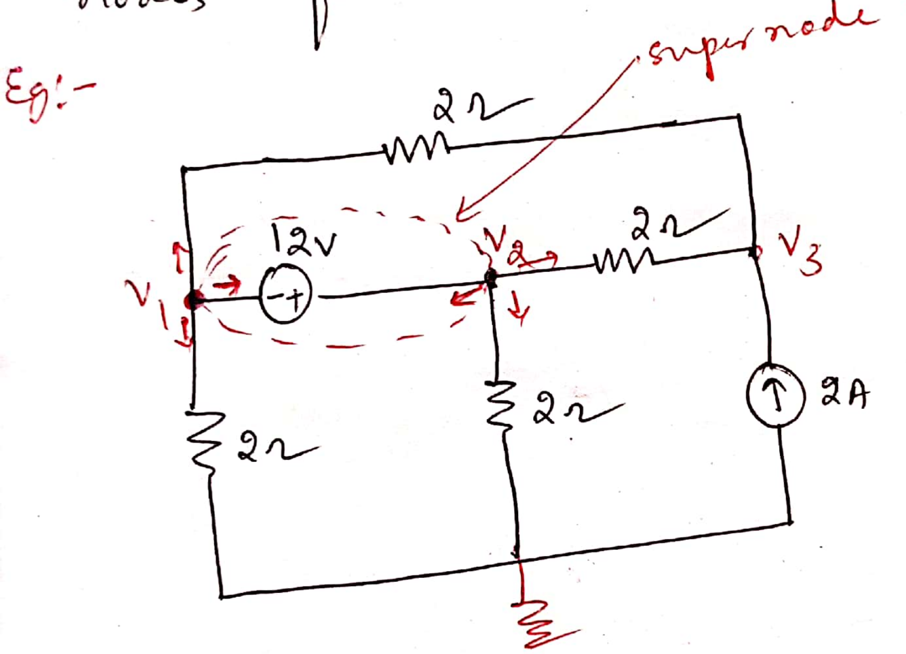
Super node :-

If the ideal voltage source [dependent or independent voltage source] is connected b/n any two non reference nodes, those nodes forms a super node.

Eg:-



→ 12v voltage source exists b/n nodes 1 & 2. Hence node 1 & node 2 forms super node

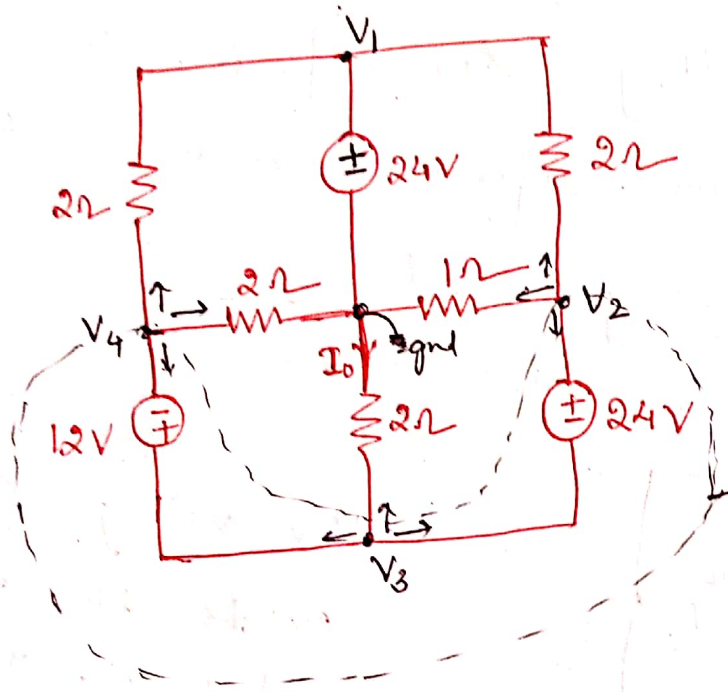
From super node

$$V_1 + 12 - V_2 = 0$$

$$\boxed{V_1 - V_2 = -12}$$

7) For the network shown below, find I_o using nodal analysis.

Jan-14 10 Marks



→ Super node 2-3-4.

From the circuit :

$V_1 - 24 = 0$

$\boxed{V_1 = 24 \text{ volts}}$ ——— ①

→ 24V is in b/n 2 & 3

→ 12V is in b/n 4 & 3.

∴ 2-3-4 forms Supernode.

From the super node,

$\boxed{V_2 - V_3 = 24\,V}$ ——— ②

$\boxed{V_3 - V_4 = 12\,V}$ ——— ③

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Apply KCL @ node '2-3-4':

$$\frac{V_4 - V_1}{2} + \frac{V_4}{2} + \frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3}{2} = 0$$

$\cancel{2V_4 - 2V_1 + 2V_4 + 2}$

$$0.5\,V_4 - 0.5\,V_1 + 0.5\,V_4 + 0.5\,V_2 - 0.5\,V_1 + V_2 + 0.5\,V_3 = 0$$

$$-\overset{24}{V_1} + 1.5\,V_2 + 0.5\,V_3 + V_4 = 0$$

$$1.5\,V_2 + 0.5\,V_3 + V_4 = 24 \quad \text{--- (4)}$$

Solving (2), (3) & (4)

$$\left\{\begin{aligned} V_2 - V_3 + 0\,V_4 &= 24 \quad \text{--- (2*)} \\ 0 + V_3 - V_4 &= 12 \quad \text{--- (3*)} \\ 1.5\,V_2 + 0.5\,V_3 + V_4 &= 24 \quad \text{--- (4*)} \end{aligned}\right.$$

$\boxed{V_2 = 24\,V}$ $\boxed{V_3 = 0\,V}$ $\boxed{V_4 = -12\,V}$

From the figure $I_0 = \dfrac{0 - V_3}{2} = \underline{\underline{0\ \text{Amp}}}$

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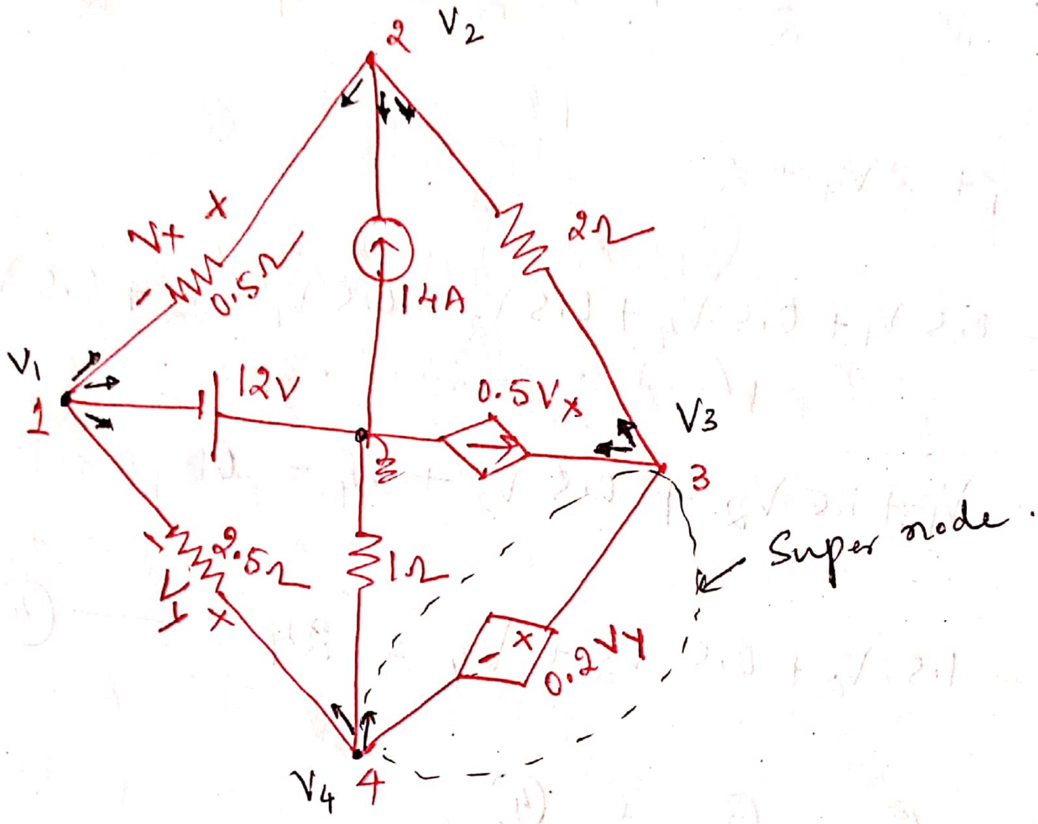
8) Find the voltages at nodes 1, 2, 3, 4 for the n/w using nodal analysis.

Jan-8



From the circuit,

$$V_1 + 12 = 0$$

$$\boxed{V_1 = -12 \text{ volts}}$$

Apply KCL @ node 2,

$$\frac{V_2 - V_1}{0.5} - 14 + \frac{V_2 - V_3}{2} = 0$$

$$2V_2 - 2V_1 - 14 + 0.5V_2 - 0.5V_3 = 0$$

-2×-12 $\quad -2V_1 + 2.5V_2 - 0.5V_3 = 14$

$\cdot 24$ $\quad 2.5V_2 - 0.5V_3 = 14 - 24$

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$$2.5V_2 - 0.5V_3 = -10 \quad \text{——} \quad (1)$$

→ $0.2V_y$ exists b/n node 3 & 4

∴ 3 & 4 forms super node.

from super node,

$$\frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0$$

$$0.5V_3 - 0.5V_2 - 0.5V_x + V_4 + 0.4V_4 - 0.4V_1 = 0$$

$$-0.4V_1 - 0.5V_2 + 0.5V_3 + 1.4V_4 - 0.5V_x = 0$$

But $V_x = V_2 - V_1$ & $\underline{\underline{V_1 = -12 \text{ volts}}}.$

$$\therefore -0.4(-12) - 0.5V_2 + 0.5V_3 + 1.4V_4 - 0.5(V_2 - V_1) = 0.$$

0.5 × −12

$$\rightarrow 4.8 - 0.5V_2 + 0.5V_3 + 1.4V_4 - 0.5V_2 + 0.5V_1 = 0$$

−12

$$\rightarrow 4.8 - V_2 + 0.5V_3 + 1.4V_4 - 6 = 0$$

$$-V_2 + 0.5V_3 + 1.4V_4 = 1.2 \quad \text{——} \quad (2)$$

from super node

$$V_3 - V_4 = 0.2V_y$$

But $V_y = V_4 - V_1$

$\rightarrow V_3 - V_4 = 0.2(V_4 - V_1)$

$\rightarrow V_3 - V_4 = 0.2V_4 - 0.2V_1$

or $0.2V_1 + V_3 - V_4 - 0.2V_4 = 0$

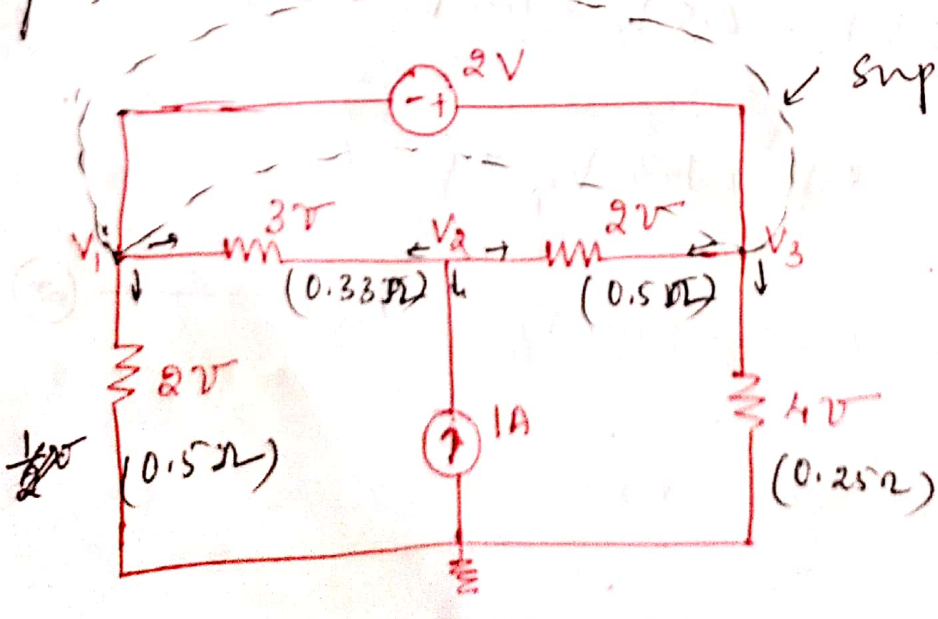
$0.2(-12) + V_3 - 1.2V_4 = 0$

$$V_3 - 1.2V_4 = 2.4 \quad \text{——— (3)}$$

Solving (1), (2), & (3)

$V_1 = -12V$ $V_2 = -4V$ $V_3 = 0V$ $V_4 = -2V$

9) In the Circuit shown below, use nodal analysis to determine the voltages V_2 & V_3



From the super node (1-3)

$$\boxed{V_3 - V_1 = 2V} \longrightarrow -V_1 + V_3 = 2V \text{ ——— } (1)$$

KCL @ super node (1-3)

$$\frac{V_1}{0.5} + \frac{V_1 - V_2}{0.33} + \frac{V_3 - V_2}{0.5} + \frac{V_3}{0.25} = 0.$$

$$2V_1 + 3V_1 - 3V_2 + 2V_3 - 2V_2 + 4V_3 = 0.$$

$$5V_1 - 5V_2 + 6V_3 = 0 \text{ ——— } (2)$$

→ Apply KCL @ node 2

$$\frac{V_2 - V_1}{0.33} - 1 + \frac{V_2 - V_3}{0.5} = 0$$

$$3V_2 - 3V_1 - 1 + 2V_2 - 2V_3 = 0$$

$$-3V_1 + 5V_2 - 2V_3 = 1 \text{ ——— } (3)$$

Solving (1), (2) & (3) we get -

$$\boxed{V_1 = -1.667 \text{ volts}} \quad \boxed{V_2 = -0.1667 \text{ volts}}$$

$$\boxed{V_3 = 0.833 \text{ volts}}$$

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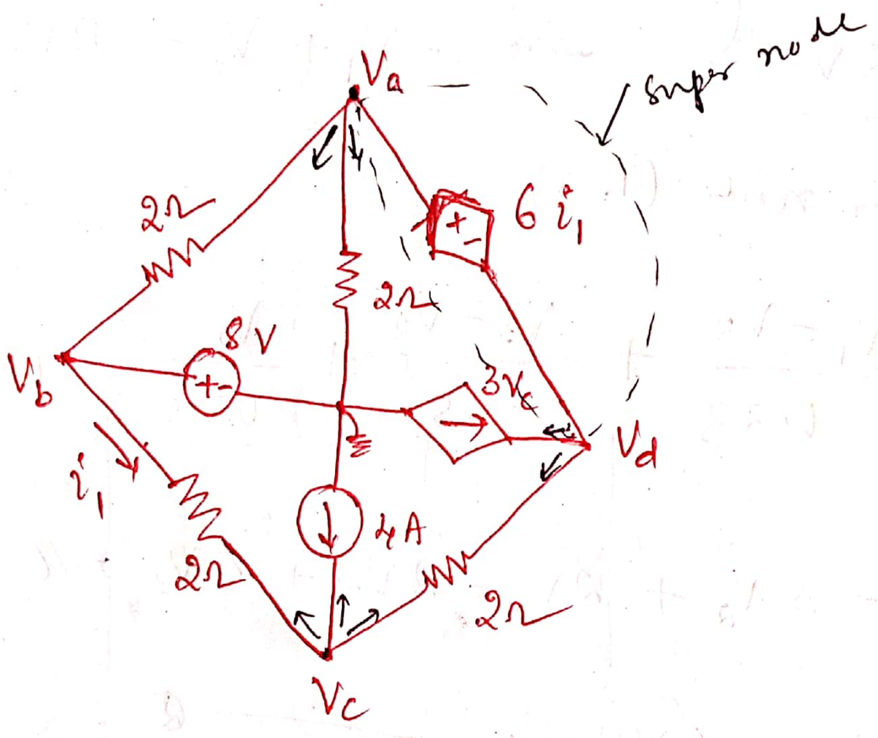
10) Find V_c & V_d using nodal analysis.



From the figure, (@ node V_b)

$$V_b - 8 = 0 \Rightarrow \boxed{V_b = 8V}$$

Apply KCL @ node c

$$\frac{V_c - V_b}{2} + \frac{V_c - V_d}{2} - 4 = 0$$

$$0.5V_c - 0.5V_b + 0.5V_c - 0.5V_d - 4 = 0$$

$$-0.5V_b + V_c - 0.5V_d - 4 = 0$$

$$-0.5(8) + V_c - 0.5V_d - 4 = 0$$

$$V_c - 0.5 V_d = 8 \quad \text{——— } ①$$

from super node, (a-d)

$$V_a - V_d = 6 i_1$$

from the figure,

$$i_1 = \frac{V_b - V_c}{2}$$

$$V_a - V_d = \cancel{6}^{3}\left[\frac{V_b - V_c}{\cancel{2}}\right]$$

$$V_a - V_d = 3V_b - 3V_c$$

$$\boxed{V_a + 3V_c - V_d = 24} \quad \text{——— } ②$$

$$V_a - V_d - 3V_b + 3V_c = 0$$

$$V_a - 3(V_b)^{8} + 3V_c - V_d = 0 \quad \text{——— } ②$$

Apply KCL to super node (a-d)

$$\frac{V_a - V_b}{2} + \frac{V_a}{2} - 3V_c + \frac{V_d - V_c}{2} = 0$$

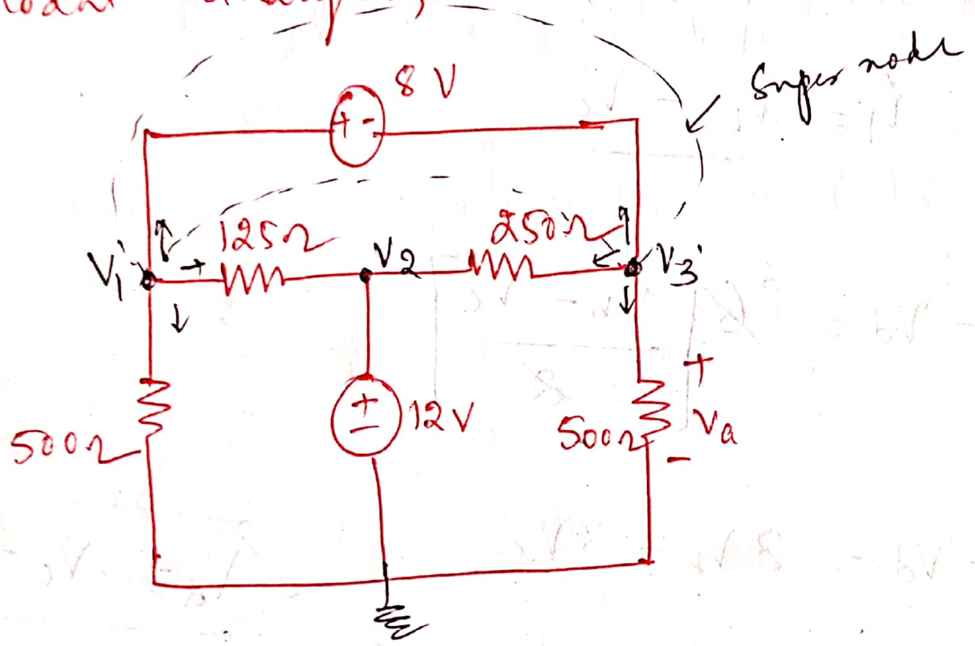
$$0.5V_a - 0.5V_b + 0.5V_a - 3V_c + 0.5V_d - 0.5V_c = 0$$

$$V_a - 0.5 \overset{8}{V_b} - 3.5V_c + 0.5V_d = 0$$

$$\boxed{V_a - 3.5V_c + 0.5V_d = 4} \quad \text{——— } ③$$

$\boxed{V_a = 9.14}$ $\boxed{V_c = -1.14}$ $\boxed{V_d = -18.28 \text{ volt}}$

11) For the electrical n/w, find V_a using nodal analysis



@ node 2 :-

$V_2 - 12 = 0 \Rightarrow \boxed{V_2 = 12V}$

From super node (1-3)

$V_1 - V_3 = 8V$ ———— ①

Apply KCL to super node,

$$\frac{V_1}{500} + \frac{V_1 - V_2}{125} + \frac{V_3 - V_2}{250} + \frac{V_3}{500} = 0$$

$$2\times10^{-3}V_1 + 8\times10^{-3}V_1 - 0.096 + 4\times10^{-3}V_3 - 0.048 + 2\times10^{-3}V_3 = 0$$

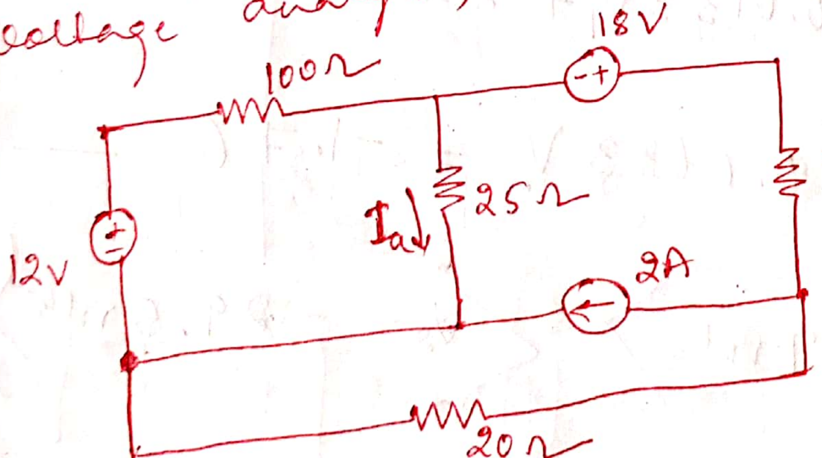
$$10\times10^{-3}V_1 + 6\times10^{-3}V_3 = 0.144 \quad \text{—— (2)}$$

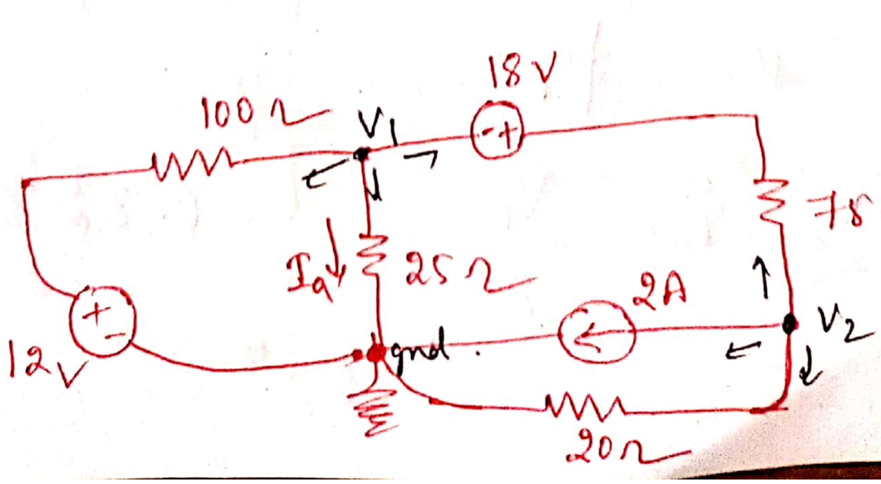
$V_1 = 12V$ $\quad$ $V_3 = 4$ volts

from the figure.

$V_a = V_3 = 4$ volts.

6) Find I_a in the circuit shown using node voltage analysis





Apply KCL @ node 1

$$\frac{V_1 - 12}{100} + \frac{V_1}{25} + \frac{V_1 + 18 - V_2}{75} = 0$$

$$0.01V_1 - 0.12 + 0.04V_1 + 0.0133V_1 + 0.24 - 0.0133V_2 = 0$$

$$0.0633V_1 - 0.0133V_2 = -0.12 \quad \text{—— (1)}$$

Apply KCL @ node 2

$$\frac{V_2 - 18 - V_1}{75} + 2 + \frac{V_2}{20} = 0$$

$$0.0133V_2 - 0.24 - 0.0133V_1 + 2 + 0.05V_2 = 0$$

$$-0.0133V_1 + 0.0633V_2 = -1.76 \quad \text{—— (2)}$$

$V_1 = -8.09$ volts

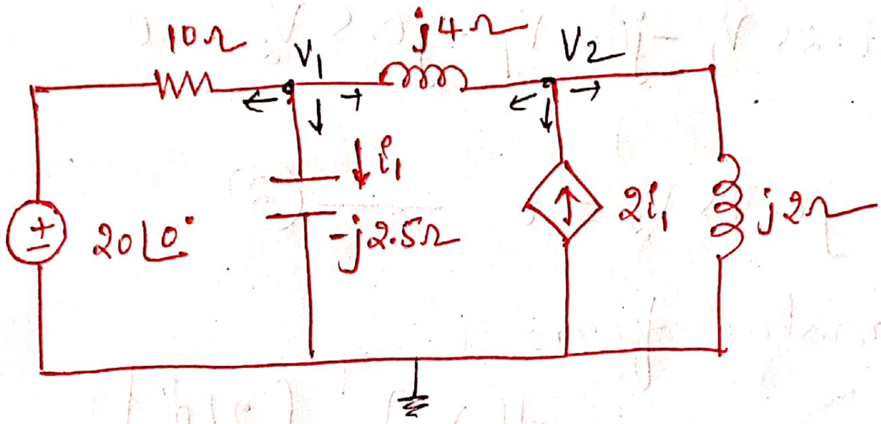
$V_2 = -29.5$ volts

From the fig.

$$i_a = \frac{V_1}{25} = -0.323 \text{ volts}$$

Problems on AC Analysis :-

1) Find i_1 in the Circuit using nodal analysis.



from the Circuit

$$i_1 = \frac{V_1}{-j2.5\Omega}$$

$$\frac{1}{j} = -j \qquad \frac{1}{-j} = j$$

Apply KCL @ node 1

$$\frac{V_1 - 20\angle 0°}{10} + \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = 0$$

$$\frac{V_1}{10} - \frac{20\angle 0°}{10} + j\frac{V_1}{2.5} + (-j)\frac{V_1}{4} + j\frac{V_2}{4} = 0$$

$$0.1V_1 - 2\angle 0° + j0.4V_1 - j0.25V_1 + j0.25V_2 = 0$$

$$[0.1 + \underbrace{j0.4 - j0.25}_{j0.15}]V_1 + j0.25V_2 = 2\angle 0° \quad \text{—— (1)}$$

Apply KCL @ node 2.

$$\frac{V_2 - V_1}{j4} - 2i_1 + \frac{V_2}{j2} = 0$$

$$-j0.25(V_2 - V_1) - \frac{2V_1}{-j2.5} + (-j0.5)V_2 = 0$$

$$-j0.25V_2 + j0.25V_1 - j0.8V_1 - j0.5V_2 = 0$$

$$-j0.55V_1 - j0.75V_2 = 0 \quad \text{——— (2)}$$

In matrix form:

$$\begin{bmatrix} 0.1+j0.15 & j0.25 \\ -j0.55 & -j0.75 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.1+j0.15 & j0.25 \\ -j0.55 & -j0.75 \end{vmatrix}$$

$$= [(0.1+j0.15)(-j0.75) + (j0.25)(j0.55)]$$

$$= 0.1125 - j0.075 - 0.1375$$

$$= -0.025 - j0.075$$

$$\Delta_1 = \begin{vmatrix} 2\angle 0^\circ & j0.25 \\ 0 & -j0.75 \end{vmatrix}$$

$$= -j1.5$$

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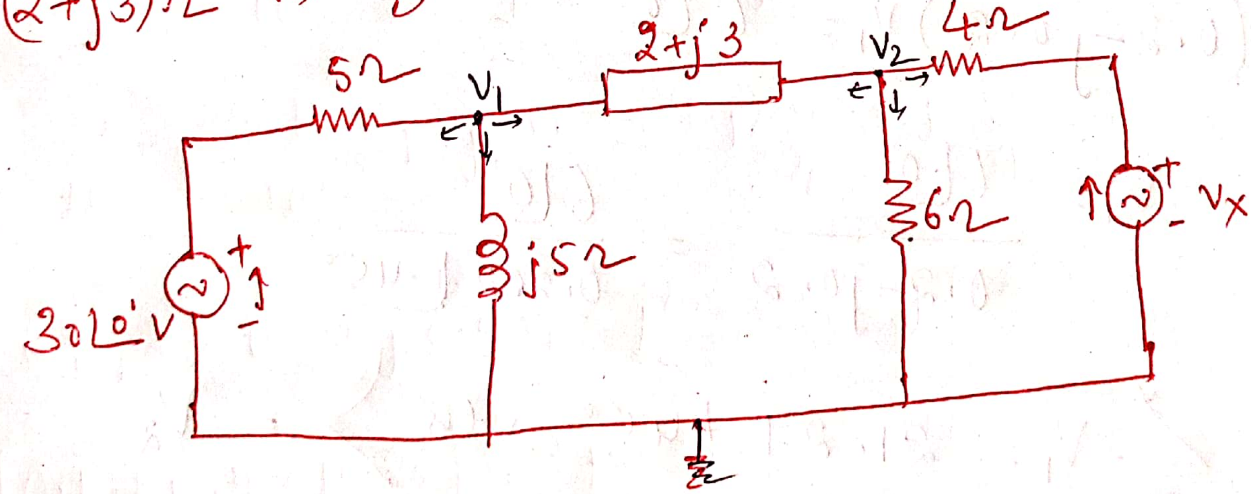
$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-1.5j}{-0.025-j0.075}$$

$$\boxed{V_1 = 18.97 \angle 18.43^\circ \text{ V}}$$

$$\therefore i_1 = \frac{18.97 \angle 18.43}{-j2.5} = \frac{18.97 \angle 18.43}{2.5 \angle -90^\circ}$$

$$\boxed{i_1 = 7.59 \angle 108^\circ.43 \text{ Amp}}$$

2) Use the nodal analysis & find the value of V_x in the circuit shown in below fig such that the current through the impedance $(2+j3)\Omega$ is zero



From the figure (data)

$$I_{(2+j3)\Omega} = \frac{V_1 - V_2}{2+j3}$$

Given the current $\boxed{I_{(2+3)\Omega} = 0}$

$$\therefore \quad 0 = \frac{V_1 - V_2}{2+j3}$$

$$0 = V_1 - V_2$$

$$\Rightarrow \boxed{V_1 = V_2} \quad (\because \text{Equipotential})$$

Apply KCL @ node 1,

$$\frac{V_1 - 30\angle 0^\circ}{5} + \frac{V_1}{j5} + \cancel{\frac{V_1 - V_2}{2+j3}}^{\,0} = 0$$

$$0.2V_1 - 6\angle 0^\circ - j0.2V_1 = 0$$

$$(0.2 - j0.2)V_1 = 6\angle 0^\circ$$

$$V_1 = \frac{6\angle 0^\circ}{0.2 - j0.2} = \frac{6\angle 0^\circ}{0.282\angle -45^\circ}$$

$$\boxed{V_1 = 21.27\angle 45^\circ \text{ volts}} = V_2$$

Apply KCL @ node 2,

$$\cancel{\frac{V_2 - V_1}{2+j3}}^{\,0} + \frac{V_2}{6} + \frac{V_2 - V_x}{4} = 0$$

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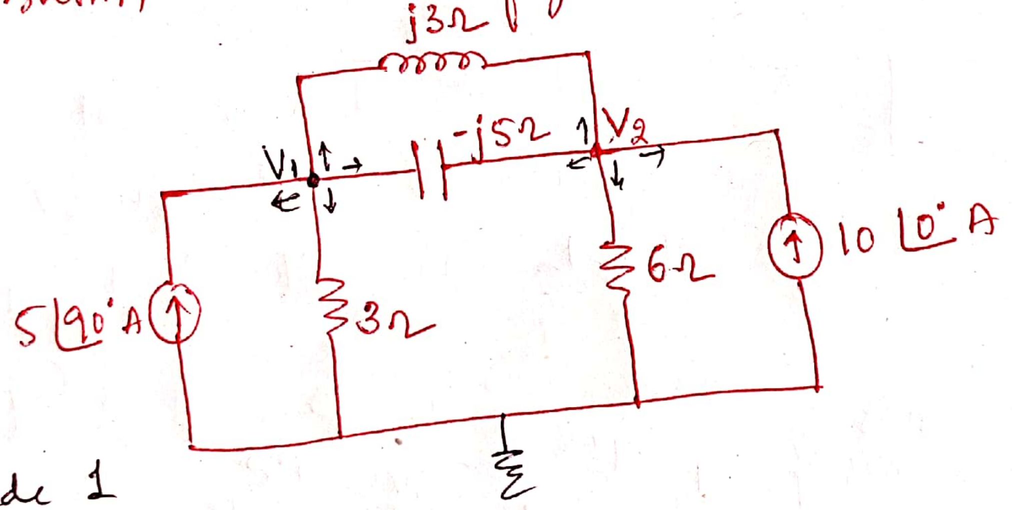
$$0.167\,V_2 + 0.25\,[V_2 - V_x] = 0$$

$$0.417\,V_2 = 0.25\,V_x$$

$$V_x = \frac{0.417\,V_2}{0.25} = \frac{0.417 \times 21.27\angle 45^\circ}{0.25}$$

$$\boxed{V_x = 35.47\angle 45^\circ \text{ volts}}$$

3) Use nodal analysis to find V_2 in the circuit shown in the figure.



At node 1

$$-5\angle 90^\circ + \frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} = 0$$

$$0.33\,V_1 + j\,0.2V_1 - j\,0.2V_2 - j0.33V_1 + j\,0.33\,V_2 = 5\angle 90^\circ$$

$$[0.33 - j0.133]\,V_1 + j\,0.133\,V_2 = 5\angle 90^\circ \quad \text{—(1)}$$

At node 2

$$\frac{V_2}{6} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} - 10\angle 0^\circ = 0$$

$$0.166 V_2 + j0.2V_2 - j0.2V_1 - j0.333V_2 + j0.33V_1$$
$$= 10\angle 0^\circ$$

$$\cancel{0.166} - j0.133V_1 + (0.166 - j0.133)V_2 = 10\angle 0^\circ \quad \text{--- (2)}$$

$$\Delta = \begin{vmatrix} 0.33 - j0.133 & 0.133j \\ -0.133j & 0.166 - j0.133 \end{vmatrix}$$

$$\Delta = [(0.33 - j0.133)(0.166 - j0.133) + (0.133j)(0.133j)]$$

$$= 0.0547 - j0.022 - j0.0442 - 0.01768$$
$$- 0.01768$$

$$= 0.0193 - j0.0658$$

$$\Delta = \underline{0.0686 \angle -73.67}$$

$$\Delta_2 = \begin{vmatrix} 0.33 - j0.133 & 5j \\ -0.133j & 10 \end{vmatrix}$$

$$= (0.33 - j0.133)10 + 5j(0.133j)$$

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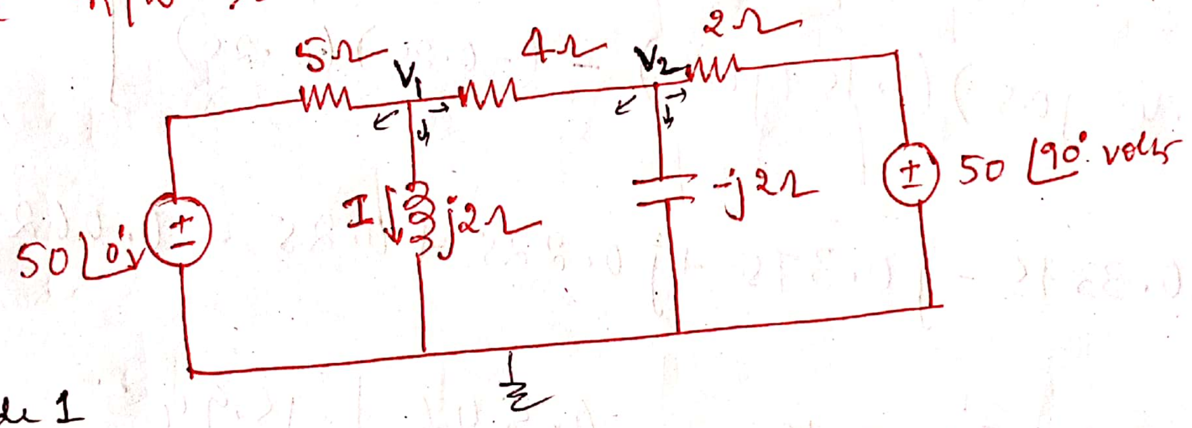
$$\Delta_2 = 3.33 - j1.33 - 0.665$$

$$\Delta_2 = 2.665 - j1.33 = 2.978 \angle -26.52$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2.978 \angle -26.52}{0.0686 \angle -73.67}$$

$$V_2 = 43.41 \angle 47.15^\circ \text{ volts}$$

4) Use node voltage technique to find I in the n/w shown.



@ node 1

$$\frac{V_1 - 50\angle 0^\circ}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} = 0$$

$$0.2V_1 - 10\angle 0^\circ - j0.5V_1 + 0.25V_1 - 0.25V_2 = 0$$

$$(0.45 - j0.5)V_1 - 0.25V_2 = 10\angle 0^\circ \quad \text{——} \ (1)$$

@ node 2

$$\frac{V_2 - V_1}{4} + \frac{V_2}{-j2} + \frac{V_2 - 50\angle 90^\circ}{2} = 0$$

$$0.25V_2 - 0.25V_1 + j\,0.5\,V_2 + 0.5\,V_2 - 25\angle 90^\circ = 0$$

$$-0.25V_1 + (0.75 + j0.5)V_2 = 25\angle 90^\circ \quad \text{—— (2)}$$

$$\Delta = \begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}$$

$$\Delta = \left[(0.45 - j0.5)(0.75 + j0.5) - (-0.25)(-0.25)\right]$$

$$= 0.3375 - j\,0.375 + j\,0.225 + 0.25 - 0.0625$$

$$= 0.525 - j\,0.15 = 0.546\angle -15.94^\circ$$

$$\Delta_1 = \begin{vmatrix} 10 & -0.25 \\ 25j & 0.75 + j0.5 \end{vmatrix}$$

$$= 7.5 + j5 + j6.25$$

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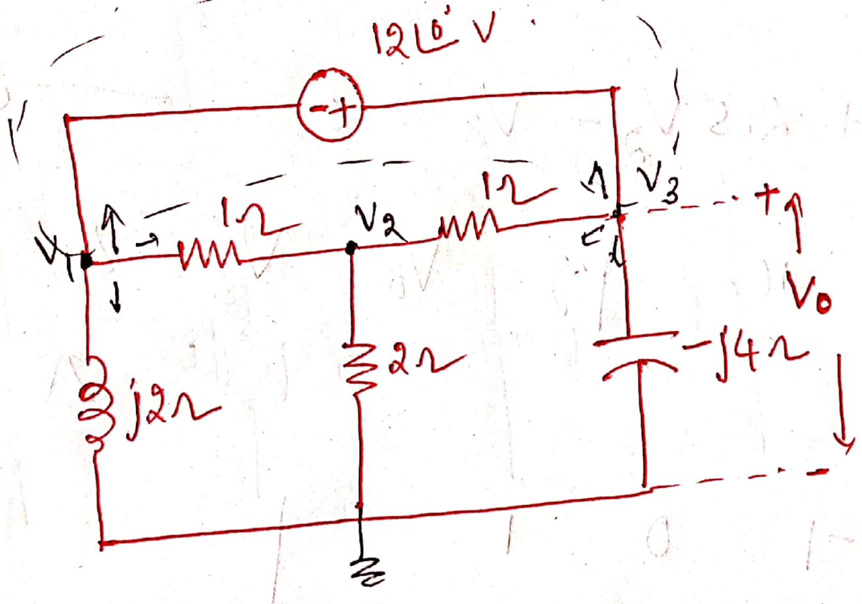
$$\Delta_1 = 7.5 + j11.25 = 13.52 \angle 56.3$$

$$V_1 = \frac{\Delta_1}{\Delta} = 24.76 \angle 72.24^\circ$$

$$I_1 = \frac{V_1}{j2} = \frac{24.76 \angle 72.24^\circ}{2 \angle 90^\circ}$$

$$\boxed{I_1 = 12.38 \angle -17.76 \text{ Amp}}$$

5) Use nodal analysis to find V_o in the ckt-



from Super node,

$$V_3 - V_1 = 12\angle 0^\circ \longrightarrow -V_1 + 0V_2 + V_3 = 12\angle 0^\circ$$

KCL @ Supernode,

$$\frac{V_1}{j2} + \frac{V_1 - V_2}{1} + \frac{V_3 - V_2}{1} + \frac{V_3}{-j4} = 0$$

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$$-j0.5V_1 + V_1 - V_2 + V_3 - V_2 + j0.25V_3 = 0$$

$$(1-j0.5)V_1 - 2V_2 + (1+j0.25)V_3 = 0 \quad \text{——— (2)}$$

KCL @ node 2

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$$

$$V_2 - V_1 + 0.5V_2 + V_2 - V_3 = 0$$

$$-V_1 + 2.5V_2 - V_3 = 0 \quad \text{——— (3)}$$

from the fig $V_0 = V_3 = \dfrac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} -1 & 0 & 1 \\ 1-j0.5 & -2 & (1+j0.25) \\ -1 & 2.5 & -1 \end{vmatrix}$$

$$\Delta = [-1(2 - 2.5 - j0.625) + 1(2.5 - j1.25 - 2)$$

$$\Delta = \left[-1(-0.5 - j0.625) + 1(0.5 - j1.25)\right]$$

$$\Delta = 0.5 + j0.625 + 0.5 - j1.25$$

$$\Delta = 1 - 0.625j = 1.179 \angle -32^\circ$$

$$\Delta_3 = \begin{vmatrix} -1 & 0 & 12 \\ 1-j0.5 & -2 & 0 \\ -1 & 2.5 & 0 \end{vmatrix}$$

$$\Delta_3 = \left[-1(\cancel{0}-0) + 12(2.5 - j\,\overset{j1.25}{\cancel{0.625}} - 2)\right]$$

$$\Delta_3 = 12(0.5 - j\,\overset{1.25}{\cancel{0.625}}) = 6 - j15$$

$$\hookrightarrow = 6 - \overset{j15}{\cancel{j7.5}} = \cancel{9.6 \angle -51.3^\circ}$$

$$16.155 \angle -68.19$$

$$\therefore V_0 = \frac{\cancel{9.6 \angle -51.3^\circ}}{1.179 \angle -32^\circ} = \cancel{8.14 \angle -19.3^\circ}$$

$$V_0 = \frac{16.155 \angle -68.19}{1.179 \angle -32^\circ} = 13.70 \angle -36.2^\circ$$

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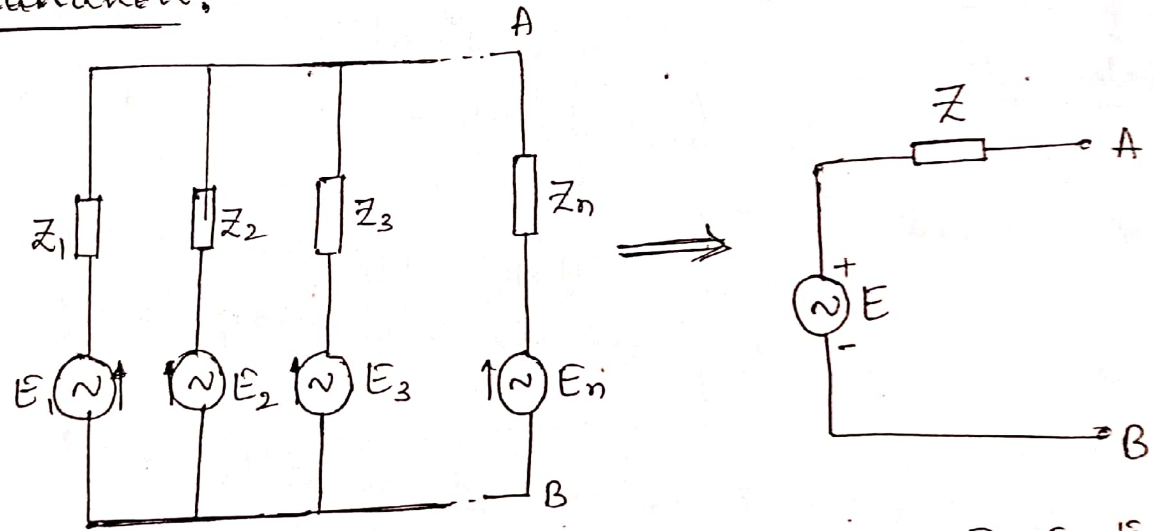
1. **Millman's theorem :-**

Statement :- "If n no of voltage source $E_1, E_2, E_3 \ldots E_n$ with their internal impedance $Z_1, Z_2 \ldots Z_n$ resp are in parallel. Then these voltage sources may be replaced by a single voltage source of voltage E with the internal impedance Z where

$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3 + \ldots E_n Y_n}{Y_1 + Y_2 + Y_3 + \ldots + Y_n}$$

And $Z = \dfrac{1}{Y_1 + Y_2 + \ldots Y_n}$

Explanation :-



Consider n number of voltage source $E_1, E_2, E_3 \ldots E_n$ with their internal impedance $Z_1, Z_2, Z_3 \ldots Z_n$ are connected in parallel are as shown in fig above.

W.K.T $I = I_1 + I_2 + I_3 + \ldots I_n$

But
$$\left.\begin{aligned} I &= EY \\ I_1 &= E_1 Y_1 \\ I_2 &= E_2 Y_2 \quad \ldots I_n = E_n Y_n \end{aligned}\right\} \text{—} ①$$

where $Y_1, Y_2, Y_3 \cdots Y_n$ are the admittences connected in parallel corresponding to impedance $Z_1, Z_2, \cdots Z_n$ respectively

$$\therefore \quad E = \frac{I}{Y}$$

$$E = \frac{I_1 + I_2 + I_3 + \cdots I_n}{Y_1 + Y_2 + \cdots Y_n}$$

$$\boxed{E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3 + \cdots E_n Y_n}{Y_1 + Y_2 + Y_3 + \cdots + Y_n}}$$

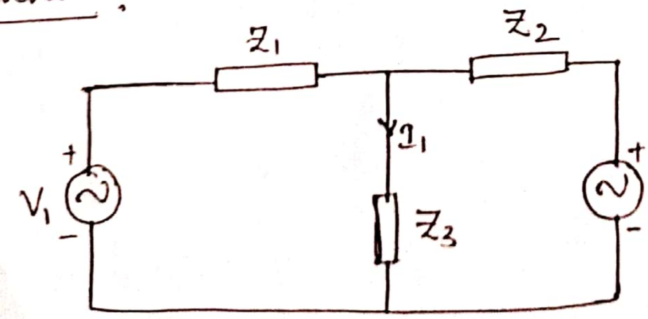
and $\boxed{Z = \frac{1}{Y} = \frac{1}{Y_1 + Y_2 + Y_3 + \cdots Y_n}}$ hence the proof.

2) **Superposition theorem:-**

Statement:- "In any linear bilateral network containing more than one independent source, the current or vg across any element in the network is equal to sum of the individual current or voltage produced by each source acting alone, setting all the other independent sources to zero."

If it is a vg src, it is replaced by short circuit. If it is a current source, it is replaced by an open circuit.

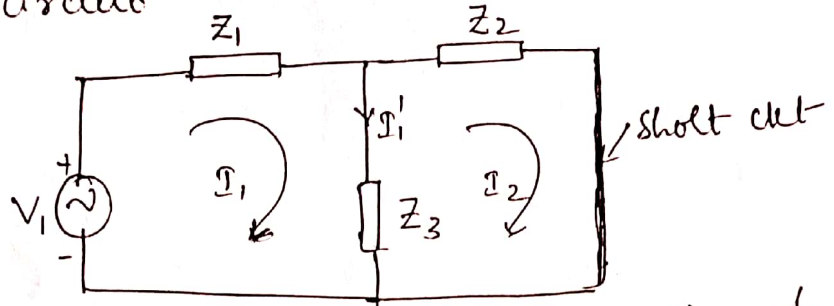
Explanation:-



Consider a linear bilateral network having two voltage source as shown in fig above.

Let I_1 be the current flowing through Z_3 when both the vg sources V_1 & V_2 are present in the circuit.

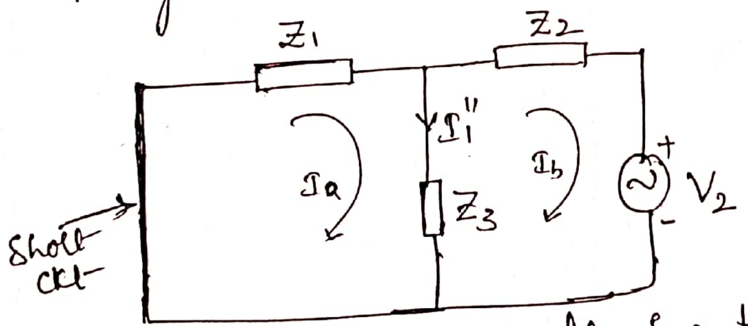
Case i) Consider the voltage source V_1 only, replacing the other V_2 by short circuit



Let I_1' is the current flowing through Z_3. Using any one of the n/w reduction method find the current I_1'

$$I_1' = I_1 - I_2.$$

Case ii) Consider the voltage source V_2 only. Replacing the other vg source V_1 by short ckt. The resulting n/w is as shown.



Let I_1'' is the current flowing through Z_3. using any one of the n/w reduction method find the current I_1''

$$\Rightarrow I_1'' = I_a - I_b$$

According to superposition theorem, the total current I_1 through Z_3 is the algebraic sum of the current through Z_3 produced by V_1 & V_2 acting alone.

i.e., $\boxed{I_1 = I_1' + I_1''}$

hence the proof.

* Reciprocity theorem :-

Statement: "In any linear, bilateral network containing only one independent source, the ratio of excitation to response remains same (constant) when their positions are interchanged."

Explanation:-

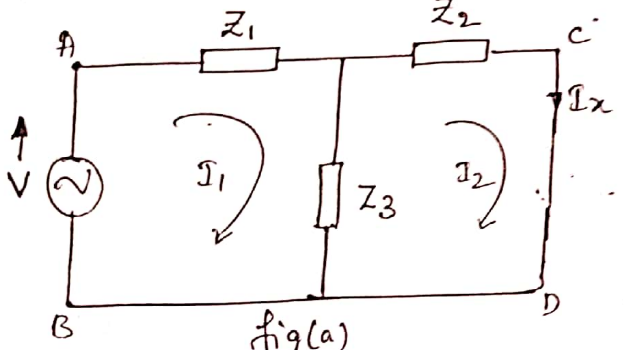


fig (a)

Consider a linear bilateral n/w as shown in fig (a). Let V volts is the excitation across AB & I_x is the response through CD, the ratio of excitation to response is $\frac{V}{I_x}$.

Now interchange the positions of excitation and response as shown in fig. (b)

If V volts is placed across CD, it produces the same current I_x through AB.

Then according to reciprocity theorem the ratio of excitation to response remains same.

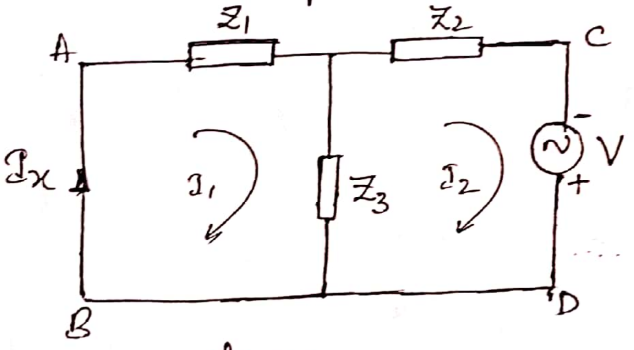


fig (b)

Proof:- For first loop (fig (a))

$$(Z_1+Z_3)I_1 - Z_3 I_2 = V \quad \text{——— (1)}$$

For second loop (fig a)

$$-Z_3 I_1 + (Z_2+Z_3) I_2 = 0 \quad \text{——— (2)}$$

$$\therefore \Delta = \begin{bmatrix} Z_1+Z_3 & -Z_3 \\ -Z_3 & Z_2+Z_3 \end{bmatrix}$$

$$\Delta = (Z_1+Z_3)(Z_2+Z_3) - Z_3^2$$

$$\Delta = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

$$\Delta_2 = \begin{bmatrix} Z_1+Z_3 & V \\ -Z_3 & 0 \end{bmatrix}$$

$$\Delta_2 = V Z_3$$

from fig (a) $I_x = I_2 = \frac{\Delta_2}{\Delta}$

$$I_x = \frac{V Z_3}{Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_1} \quad \text{——— (A)}$$

Now consider fig (b) :-

for 1st loop.

$$-Z_1 I_1 - Z_3 (I_1 - I_2) = 0$$

$$(Z_1 + Z_3) I_1 - Z_3 I_2 = 0 \quad \text{——— (1)}$$

for second loop apply KVL,

$$-Z_2 I_2 + V - Z_3 (I_2 - I_1) = 0$$

$$\cancel{(Z_2+Z_3)I} \; - Z_3 I_1 + (Z_2 + Z_3) I_2 = V \quad \text{——— (2)}$$

$$\Delta = \begin{bmatrix} (Z_1+Z_3) & -Z_3 \\ -Z_3 & (Z_1+Z_3) \end{bmatrix}$$

$$\Delta = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

$$\Delta_1 = \begin{bmatrix} 0 & -Z_3 \\ V & Z_1+Z_3 \end{bmatrix}$$

$$\Delta_1 = Z_3 V$$

from fig (b)

$$I_1 = I_x = \frac{\Delta_1}{\Delta} = \frac{VZ_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \quad \text{——— (B)}$$

from (A) & (B), reciprocity theorem is verified.

——— x ———

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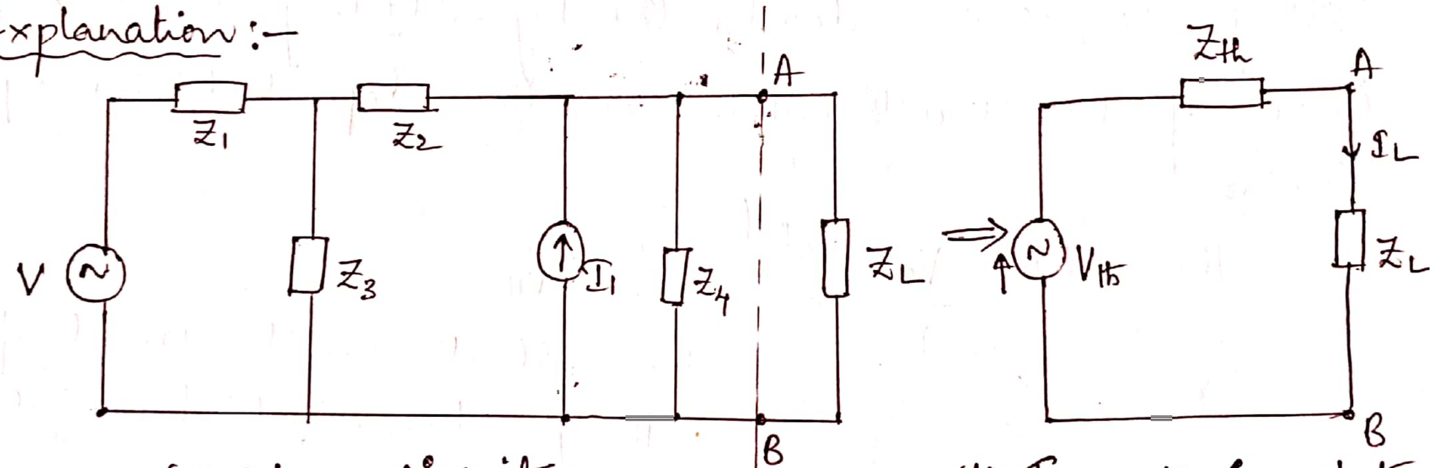
UNIT-4 Network Theorems - II

Thevenin's Theorem:-

Statement:- "In any linear bilateral complicated network connected to load may be replaced by a simple equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open-circuit voltage at the terminals and R_{th} is the input or equivalent resistance at the terminals when the independent sources are turned off or R_{th} is the ratio of open-circuit voltage to the short circuit current at the terminal pair."

Explanation:-



(a) given circuit (b) Thevenin's Equivalent circuit

Consider a linear bilateral complicated n/w is as shown in fig (a). According to Thevenin's theorem, the above complicated network can be reduced to a simple network as shown in fig (b).

The load current is calculated by,

$$\boxed{I_L = \frac{V_{th}}{Z_{th} + Z_L}}$$

Where, $V_{th} \rightarrow$ Thevenin's Equivalent voltage or open circuit voltage across the terminals A & B.

Z_{th} → Equivalent impedance b/w the terminals A & B.

Z_L → Load impedance.

Procedure :-

1) Remove the load impedance & create a open circuit across the load terminals A & B

2) Calculate open circuit voltage V_{th} across the load terminals

3) To find R_{th} :-

Case i) If the circuit contains only independent sources & resistors then deactivate the sources i.e, independent current srs are deactivated by opening them while independent voltage sources are deactivated by shorting them.

Case ii) If the circuit contains resistors, dependent & independent sources

then $R_{th} = \dfrac{V_{oc}}{i_{sc}}$

where,

$V_{oc} = V_{th}$

i_{sc} → Short circuit current flowing through the short terminal a-b.

Case iii) If the circuit contains resistors & only dependent sources

a) $V_{oc} = 0$ [Since there is no energy source]

b) Connect 1A current src to terminals a-b & determine V_{ab}

c) $R_{th} = \dfrac{V_{ab}}{1A}$

4) After finding V_{th} & R_{th}, write the Thevenin's equivalent circuit.

Maximum Power transfer theorem :-

Statement :- "In any linear bilateral network, the maximum power is transferred from source to load when

i) Load resistance is equal to source resistance.

i.e, $\boxed{R_L = R_0}$

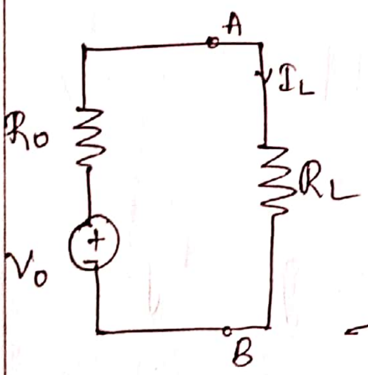
ii) Load resistance is equal to magnitude of source impedance

i.e, $\boxed{R_L = |Z_0|}$

iii) Load impedance is equal to complex conjugate of the source impedance.

i.e, $\boxed{Z_L = Z_0^*}$

Proof :- case i) when both source & load has resistance [DC circuits]



where, $V_0 \rightarrow$ Source voltage.

$R_0 \rightarrow$ Source resistance.

$R_L \rightarrow$ Load resistance.

The current flowing through the load is given by,

$$I_L = \frac{V_0}{R_0 + R_L} \quad \text{——} ①$$

The power delivered to the load is given by.

$$P = I_L^2 R_L \quad \text{——} ②$$

$$P = \frac{V_0^2}{(R_0+R_L)^2} \times R_L$$

Power delivered to the load is maximum when, $\frac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = \frac{(R_0+R_L)^2 \times V_0^2 - V_0^2 R_L \times 2(R_0+R_L)}{[(R_0+R_L)^2]^2} = 0$$

$$\Rightarrow (R_0+R_L)^2 V_0^2 - V_0^2 R_L \times 2(R_0+R_L) = 0$$

$$\Rightarrow (R_0^2 + R_L^2 + 2R_0R_L)V_0^2 - V_0^2[2R_0R_L + 2R_L^2] = 0$$

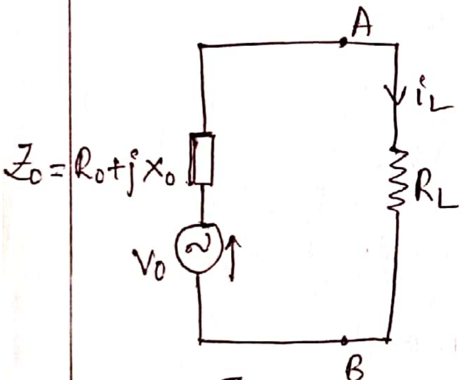
$$\Rightarrow (R_0^2 + R_L^2 + 2R_0R_L)\cancel{V_0^2} = \cancel{V_0^2}(2R_0R_L + 2R_L^2)$$

$$R_0^2 + R_L^2 = 2R_L^2$$

$$R_0^2 = 2R_L^2 - R_L^2$$

$$\boxed{R_0 = R_L}$$

Case ii) When source has impedance and load has resistance.



where,

$V_0 \rightarrow$ Source voltage.

$Z_0 \rightarrow$ Source impedance.

$R_L \rightarrow$ load resistance.

The current flowing through load is given by,

$$I_L = \frac{V_0}{Z_0 + R_L} = \frac{V_0}{R_0 + jX_0 + R_L}$$

$$I_L = \frac{V_0}{(R_0+R_L) + jX_0}$$

$$I_L = \frac{V_0}{\sqrt{(R_0+R_L)^2 + X_0^2}} \quad \text{—— (1)}$$

The power delivered to the load is given by

$$P = I_L^2 R_L \quad \text{—— (2)}$$

$$\therefore P = \frac{V_0^2 R_L}{(R_0+R_L)^2 + X_0^2}$$

The power transformed to the load is max. when

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{[(R_o+R_L)^2 + X_o^2] \times V_o^2 - V_o^2 R_L \times 2(R_o+R_L)}{[(R_o+R_L)^2 + X_o^2]^2} = 0$$

$$\Rightarrow [(R_o+R_L)^2 + X_o^2]\cancel{V_o^2} = 2\cancel{V_o^2} R_L(R_o+R_L)$$

$$\Rightarrow R_o^2 + R_L^2 + \cancel{2R_oR_L} + X_o^2 = \cancel{2R_LR_o} + 2R_L^2$$

$$\Rightarrow R_o^2 + X_o^2 = 2R_L^2 - R_L^2$$

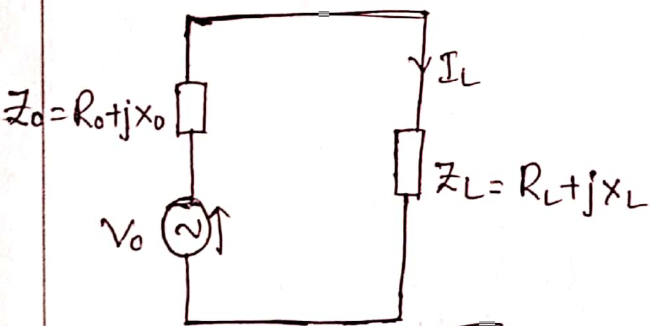
$$\Rightarrow R_o^2 + X_o^2 = R_L^2$$

$$\therefore R_L = \sqrt{R_o^2 + X_o^2}$$

$$R_L = R_o + jX_o \qquad \Rightarrow \boxed{R_L = |Z_o|}$$

Hence, the load resistance is equal to magnitude of src impedance is proved.

case iii) Both Source & load has impedances (AC circuits):-



where, $Z_o \rightarrow$ Source impedance

$V_o \rightarrow$ Src voltage.

$Z_L \rightarrow$ Load impedance.

The current flowing through the load is

$$I_L = \frac{V_o}{Z_o + Z_L} = \frac{V_o}{R_o + jX_o + R_L + jX_L}$$

$$I_L = \frac{V_o}{R_o + R_L + j(X_o + X_L)} = \frac{V_o}{\sqrt{(R_o+R_L)^2 + (X_o+X_L)^2}} \quad \text{—①}$$

The power delivered to the load is,

$$P = I_L^2 Z_L$$

$$P = I_L^2 \times (R_L + \cancel{jX_L})$$ (Since power consumed in inductor is zero)

$$P = I_L^2 R_L \quad \text{---} ②$$

Substitute ① in ②, we get-

$$P = \left[\frac{V_0^2}{\sqrt{(R_0+R_L)^2 + (X_0+X_L)^2}} \right]^2 \times R_L$$

$$P = \frac{V_0^2 R_L}{(R_0+R_L)^2 + (X_0+X_L)^2}$$

Power delivered to the load is max when $\frac{dP}{dR_L} = 0$

$$\therefore \frac{dP}{dR_L} = \frac{\left[(R_0+R_L)^2 + (X_0+X_L)^2\right] V_0^2 - V_0^2 R_L \times 2(R_0+R_L)}{\left[(R_0+R_L)^2 + (X_0+X_L)^2\right]^2} = 0$$

$$\Rightarrow \left[(R_0+R_L)^2 + (X_0+X_L)^2\right] \cancel{V_0^2} = 2\cancel{V_0^2} R_L (R_0+R_L)$$

$$\Rightarrow R_0^2 + R_L^2 + \cancel{2R_0R_L} + (X_0+X_L)^2 = \cancel{2R_LR_0} + 2R_L^2$$

$$\Rightarrow R_0^2 + (X_0+X_L)^2 = 2R_L^2 - R_L^2$$

$$\Rightarrow R_0^2 + (X_0+X_L)^2 = R_L^2$$

$$\therefore R_L = \sqrt{R_0^2 + (X_0+X_L)^2}$$

$$R_L = R_0 + j(X_0+X_L)$$

$$R_L = R_0 + jX_0 + jX_L$$

$$R_L - jX_L = R_0 + jX_0$$

$$Z_L^* = Z_0$$

$$\left\{ \begin{array}{l} x+jy = \sqrt{x^2+y^2} \\ Z_0 = R_0 + jX_0 \\ Z_L = R + jX_L \\ Z_L^* = R - jX_L \end{array} \right.$$

$$\therefore \boxed{Z_L = Z_0^*}$$

Norton's Theorem :-

Statement :- "In any linear bilateral complicated n/w connected to load may be replaced by a simple network containing a current source & an impedance in parallel with it. The current source 'I_{sc}' is the short circuit current in load terminals & Z_{th} is the value of impedance looking from the load terminals replacing all the voltage sources by short circuit & all the current sources by open circuit."

Explanation :-

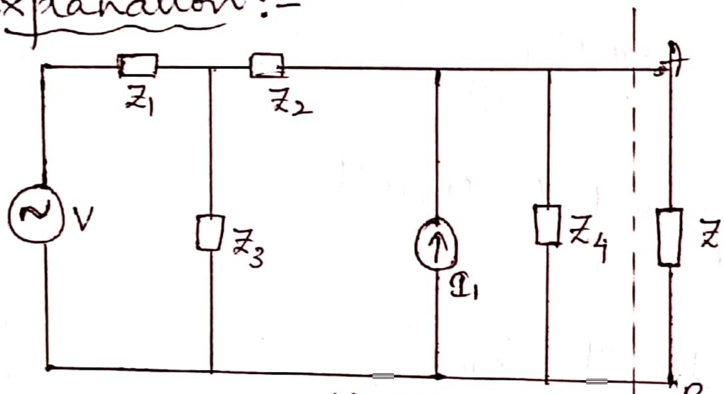
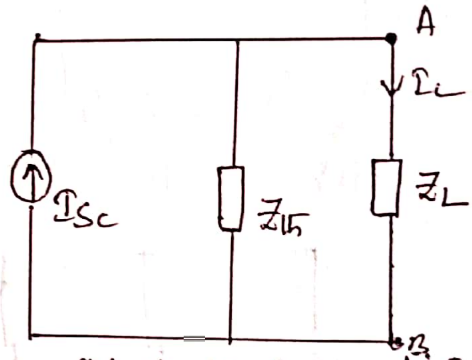


fig (1)



Norton's Equivalent circuit (fig 2)

Consider a linear bilateral n/w as shown in fig(1)

According to Norton's theorem. The above complicated network can be reduced into a simple n/w as shown in fig (2)

The load current is calculated by using

$$I_L = \frac{I_{sc} \times Z_{th}}{Z_{th} + Z_L}$$

Where I_{sc} → Short circuit current or Norton's current

Z_{th} → Norton's Equivalent imp. &

Z_L → load impedances.

Procedure :- 1) Remove the load impedance & short circuit the load terminals A & B.

OC → Open Circuit   SC → Short circuit

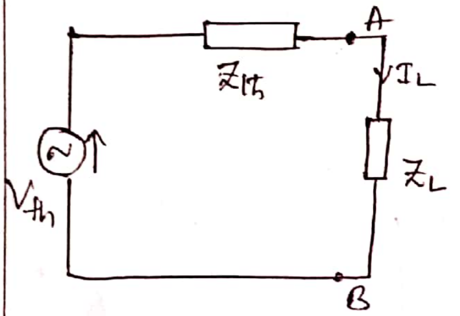
2) Calculate the short ckt current I_{sc} through the load terminal

3) Replace all the vg source by SC & all the current sources by OC.

4) Find the equivalent impedance Z_{th} (Z_N) as looking from the load terminals

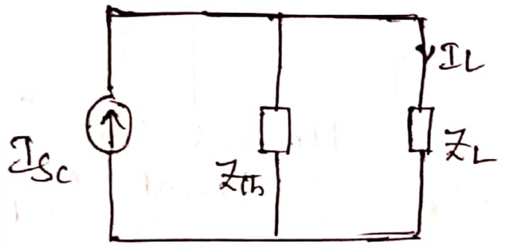
"Thevenin's equivalent is the dual of Norton's equivalent" Comment on the above statement

Soln Consider the Thevenin's equivalent circuit



$$I_L = \frac{V_{th}}{Z_{th}+Z_L} \quad \text{——— (1)}$$

Consider the Norton's equivalent circuit,



$$I_L = \frac{I_{sc}\, Z_{th}}{Z_{th}+Z_L} \quad \text{——— (2)}$$

from (1) & (2)

$$\frac{V_{th}}{Z_{th}+Z_L} = \frac{I_{sc}\, Z_{th}}{Z_{th}+Z_L}$$

where,
V_{th} → Thevenin's vg
I_{sc} → Norton's current
Z_{th} → Thevenin's equivalent impedance

$$\Rightarrow \boxed{V_{th} = I_{sc} \cdot Z_{th}} \quad \text{——— (3)}$$

or

$$\boxed{I_{sc} = \frac{V_{th}}{Z_{th}}} \quad \text{——— (4)}$$

∴ Norton's equivalent circuit can be converted into thevenin's equivalent circuit using equation (3) & the Thevenin's equivalent circuit can be converted into Norton's equivalent circuit by using eqn (4).

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Nortons theorem:

Statement:

Any Linear, bilateral Complicated N/w. Connected to a load impedance Can be replaced by a Simple equivalent CKt Containing a Current Source of Current I_{sc} in parallel with impedance Z_{th}.

Where, $I_{sc} \rightarrow$ Short CKt Current in the Load terminals.

$Z_{th} \rightarrow$ Equivalent impedance of the N/w as looking from the load terminals, replacing all the voltage Sources by Short CKt & all Current Sources by open CKt.

Explanation:

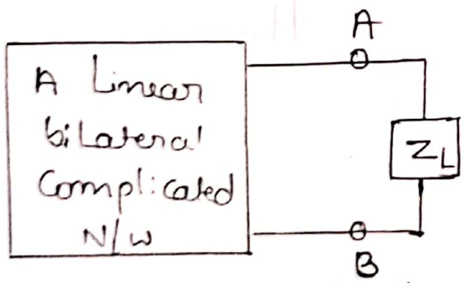


Fig (a) Complicated N/w

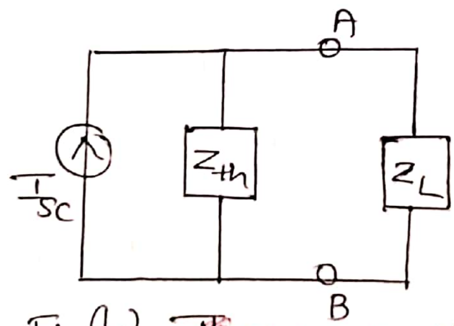


Fig (b). ~~Thevenins~~ Norton's N/w.

Consider a Linear, bilateral, Complicated N/w as shown in fig (a). According to ~~thevenins~~ Norton's theorem the Complicated N/w reduced to a Simple as shown in fig (b).

The load Current is $I_L = \frac{I_{sc} \times Z_{th}}{Z_{th} + Z_L}$

where, $I_{SC} \rightarrow$ Short ckt current or Nortons current.

$Z_{th} \rightarrow$ Equivalent impedance b/w A & B

$Z_L \rightarrow$ Load impedance.

Procedure :-

1) Remove the load impedance & short the load terminals A & B.

2) Calculate the short ckt current $[I_{SC}]$ through the load terminals.

3) Replace all the independent voltage sources by short ckt & all the independent current sources by open ckt.

4) Find the equivalent impedance b/w A & B.

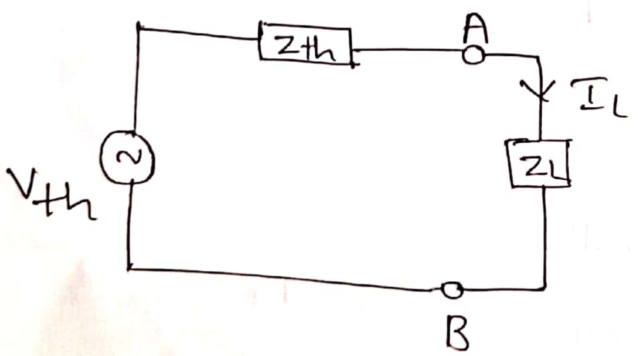
5) Write the Nortons equivalent ckt.

6) Calculate the load current $I_L = \dfrac{I_{SC} \times Z_{th}}{Z_{th} + Z_L}$

Jan SMK

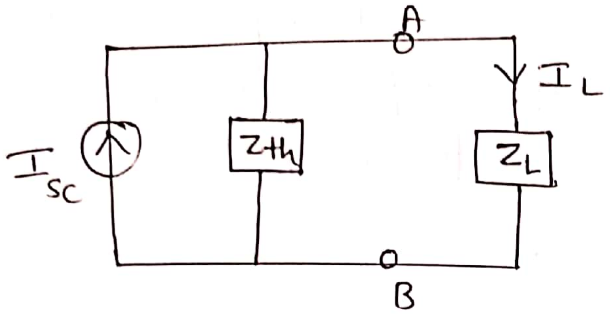
* "Thevenins equivalent is the dual of Nortons equivalent". Comment on the above statement & substantiate the same.

Thevenin's equivalent ckt is



$$I_L = \frac{V_{th}}{Z_{th} + Z_L} \longrightarrow (1)$$

Nortons equivalent ckt is



$$I_L = \frac{I_{SC} \times Z_{th}}{Z_{th} + Z_L} \longrightarrow (2)$$

From (1) & (2)

$$\frac{V_{th}}{Z_{th} + Z_L} = \frac{I_{SC} \cdot Z_{th}}{Z_{th} + Z_L}$$

$$V_{th} = I_{SC} \cdot Z_{th} \quad \text{---} (3)$$

$$I_{SC} = \frac{V_{th}}{Z_{th}} \quad \text{---} (4)$$

Where $V_{th} \rightarrow$ Thevenins Voltage

$I_{SC} \rightarrow$ Nortons Current

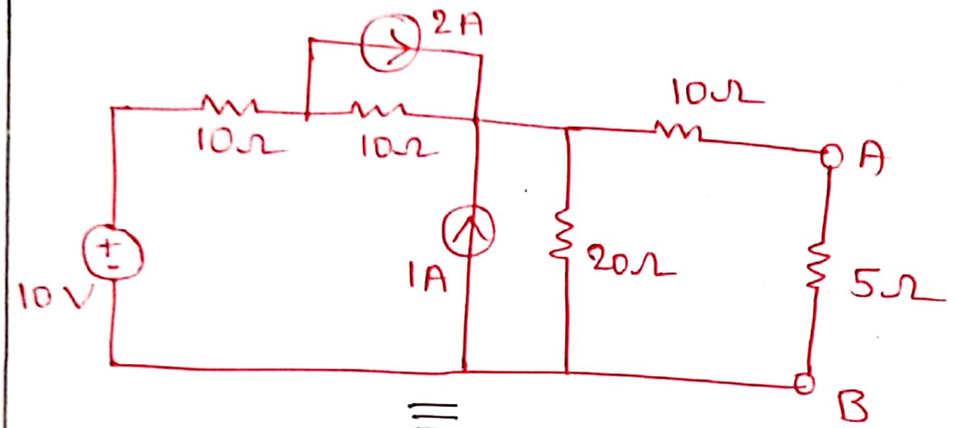
$Z_{th} \rightarrow$ Equivalent impedance

* Nortons equivalent ckt can be converted into thevenins equivalent ckt by eqn (3)

* Thevenins equivalent ckt can be converted into Nortons equivalent ckt by eqn (4)

Problems :

1) Draw the thevenins equivalent N/W & Nortons equivalent N/W for the N/W shown & also find current flowing through 5 Ω resistor connected b/w A & B.



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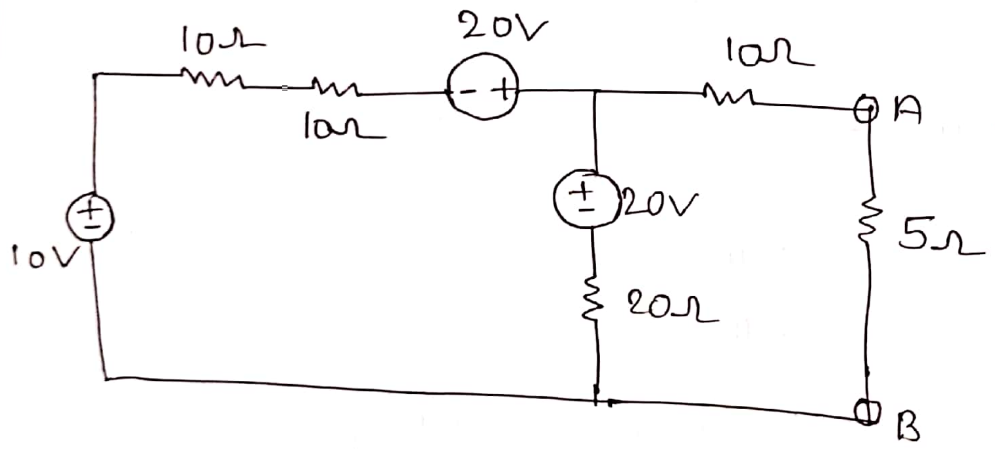
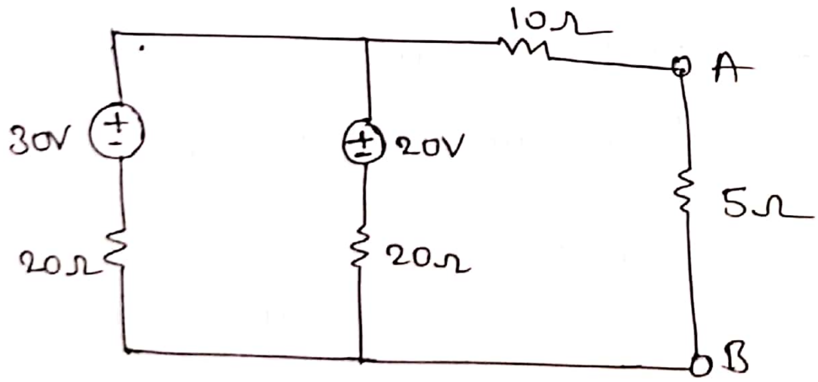
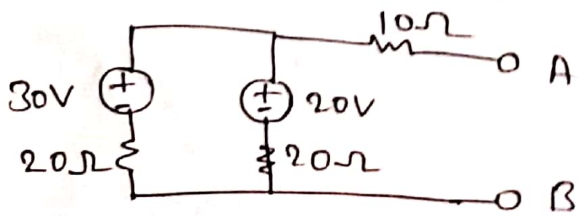
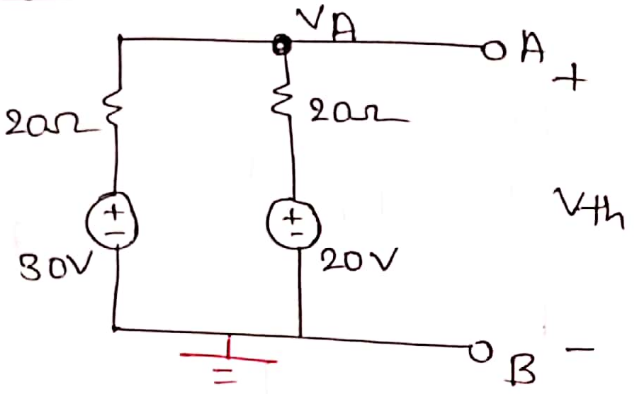




Remove the Load & create open ckt b/w A & B



To find Vth :



Apply KCL at node V_A

$$\frac{V_A - 30}{20} + \frac{V_A - 20}{20} = 0$$

$$V_A - 30 + V_A - 20 = 0$$

$$2V_A - 50 = 0$$

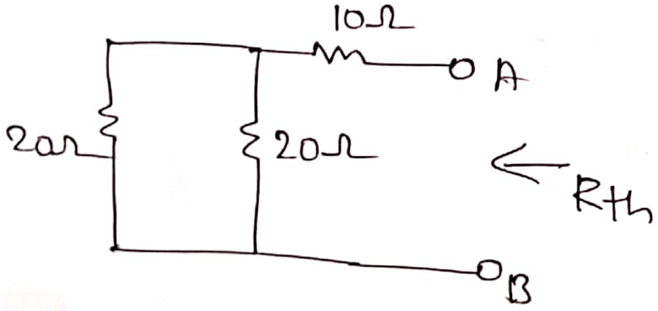
$$V_A = \frac{50}{2} = 25\,V$$

$$\therefore V_{th} = V_{AB} = V_A - V_B$$

$$= 25 - 0$$

$$= \underline{\underline{25\,V}}$$

To find Rth :

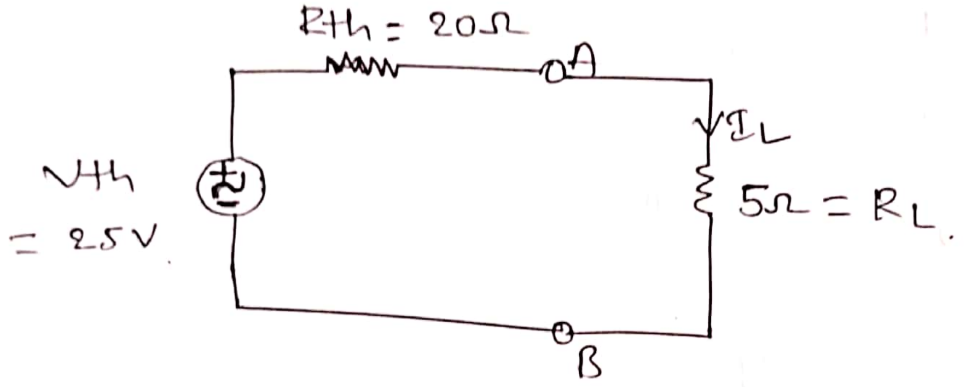


$$R_{th} = (20 \| 20) + 10$$

$$= 10 + 10$$

$$= 20\,\Omega$$

Thevenins equivalent N/w :-



$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

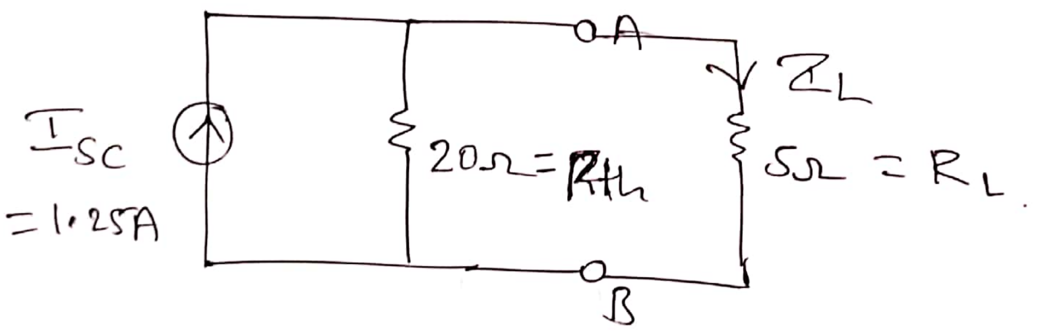
$$I_L = \frac{25}{20+5}$$

$$\boxed{I_L = 1A}$$

Nortons equivalent N/w :-

W.K.T

$$I_{SC} = \frac{V_{th}}{Z_{th}} = \frac{25}{20} = 1.25\,A$$

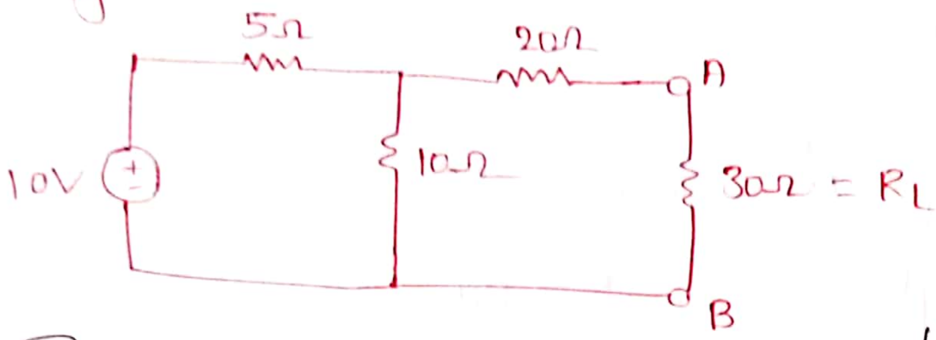


$$I_L = \frac{I_{SC}\, R_{th}}{R_{th} + R_L}$$

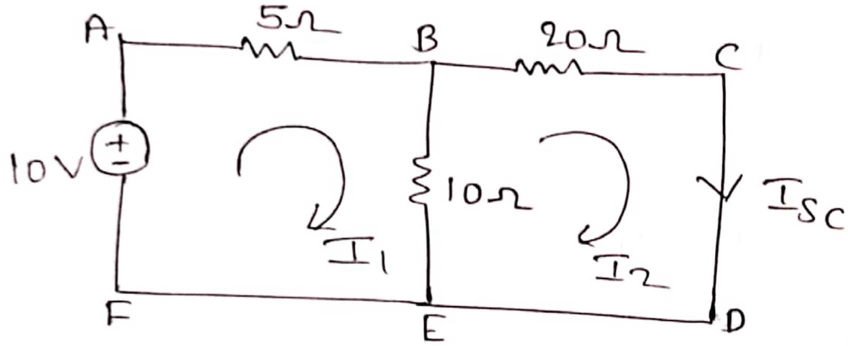
$$= \frac{1.25 \times 20}{20 + 5}$$

$$\boxed{I_L = 1A}$$

2) Find the Current through 30Ω load resistor using Nortons theorem.



Remove the load & Creat the short ckt.



From the CKt

$\therefore I_{SC} = ?$ $\qquad$ $I_{SC} = I_2$

KVL to the 1st Loop :

$$10 - 5I_1 - 10[I_1 - I_2] = 0$$

$$10 - 5I_1 - 10I_1 + 10I_2 = 0$$

$$-15I_1 + 10I_2 = -10 \longrightarrow (1)$$

KVL to the 2nd Loop :

$$-20I_2 - 10[I_2 - I_1] = 0$$

$$-20I_2 - 10I_2 + 10I_1 = 0$$

$$10I_1 - 30I_2 = 0 \longrightarrow (2)$$

Solving (1) & (2).

$I_1 = 0.857A$ $\qquad$ $I_2 = 0.2857A$

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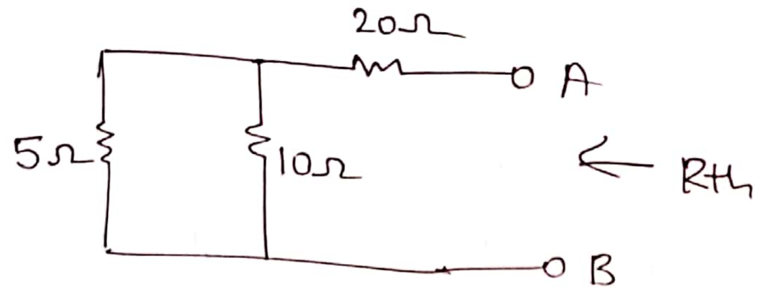
$\therefore \; I_{sc} = I_2 = 0.2857A$

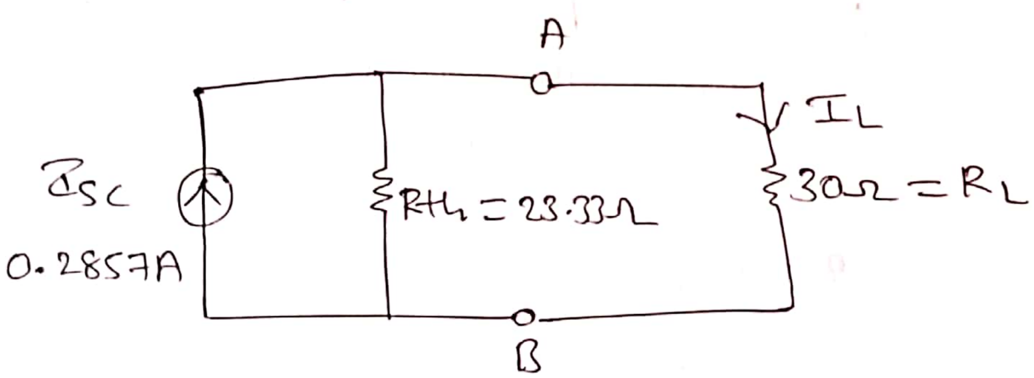
To find Rth :



$$R_{th} = (5 \parallel 10) + 20$$

$$= 23.33\,\Omega$$

Nortons equivalent N/W :-

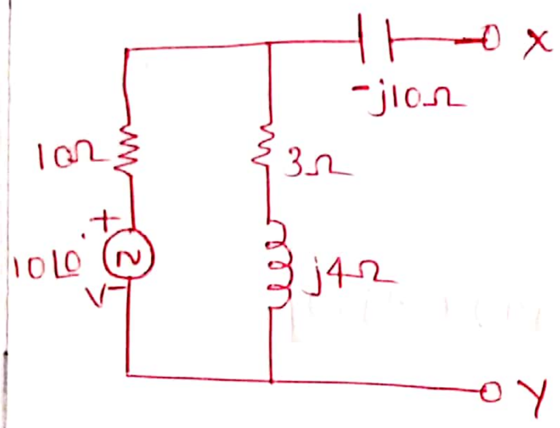


$$I_L = \frac{I_{sc} \times R_{th}}{R_{th} + R_L}$$

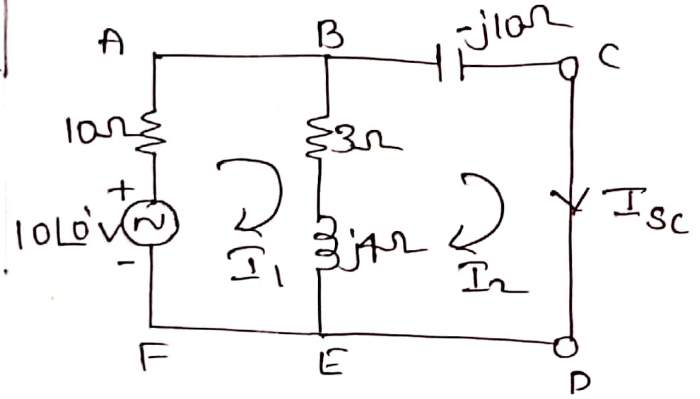
$$= \frac{0.2857 \times 23.33}{23.33 + 30}$$

$$\underline{I_L = 0.125\,A}$$

3. Obtain the Nortons equivalent ckt for the N/w shown



To find I_{sc} :



$$I_{sc} = I_2$$

Apply KVL to the 1st Loop

$$10\angle 0 - 10I_1 - 3[I_1 - I_2] - j4[I_1 - I_2] = 0$$

$$10\angle 0 - 10I_1 - 3I_1 + 3I_2 - j4I_1 + j4I_2 = 0$$

$$-13I_1 - j4I_1 + 3I_2 + j4I_2 = -10\angle 0$$

$$(-13 - j4)I_1 + (3+j4)I_2 = -10\angle 0 \rightarrow (1)$$

Apply KVL to the 2nd Loop..

$$+j10\Omega I_2 - (3+j4)(I_2 - I_1) = 0$$

$$j10\Omega I_2 - (3+j4)I_2 + (3+j4)I_1 = 0$$

$$(3+j4)I_1 + (-3+j6)I_2 = 0 \rightarrow (2)$$

$$\Delta = \begin{vmatrix} -13-j4 & 3+j4 \\ 3+j4 & -3+j6 \end{vmatrix}$$

$$= [(-13-j4)(-3+j6) - (3+j4)(3+j4)]$$

$$= 70 - 90j$$

$$\Delta_2 = \begin{vmatrix} -13-j4 & -10\angle 0 \\ 3+j4 & 0 \end{vmatrix}$$

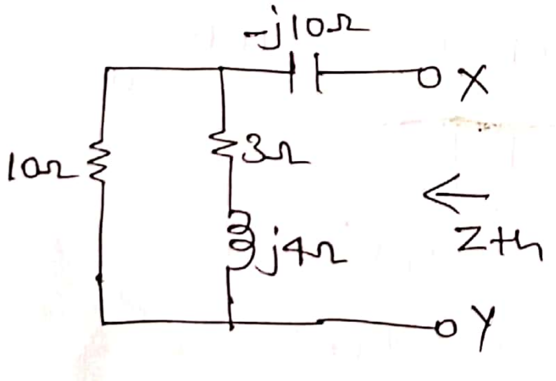
$$= +(3+j4)(10\angle 0)$$

$$= 30 + 40j$$

$$\therefore I_{SC} = I_2 = \frac{\Delta_2}{\Delta} = \frac{30+40j}{70-90j} = -0.115 + 0.42j$$

$$= 0.439\angle 105.3^\circ \text{ A}$$

<u>To find Z_{th}:</u>

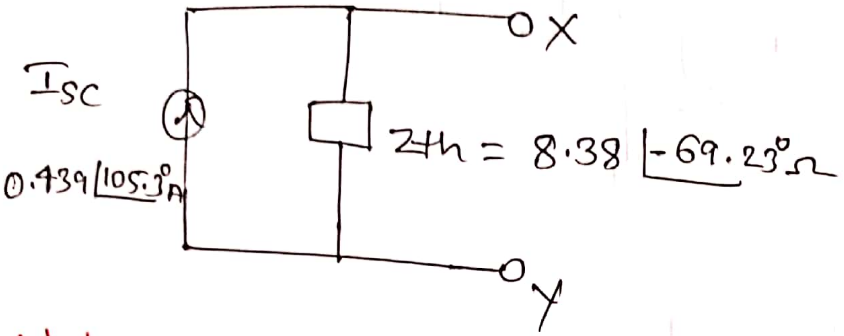


$$Z_{th} = \frac{10 * (3+j4)}{10+3+j4} - j10$$

$$= 2.97 - 7.84j$$

$$= 8.38\angle -69.23^\circ \ \Omega$$

Nortons equivalent N/w :



Note: If the given N/w Consists of some dependent source, then these dependent source must be kept as it is while Calculating Zth & should not be shorted or open CKted wether it is voltage or Current source.

In such cases, Zth is given by

$$Z_{th} = \frac{V_{th}}{I_{sc}}$$

Where $I_{sc} \rightarrow$ Nortons Current

$V_{th} \rightarrow$ thevenins voltage.

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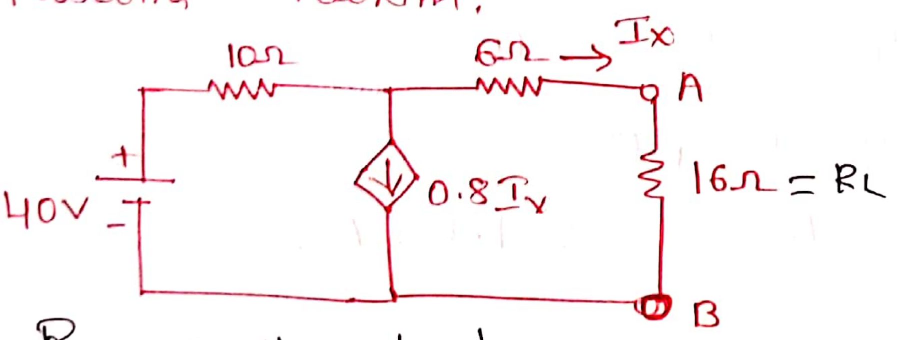
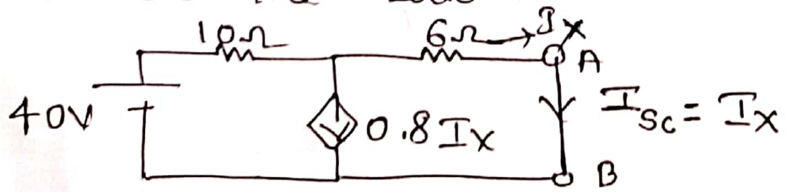
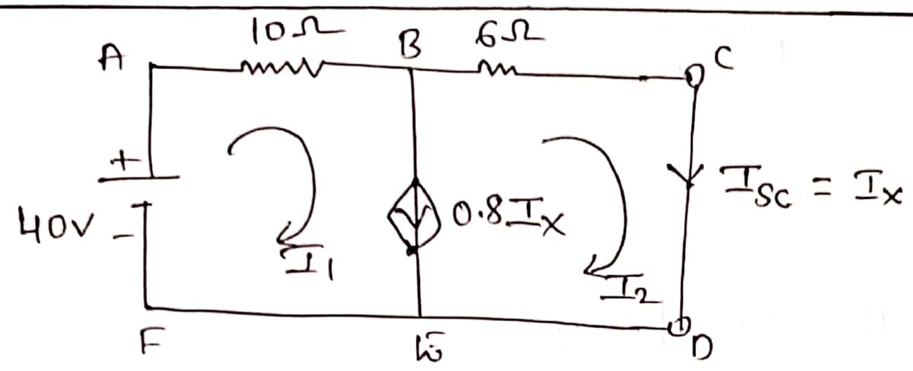
4) Find the Current through 16 Ω resistor using Nortons theorem.



Remove the Load





$$I_{sc} = I_x = I_2$$

0.8 I_x is in b/w two meshes ∴ it forms supermesh ABCDEFA.

$$I_1 - I_2 = 0.8\, I_x$$

$$I_1 - I_2 = 0.8\, I_2$$

$$I_1 - I_2 - 0.8\, I_2 = 0$$

$$I_1 - 1.8\, I_2 = 0 \longrightarrow (1)$$

Apply KVL to the supermesh

$$-10 I_1 - 6 I_2 + 40 = 0$$

$$-10 I_1 - 6 I_2 = -40 \longrightarrow (2)$$

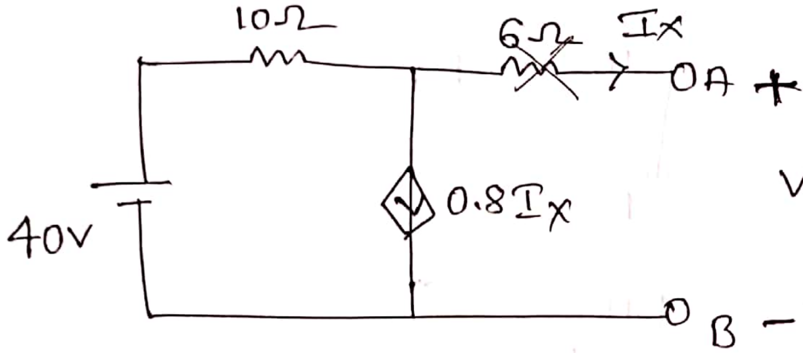
Solving (1) & (2)

$I_1 = 3A \qquad I_2 = 1.67\, A.$

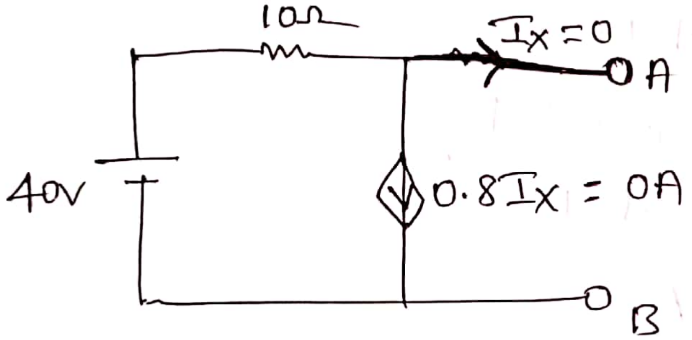
$$\therefore\ I_{sc} = I_2 = 1.67\, A.$$

To find Z_{th} :

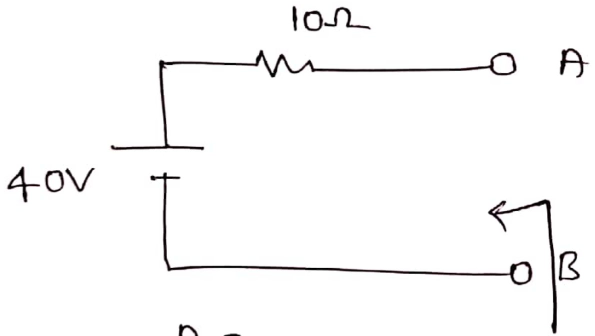
Creat open CKt b/w A & B



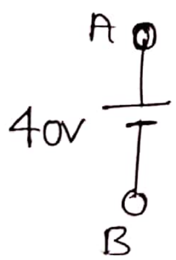
$V_{th} = V_{OC}$



NO Current flows through 6Ω



No Current flows through 10Ω

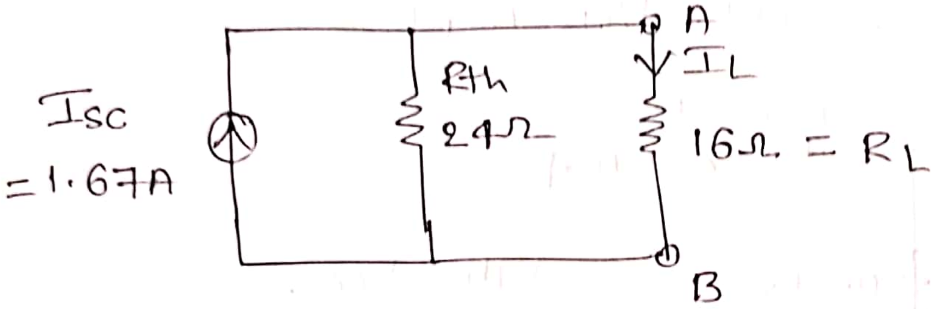


$\therefore V_{th} = V_{AB} = 40\,V$

$$\therefore Z_{th} = \frac{V_{th}}{I_{sc}}$$

$$= \frac{40}{1.67} = 24\,\Omega = R_{th}$$

∴ Nortons Equivalent Ckt



$$\therefore I_L = \frac{I_{sc} \times R_{th}}{R_{th} + R_L}$$

$$= \frac{1.67 \times 24}{24 + 16}$$

$$\underline{I_L = 1\,A}$$

Maximum power transfer theorem :

Statement :

In any linear bilateral N/w, the Maximum power is transferred from source to load when

1) Load resistance = Source resistance ie $R_L = R_S$

2) Load resistance = Magnitude of source impedance

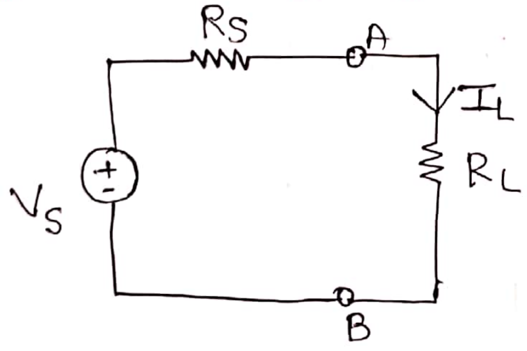
ie $R_L = |Z_S|$

3) Load impedance = Complex Conjugate of Source impedance

ie $Z_L = Z_S^*$

Proof :

Case (1) : P.T $R_L = R_S$



$V_S \rightarrow$ Source Voltage

$R_S \rightarrow$ Source resistance

$R_L \rightarrow$ load resistance

The power delivered to the load is

$$P = I_L^2 R_L \longrightarrow (1)$$

But $$I_L = \frac{V_S}{R_S + R_L} \longrightarrow (2)$$

Substitute (2) in (1)

$$P = \frac{V_S^2}{(R_S + R_L)^2} \cdot R_L \longrightarrow (3)$$

The power delivered to the load is maximum when $\frac{dp}{dR_L} = 0 \rightarrow$ Maxima theorem.

$$\frac{dp}{dR_L} = 0$$

$$\frac{d}{dR_L}\left[\frac{V_S^2}{(R_S+R_L)^2} R_L\right] = 0$$

$$\frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L \, 2\left[R_S+R_L\right]}{(R_S+R_L)^4} = 0$$

$$(R_S+R_L)^2 V_S^2 - 2V_S^2 R_S R_L - 2V_S^2 R_L^2 = 0$$

$$(2R_SR_L + R_S^2 + R_L^2) V_S^2 - 2V_S^2 R_S R_L - 2V_S^2 R_L^2 = 0$$

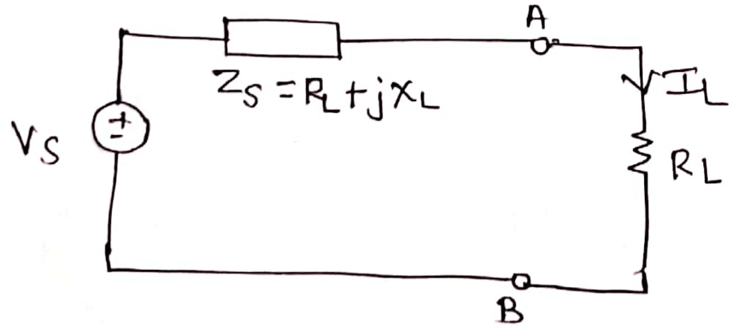
$$\cancel{2R_SR_LV_S^2} + R_S^2V_S^2 + R_L^2V_S^2 - \cancel{2V_S^2R_SR_L} - 2V_S^2R_L^2 = 0$$

$$R_S^2V_S^2 - R_L^2V_S^2 = 0$$

$$R_S^2 \cancel{V_S^2} = R_L^2 \cancel{V_S^2}$$

$$\therefore \boxed{R_L = R_S}$$

Case (2) : P.T $R_L = |Z_g|$



The power delivered to the load is

$$P = I_L^2 R_L \longrightarrow ①$$

The load current is given by

$$I_L = \frac{V_S}{Z_S + R_L}$$

$$= \frac{V_S}{R_S + jX_S + R_L}$$

$$= \frac{V_S}{(R_S + R_L) + jX_S}$$

$$I_L = \frac{V_S}{\sqrt{(R_S+R_L)^2 + X_S^2}} \longrightarrow ②$$

Substitute ② in ①

$$P = \frac{V_S^2 R_L}{(R_S+R_L)^2 + X_S^2}$$

The power delivered to the load is maximum when

$$\frac{dP}{dR_L} = 0 \longrightarrow \text{Maxima theorem}$$

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2 + X_S^2}\right] = 0$$

$$\left(\left[(R_S+R_L)^2 + X_S^2\right] V_S^2 - V_S^2 R_L \, 2\left[R_S + R_L\right]\right) \Big/ \left[(R_S+R_L)^2 + X_S^2\right]^2 = 0$$

$$(R_S + R_L)^2 + X_S^2 - 2R_L(R_S + R_L) = 0$$

$$R_S^2 + R_L^2 + \cancel{2R_S R_L} + X_S^2 - \cancel{2R_S R_L} - 2R_L^2 = 0$$

$$R_S^2 + X_S^2 - R_L^2 = 0$$

$$R_L^2 = R_S^2 + X_S^2$$

$$R_L = \sqrt{R_S^2 + X_S^2}$$

$$R_L = |Z_S|$$

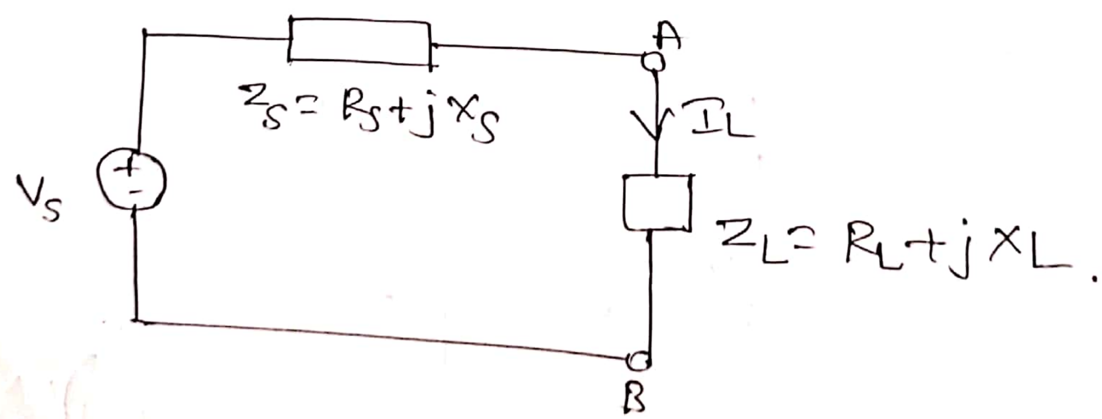
<u>Case 3</u> : P.T $Z_L = Z_S^*$

<u>or</u>

State & prove maximum power transfer theorem for AC Ckt.

<u>or</u>

P.T an alternating voltage source transfer maximum power to the load when the load impedance is equal to complex conjugate of the source impedance.



The power delivered to the load is

$$P = I_L^2 Z_L$$

$$P = I_L^2 [R_L + jX_L]$$

Power consumed by the inductor or Capacor is zero

$$\therefore \quad P = I_L^2 R_L \longrightarrow (1)$$

The load current is given by

$$I_L = \frac{V_S}{Z_S + Z_L}$$

$$= \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)}$$

$$= \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$I_L = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}} \longrightarrow (2)$$

(2) in (1)

$$P = \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

The power delivered to the load is maximum when $\frac{dP}{dR_L} = 0$.

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2}\right] = 0$$

$$\frac{\left[(R_S+R_L)^2+(X_S+X_L)^2\right]V_S^2 - V_S^2 R_L\, 2(R_S+R_L)}{\left[(R_S+R_L)^2+(X_S+X_L)^2\right]^2} = 0$$

$$(R_S+R_L)^2+(X_S+X_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2+R_L^2+\cancel{2R_SR_L}+X_S^2+X_L^2+2X_SX_L-\cancel{2R_LR_S}-2R_L^2=0$$

$$R_S^2+R_L^2+(X_S+X_L)^2-2R_L^2=0$$

$$R_S^2+(X_S+X_L)^2-R_L^2=0$$

$$R_L^2 = R_S^2+(X_S+X_L)^2$$

$$R_L = \sqrt{R_S^2+(X_S+X_L)^2}$$

$$R_L = R_S + j(X_S+X_L)$$

$$R_L = R_S + jX_S + jX_L$$

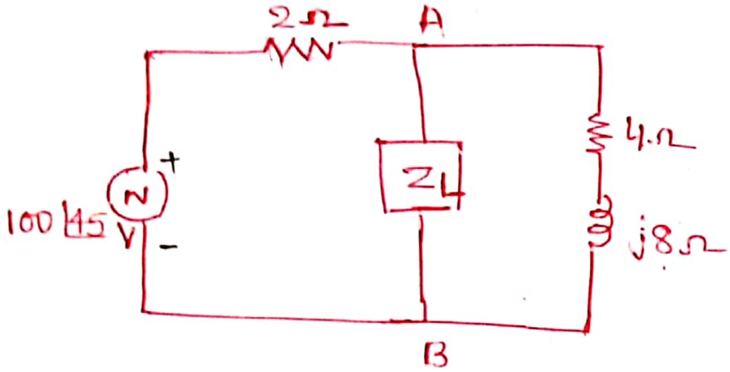
$$R_L - jX_L = R_S + jX_S$$

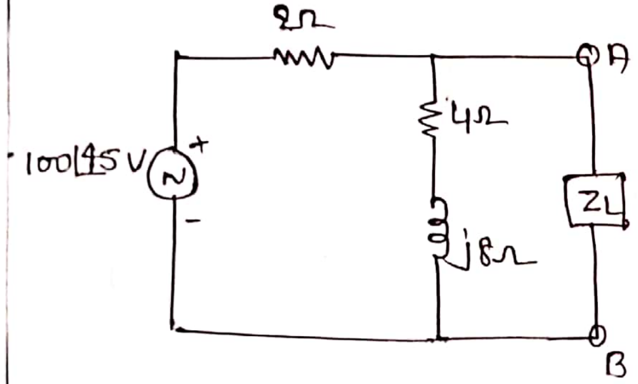
$$Z_L^* = Z_S$$

or $\boxed{Z_L = Z_S^*}$

6 times Problems :-

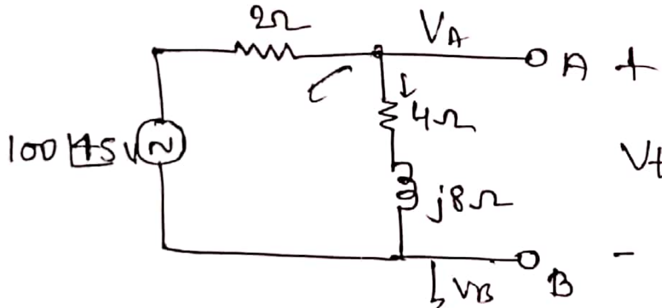
1) The N/W shown in figure determine Z_L for which power transfer is Maximum. Calculate the maximum power transfered to the load.





Remove the load

$V_{th} = V_A - V_B$ and $\boxed{V_B = 0}$



KCL

$$\frac{V_A - 100\angle 45^\circ}{2} + \frac{V_A}{(4+j8)} = 0$$

$$0.5V_A - 50\angle 45^\circ + \frac{V_A}{8.94\angle 63.43} = 0$$

$$0.5V_A - 50\angle 45^\circ + 0.1118\angle -63.43\, V_A = 0$$

$$\cancel{V_{th} = \frac{100\angle 45 \times (4+j8)}{2+4+j8}}$$

$$\cancel{V_{th} = 89.44\angle 55.3\ V}$$

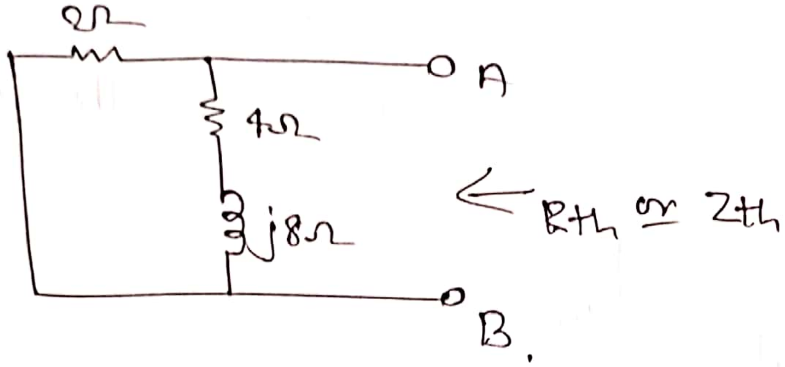
$$\boxed{V_A = 89.4\angle 55.3^\circ \text{ volts}}$$

$\rightarrow 0.5V_A + 0.05V_A - j0.099V_A = 50\angle 45^\circ$

$$(0.55 - j0.099)V_A = 50\angle 45^\circ$$

$$V_A = \frac{50\angle 45^\circ}{0.558\angle -10.2}$$

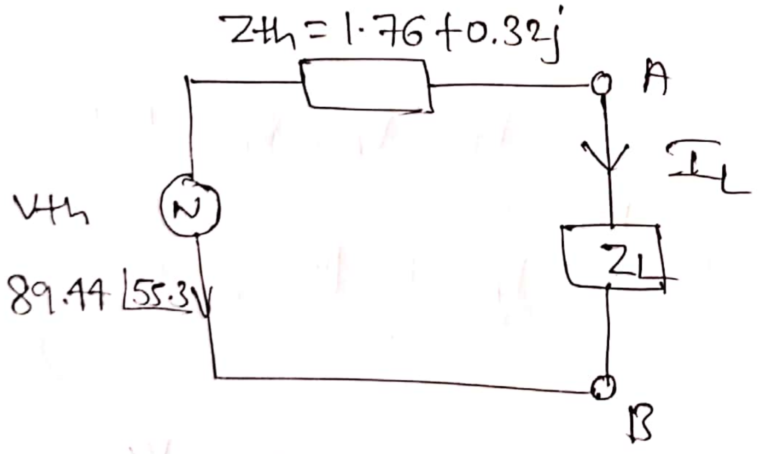
To find Z_{th} :-



$$Z_{th} = \frac{2 \times (4+j8)}{2+4+j8}$$

$$Z_{th} = 1.76 + 0.32j$$

Thevenins equivalent N/w.



The power is maximum.

when $Z_L = Z_S^*$

$Z_L = Z_{th}^*$

$$Z_L = 1.76 - 0.32j\ \Omega$$

$$\therefore I_L = \frac{V_{th}}{Z_{th}+Z_L} = \frac{89.44\angle 55.3}{1.76+\cancel{0.32j}+1.76-\cancel{0.32j}}$$

$$I_L = 25.41 \angle 55.3 \ A$$

$\therefore$ Maximum power delivered is

$$P = I_L^2 \times R_L$$

$$= 25.41 \angle 55.3 \times 1.76$$

$$P = \underline{1.136 \ KW}$$

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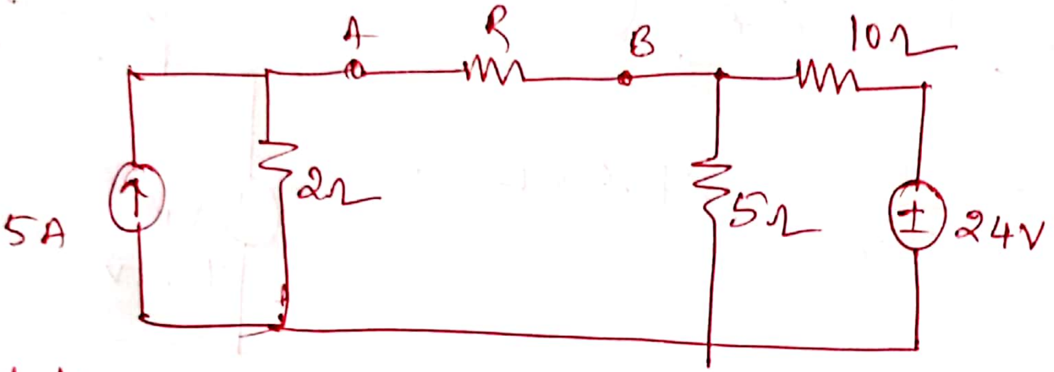
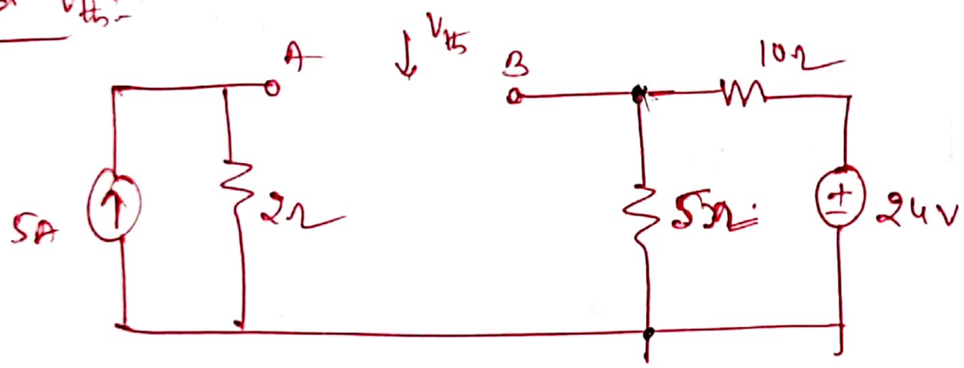
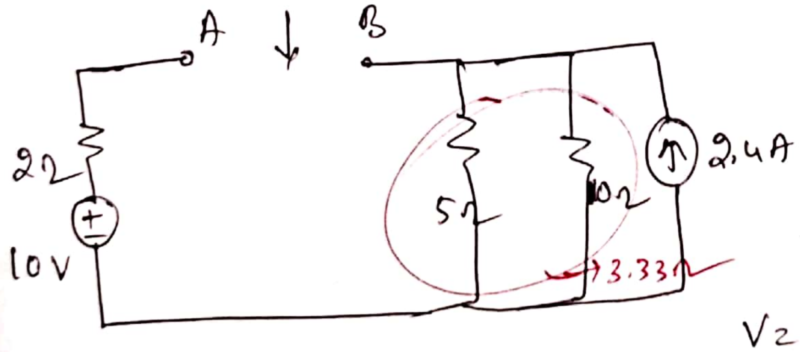
2) What should be the value of R such that Max power transfer can take place from the rest of the n/w to R. Obtain the amount of this power.



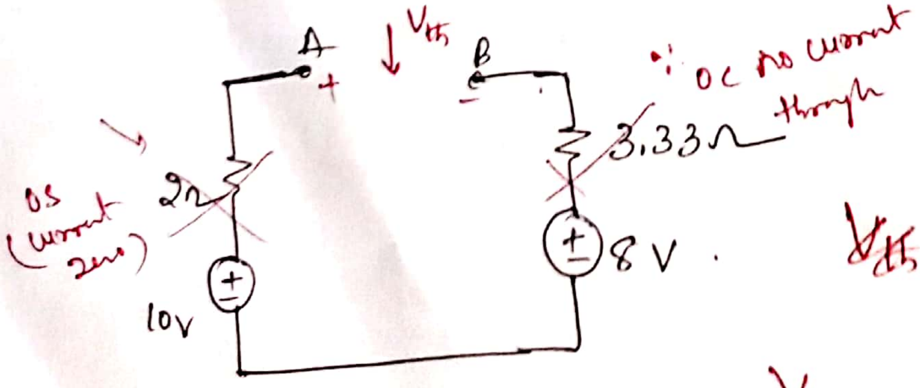
To find V_{th}:-





$$\frac{10 \times 5}{10+5} = \frac{50}{15}$$

$V = 2.4 \times 3.33$



$\cancel{V_{th}/2/-10}$

$$V_{th} - 10 + 8 = 0$$

$$\boxed{V_{th} = 2V}$$

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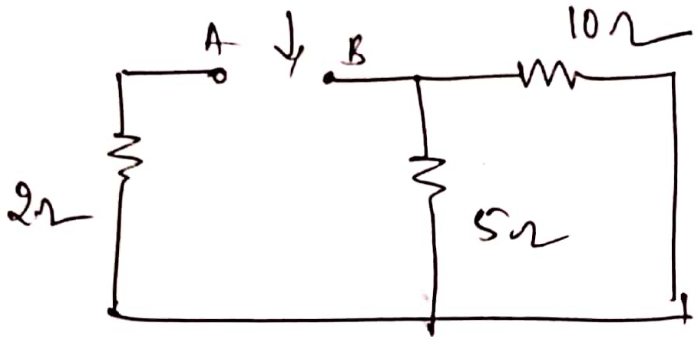
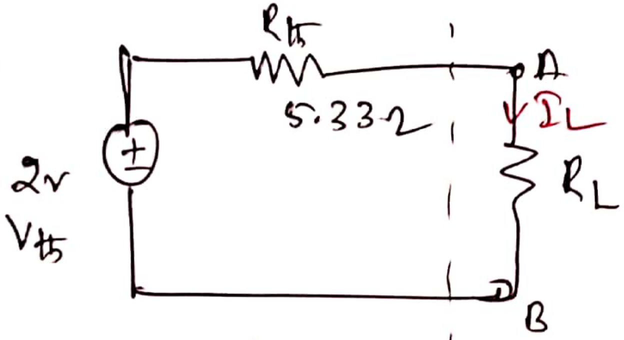
To find R_{th} :-



$$R_{th} = 2 + \frac{(10 \times 5)}{10+5}$$

$$\boxed{R_{th} = 5.33\,\Omega}$$

∴



Th Equivalent ckt

Condition for Max power transfer theorem.

$R_S = R_L$ (Here R_{th} is R_s, V_{th} is V_s)

$\therefore R_L = 5.33\,\Omega$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{2}{5.33 + 5.33}$$

$$\boxed{I_L = 0.188\,A}$$

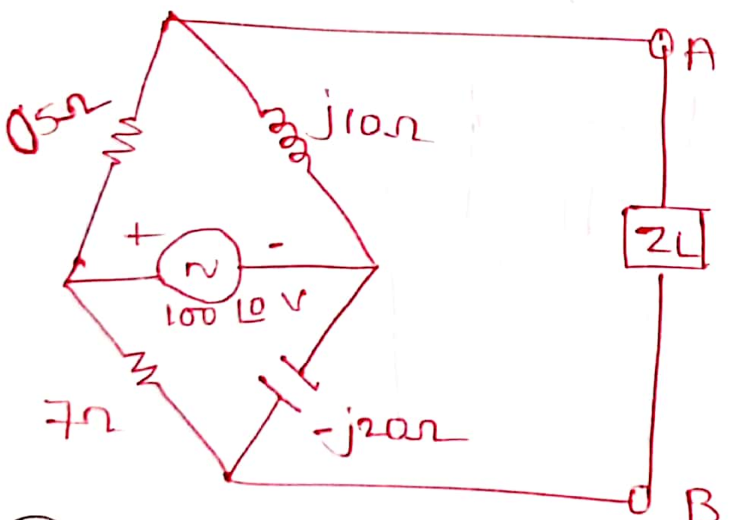
$\therefore P = I_L^2 \times R.$

$P = (0.188)^2 \times 5.33$

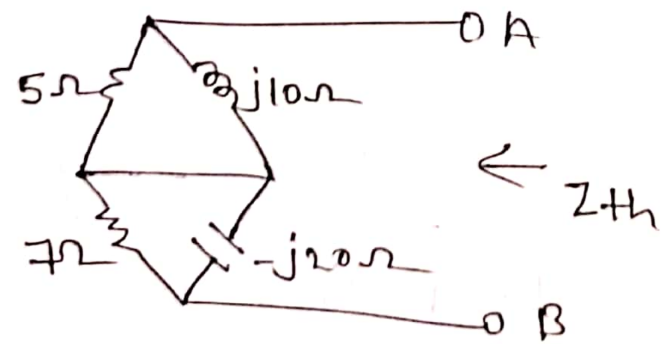
$P = 0.188$ Watts.

Jan 6 Mks

3) Find the Value of Z_L for which Maximum power is transfered to the Load ZL form the N/w in the below fig.



Remove the Load & Create open CKt b/w A & B & find out Zth.



$$Z_{th} = \left(\frac{5 \times j10}{5 + j10}\right) + \left(\frac{7 \times -j20}{7 - j20}\right)$$

$$Z_{th} = 10.24 - 0.183j \; \Omega = Z_S$$

Power is Maximum.

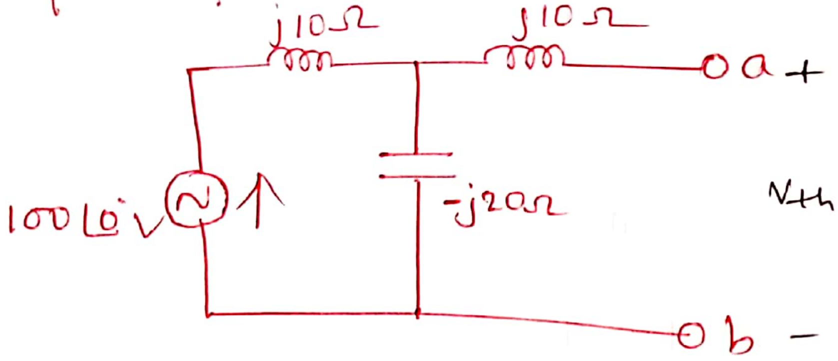
when $Z_L = Z_S^*$

$$Z_L = 10.24 + 0.183j \; \Omega$$

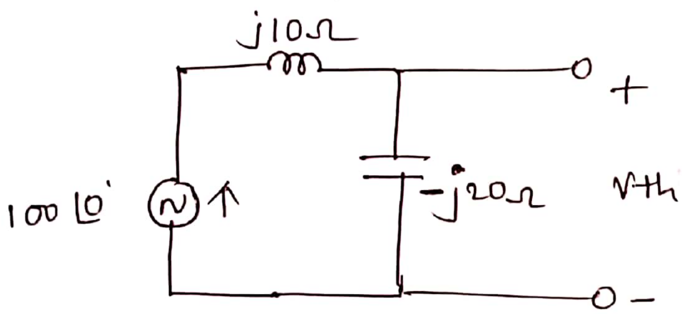
Ref 12

March 00
Jan 07 } 10MKS
Jan 10

4) What should be the value of a pure resistance to be connected across the terminals a & b in the ckt of below fig. so that maximum power is transferred to the Load. What is the max power?



To find Vth:



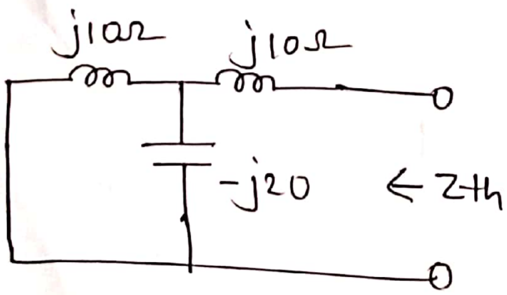
No current flows through j10Ω

$$\therefore V_{th} = \frac{100\angle 0^\circ \times (-j20)}{j10 - j20}$$

$$= 200 + 0j$$

$$= 200\angle 0 \text{ V}$$

To find Zth:

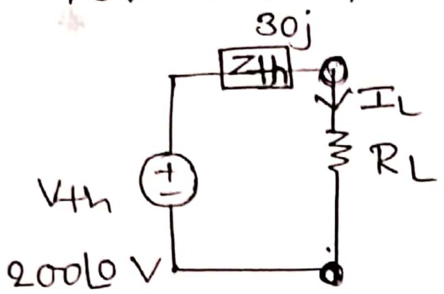


$$Z_{th} = [j10 \| -j20] + j10$$

$$= \frac{j10 \times -j20}{j10 - j20} + j10$$

$$= 30j = 30\angle 90^\circ\ \Omega$$

Tevenins equivalent N/W:



Power is maximum

When $R_L = |Z_{th}|$

$= |j30|$

$= 30\,\Omega$

$$I_L = \frac{V_{th}}{Z_{th} + R_L}$$

$$= \frac{200\angle 0}{30j + 30}$$

$$I_L = 4.714\angle{-45^\circ}\,A$$

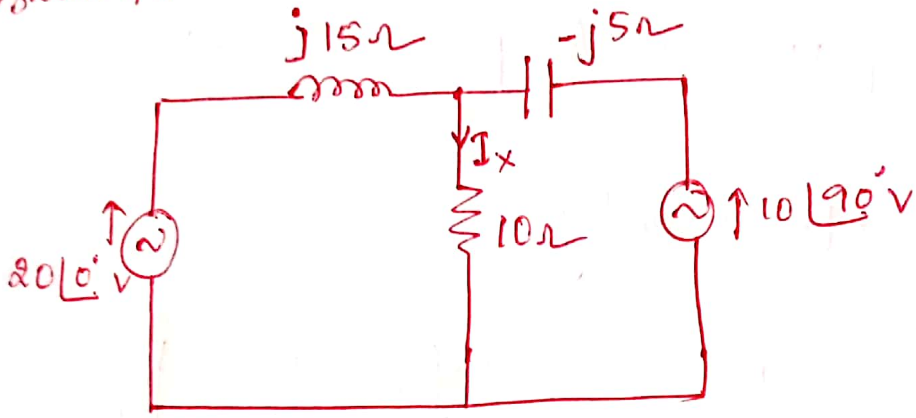
Max. Power is

$$P = I_L^2 R_L$$

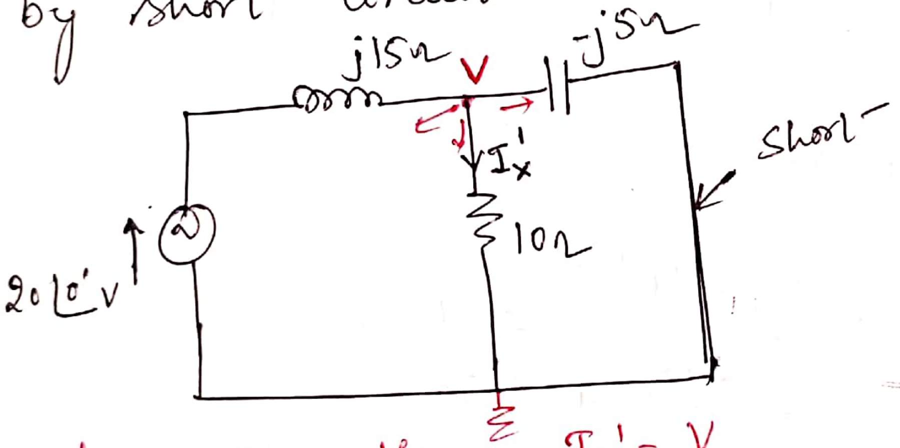
$$= (4.714)^2(30)$$

$$P = 666.653\,W$$

1) Using Superposition theorem determine the current flowing through 10Ω resistance of the n/w shown.



Case 1: Consider only $20\angle 0°$ V & replace $10\angle 90°$ V by short circuit.



from the figure, $I_x' = \frac{V}{10}$

KCL @ node V,

$$\frac{V - 20\angle 0°}{j15} + \frac{V}{10} + \frac{V}{-j5} = 0$$

$$-j0.067V - 1.33\angle -90° + 0.1V + j0.2V = 0$$

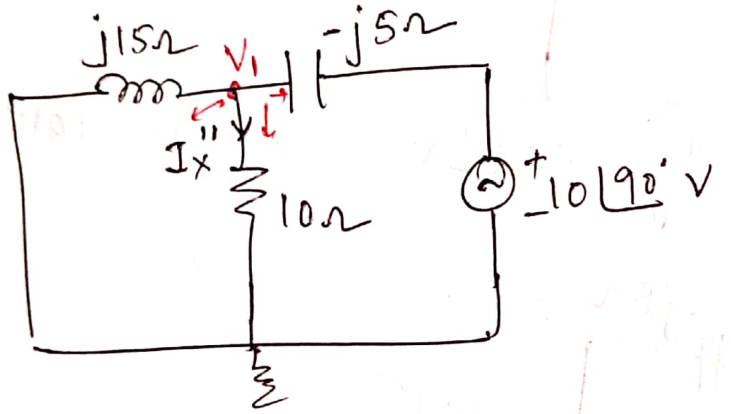
$$(0.1 + j0.133)V = 1.33\angle -90°$$

$$V = \frac{1.33\angle -90°}{0.166\angle 53.06°} = 8.01\angle -143.06°$$

$$I_x' = \frac{8.012 \angle -143.06}{10}$$

$$\boxed{I_x' = 0.8 \angle -143.06 \text{ Amp}} \quad \text{—— (1)}$$

Case 2: Consider $10\angle 90^\circ$ V only.



from the fig

$$I_x'' = \frac{V_1}{10}$$

KCL @ node V_1

$$\frac{V_1}{j15} + \frac{V_1}{10} + \frac{V_1 - 10\angle 90^\circ}{-j5 \to 5\angle -90^\circ} = 0$$

$$-j0.067 V_1 + 0.1 V_1 + j0.2 V_1 - 2\angle 180^\circ = 0$$

$$(0.1 + 0.133j) V_1 = 2\angle 180^\circ$$

$$V_1 = \frac{2\angle 180^\circ}{0.166\angle 53.13^\circ} = 12\angle 126.93$$

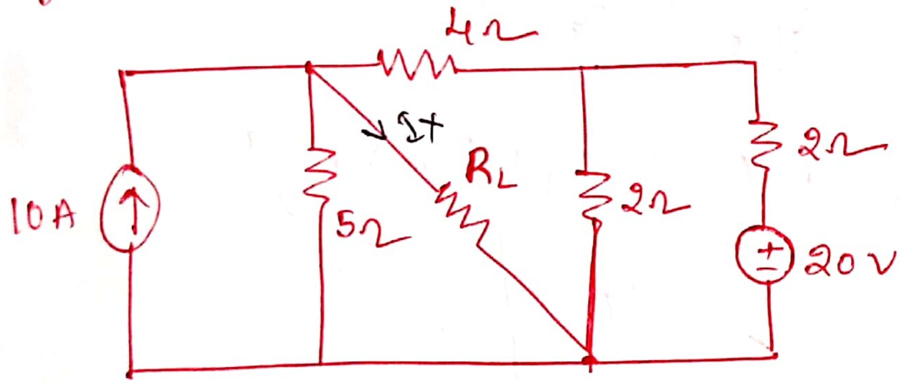
$$I_x'' = \frac{V_1}{10} = 1.2\angle 126.93 \text{ Amp} \quad \text{—— (2)}$$

$I_x = I_x' + I_x''$

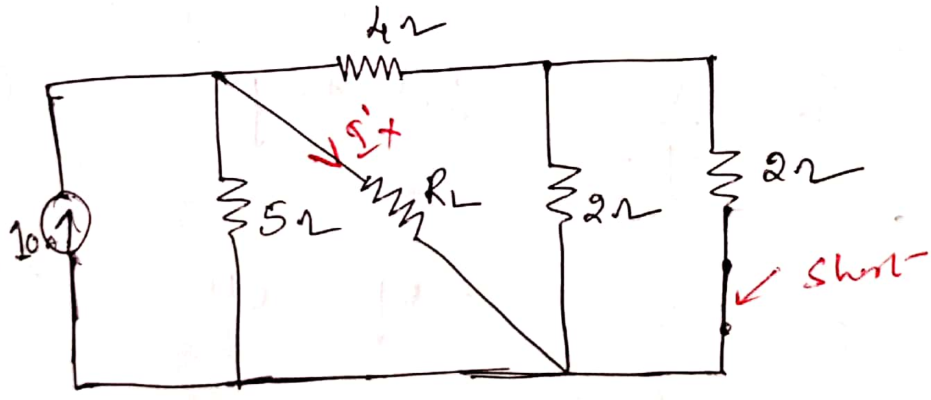
$I_x = 0.8 \angle -143.17 + 1.2 \angle 26.93$

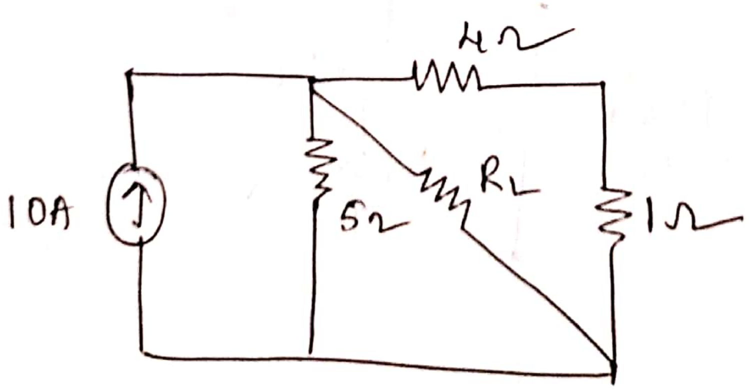
$$\boxed{I_x = 1.43 \angle 160^\circ \text{ Amp.}}$$

2) Using Superposition theorem, find the current through $R_L = 7.5\Omega$



Consider 10A source alone, short 20V source.





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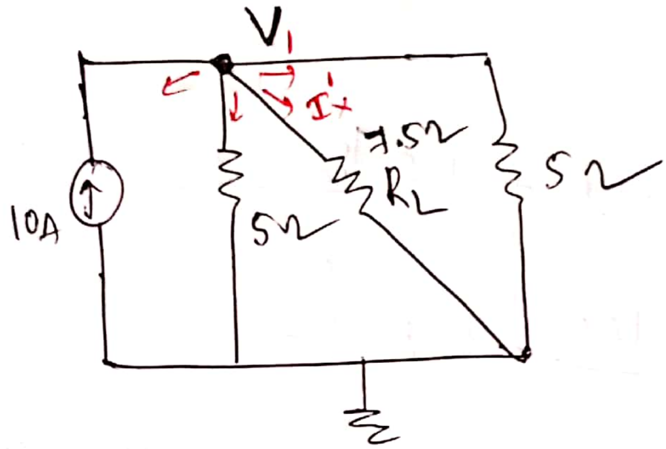


from the figure

$I_x' = \frac{V_1}{7.5}$

Apply KCL @ node V

$$-10 + \frac{V_1}{5} + \frac{V_1}{7.5} + \frac{V_1}{5} = 0$$

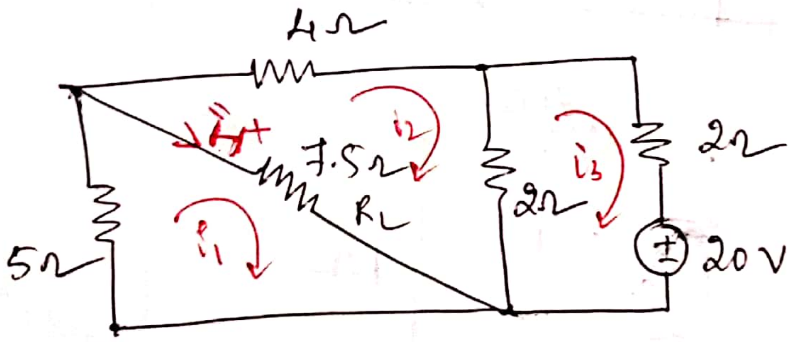
$$-10 + 0.2V_1 + 0.133V_1 + 0.2V_1 = 0$$

$$0.533V_1 = 10$$

$$V_1 = 18.76 \text{ volts}$$

$$\boxed{I_x' = \frac{V_1}{7.5} = 2.5 \text{ Amp}} \quad \text{—} \ (1)$$

Consider 20V src & open current source.



loop 1 $\rightarrow$ $-7.5(i_1 - i_2) - 5i_1 = 0$

$-12.5i_1 + 7.5i_2 + 0i_3 = 0$ —— (a)

loop 2 $\rightarrow$ $-4i_2 - 2(i_2 - i_3) - 7.5(i_2 - i_1) = 0$

$-4i_2 - 2i_2 + 2i_3 - 7.5i_2 + 7.5i_1 = 0$

$7.5i_1 - 13.5i_2 + 2i_3 = 0$ —— (b)

KVL to loop 3

$-2i_3 - 20 - 2(i_3 - i_2) = 0$

$-2i_3 - 20 - 2i_3 + 2i_2 = 0$

$0i_1 + 2i_2 - 4i_3 = 20$ —— (c)

$i_1 = -0.75A$ $i_2 = -1.25A$ $i_3 = -5.62$ amp

current through 7.5Ω $I_x'' = I_1 - I_2$

$I_x'' = -0.75 - (-1.25)$

$\boxed{I_x'' = 0.5 \text{ Amp}}$ —— (2)

$\therefore$ $I_x = I_x' + I_x''$

$\hookrightarrow = 2.5 + 0.5$

$\boxed{I_x = 3 \text{ Amp}}$

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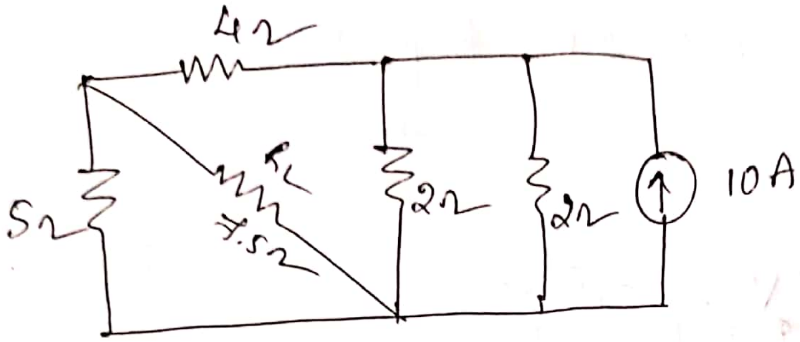
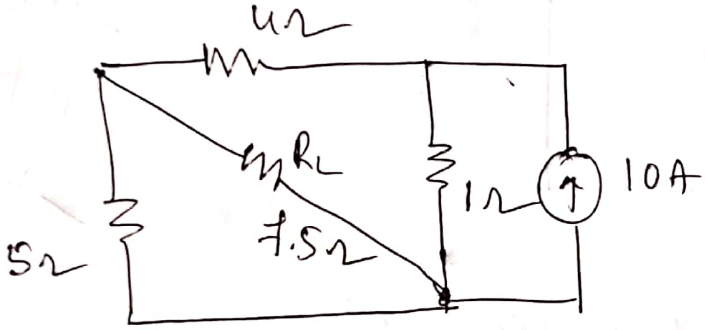
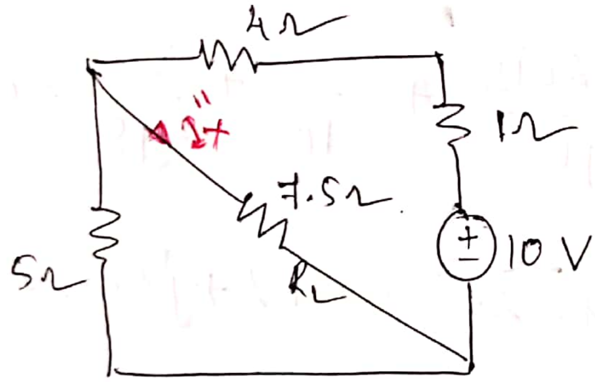
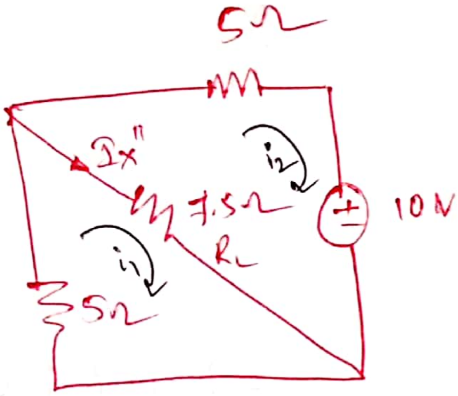








$$-7.5(i_1 - i_2) - 5i_1 = 0$$

$$-12.5i_1 + 7.5i_2 = 0 \quad \text{——} \ (1)$$

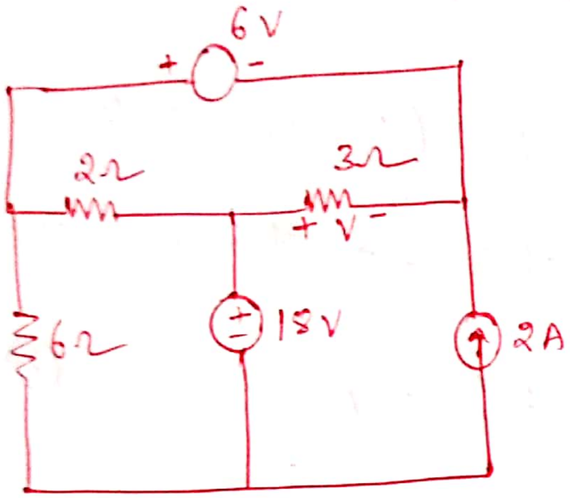
$$-5i_2 - 10 - 7.5(i_2 - i_1) = 0$$

$$7.5i_1 - 12.5i_2 = 10 \quad \text{——} \ (2)$$

$i_1 = \underline{\underline{-0.75}}$ amp $\quad i_2 = \underline{-1.25}$ amp $\quad \therefore I_x'' = i_1 - i_2$

$I_x'' = 0.5$ amp

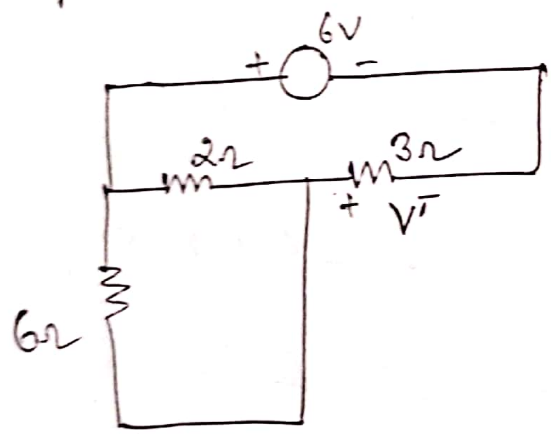
③ Find the voltage V across 3Ω resistor using Superposition theorem for the ckt shown.

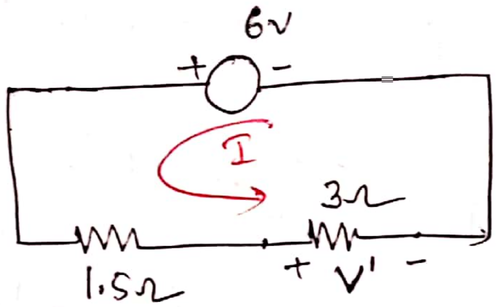


Case 1:

Consider 6V vg src, short ckt 18V src &

open ckt 2A src.





$$6 - 1.5I - 3I = 0$$

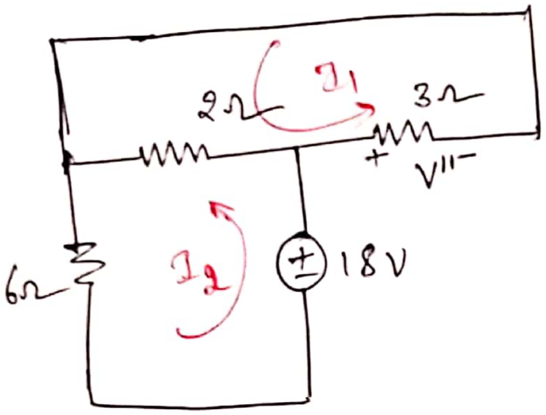
$$4.5I = 6$$

$$I = 1.33A$$

$$\therefore \boxed{V' = 3I = 4V} \quad \text{———} \quad ①$$

Case 2:

Consider 18V vg src



from the ckt

$V'' = 3I_1$

KVL to 1st loop

$-2(I_1 - I_2) - 3I_1 = 0$

$-2I_1 + 2I_2 - 3I_1 = 0$

$-5I_1 + 2I_2 = 0$ —— (a)

KVL to 2nd loop

$-2(i_2 - i_1) - 6i_2 + 18 = 0$

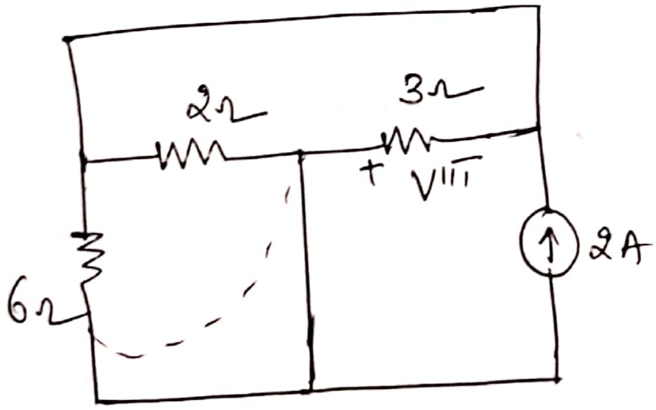
$2i_1 - 8i_2 = -18$ —— (b)

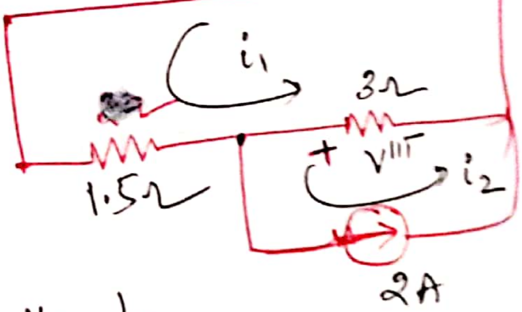
we get

$I_1 = 1A \qquad I_2 = 2.5A$

$\therefore \quad V'' = 3I_1 \qquad \boxed{V'' = 3V}$ —— (2)

Case 3 :- Consider 2A current source





from the fig

$V''' = 3(i_1 - i_2)$

but $\boxed{i_2 = 2A}$

$$-1.5 i_1 - 3(i_1 - i_2) = 0.$$

$$-4.5 i_1 + 3 i_2 = 0$$

$$\cancel{+}4.5 i_1 = \cancel{+}3 \times 2$$

$$i_1 = \frac{6}{4.5} = \cancel{2\,\text{Amp}}\ 1.33 \text{ Amp}$$

$\therefore$ $\cancel{V''' = 3(2-3) =}$

$$V''' = 3(2 - 1.33)$$

$$\boxed{V''' = -2 \text{ volt}} \text{ ——— } (3)$$

$\therefore$ $V = V' + V'' + V''' = 9$ volts

$$V''' = 3(i_2 - i_1) = \cancel{3(1.33}$$

$$= 3(2 - 1.33)$$

$$= -2 \text{ volts}$$

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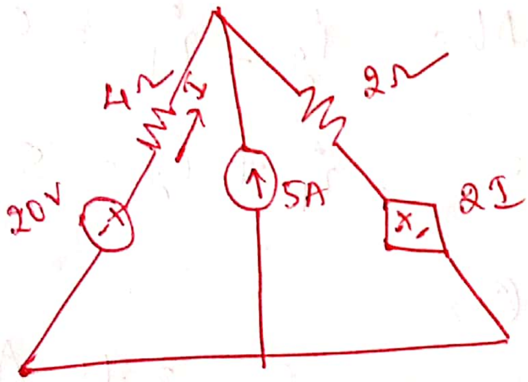
$\therefore \quad V = V' + V'' + V'''$

$V = 4 + 3 - 2$

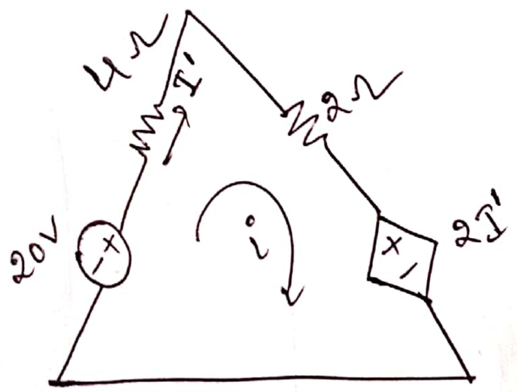
$\boxed{V = 5 \text{ volts}}$

5) For the ckt shown in below fig, find the current I using superposition theorem.



Case i)
Consider 20V src, open ckt 5A current src & keep dependent src as it is,

KVL to loop



$20 - 4i - 2i - 2I' = 0$

from the fig $\boxed{i = I'}$

$20 - 6i - 2i = 0$

$8i = 20 \qquad i = 2.5 \text{ Amp}$

$\therefore \boxed{I' = 2.5 \text{ amp}}$ —— (A)

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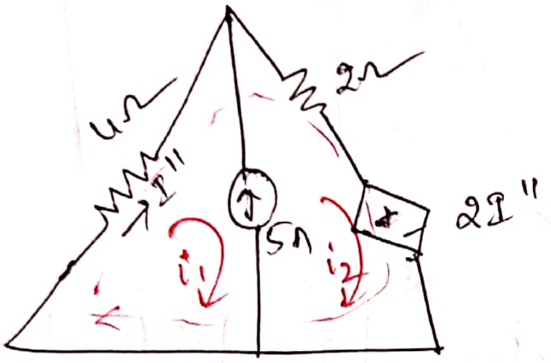
Case ii)

Consider 5A current src & short 20V vg src

Supermesh



$$-4i_1 - 2i_2 - 2I'' = 0$$

from fig $I'' = i_1$

$$-4i_1 - 2i_2 - 2i_1 = 0$$

5A is b/w 1st & 2nd loop hence Supermesh.

$$-6i_1 - 2i_2 = 0 \quad \text{--- (1)}$$

Also $i_2 - i_1 = 5$

or $-i_1 + i_2 = 5$ --- (2)

Solve (1) & (2),

$I_1 = -1.25A$ & $i_2 = 3.75$ Amp

$\therefore$ $\boxed{I'' = -1.25 \text{ Amp}}$ --- (B)

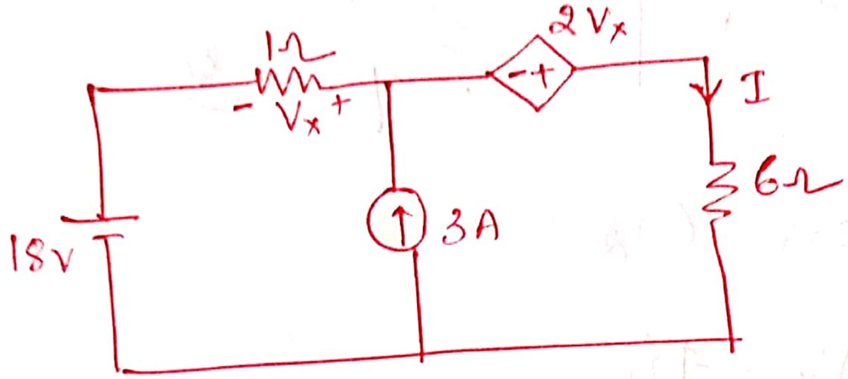
$\therefore$ from Superposition theorem.

$I = I' + I''$

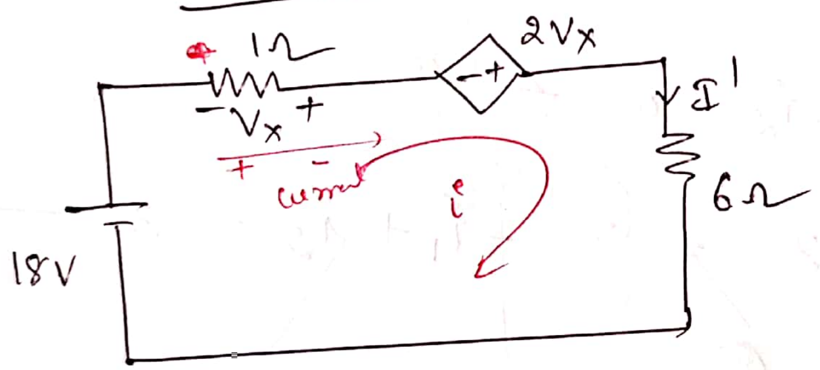
$I = 2.5 - 1.25$

$\boxed{I = 1.25 \text{ Amp}}$

6) Using Superposition theorem find the current in 6Ω resistor in the n/w shown.



case i) Consider 18V src



$$18 - 1i + 2V_x - 6i = 0$$

from the fig $V_x = -1i$

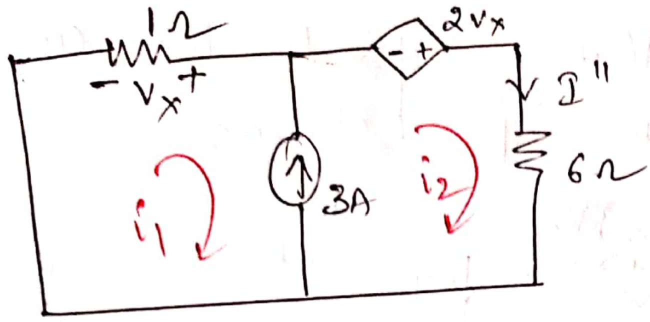
$$18 - i + 2(-1i) - 6i = 0$$

$$-9i = -18$$

$$\boxed{i = 2A}$$

$\therefore$ $\boxed{I' = i = 2\,Amp}$ — (a)

case ii) Consider 3A current src



3A in b/n 1st & 2nd loop hence Supermesh

$$-1i_1 + 2V_x - 6i_2 = 0$$

but $V_x = -1i_1$

$$-i_1 + 2(-1i_1) - 6i_2 = 0$$

$$-3i_1 - 6i_2 = 0 \quad \text{—— (1)}$$

& $i_2 - i_1 = 3 \implies -i_1 + i_2 = 3 \quad \text{—— (2)}$

solve (1) & (2)

$\boxed{i_1 = -2A}$ $\boxed{i_2 = 1A}$

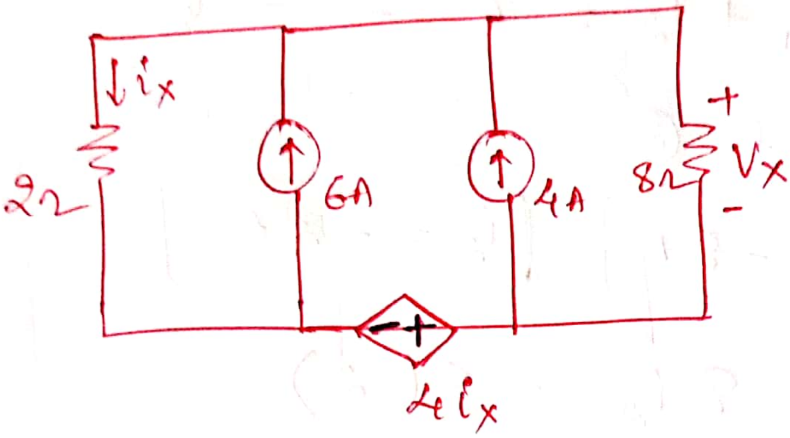
$\therefore \boxed{I'' = i_2 = 1A} \quad \text{—— (b)}$

from Superposition theorem.

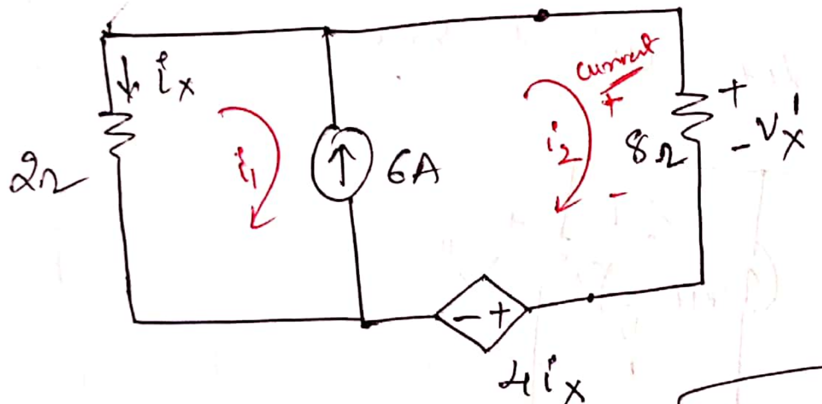
$I = I' + I''$

$I = 2 + 1 = 3$ Amp

7) Use Super position theorem to find V_x in the ckt shown below.



Case i) Consider 6A current src. Open ckt 4A src



from the fig $\boxed{i_x = -i_1}$

$$\boxed{V_x' = 8\, i_2}$$

& 6A src is b/n 1st & 2nd loop ∴ Supermesh

$$-2i_1 - 8i_2 - 4i_x = 0$$

$$-2i_1 - 8i_2 - 4(-i_1) = 0$$

$$2i_1 - 8i_2 = 0 \quad \text{——— (1)}$$

& $i_2 - i_1 = 6$

or $-i_1 + i_2 = 6$ —— (2)

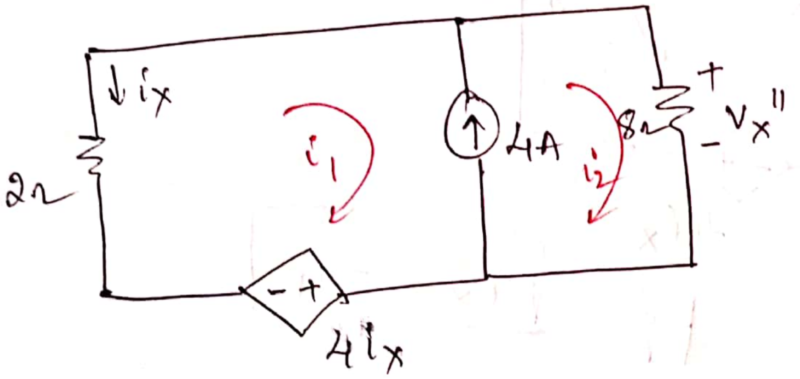
Solve (1) & (2)

$\boxed{i_1 = -8A}$ $\boxed{i_2 = -2A}$

$\therefore V_x' = 8 i_2 = 8(-2)$

$\boxed{V_x' = -16 \text{ volts}}$ —— (a)

Case ii) Consider 4A current src



from the fig $i_1 = -i_x \Rightarrow \boxed{i_x = -i_1}$

& $\boxed{V_x'' = 8 i_2}$

→ 4A Source lies b/w 1st & 2nd loop hence Super mesh

from the fig $i_2 - i_1 = 4 \rightarrow -i_1 + i_2 = 4$ —— (1)

Supermesh (KVL) $-8i_2 - 4i_x - 2i_1 = 0$

$-8i_2 - 4(-i_1) - 2i_1 = 0$

$2i_1 - 8i_2 = 0$ —— (5)

Solve (1) & (2) $\boxed{i_1 = -5.33 \text{ Amp}}$

$\boxed{i_2 = -1.33 \text{ Amp.}}$

$\therefore \quad V_x'' = 8 \times -1.33$

$\boxed{V_x'' = -10.64 \text{ volts}}$ —— (6)

$\therefore \quad V_x = V_x' + V_x''$

$V_x = -16 - 10.64$

$\boxed{V_x = -26.64 \text{ volts}}$

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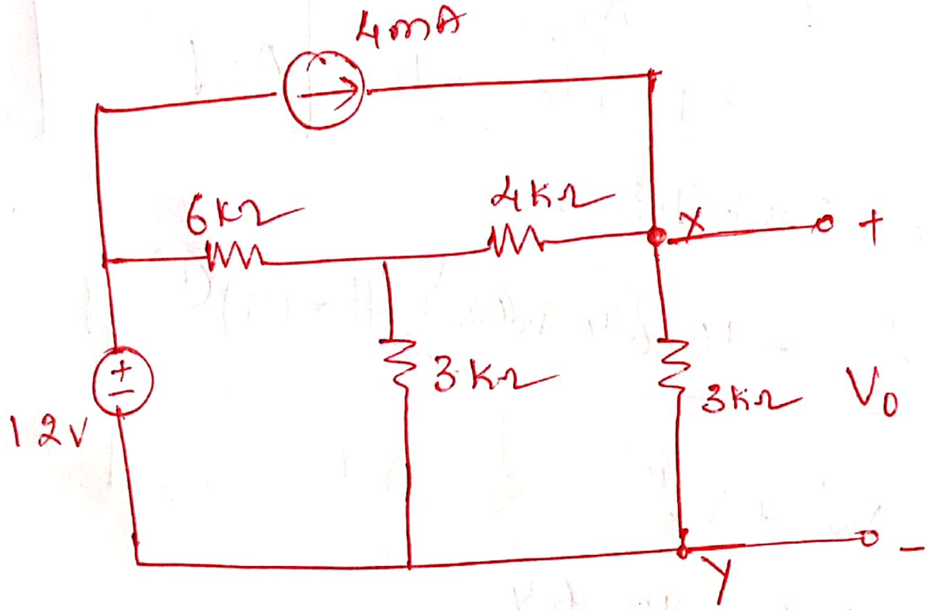
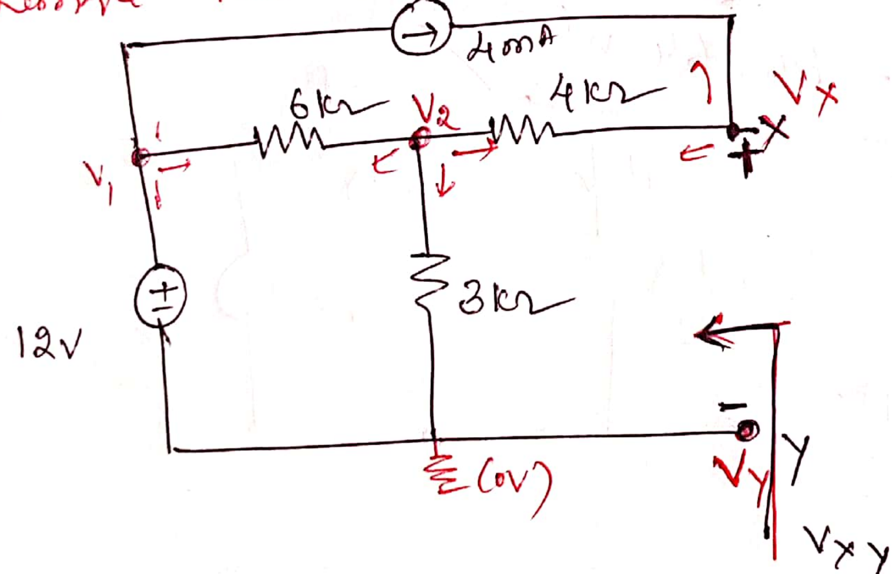
Thevenins theorem :-

1) Obtain the Thevenins equivalent of n/w shown in below fig b/n terminals X & Y. Also find V_0



Remove the load & create open ckt



V_{xy} which is nothing but V_{th}

$$V_{xy} = V_x - V_y$$

$$\boxed{V_{th} = V_{xy}}$$

KCL @ node 1

$$\boxed{V_1 = 12V}$$

KCL @ node 2

$$\frac{V_2 - V_1}{6k} + \frac{V_2}{3k} + \frac{V_2 - V_x}{4k} = 0$$

$$V_2\left[\frac{1}{6k} + \frac{1}{3k} + \frac{1}{4k}\right] - \frac{V_2}{6k} - \frac{V_x}{4k} = 0$$

$$0.75 \times 10^{-3} V_2 - 0.25 \times 10^{-3} V_x = 2 \times 10^{-3} \quad \text{--- (1)}$$

KCL @ node Vx

$$-4 \times 10^{-3} + \frac{V_x - V_2}{4k} = 0$$

$$-0.25 \times 10^{-3} V_2 + 0.25 \times 10^{-3} V_x = 4 \times 10^{-3} \quad \text{--- (2)}$$

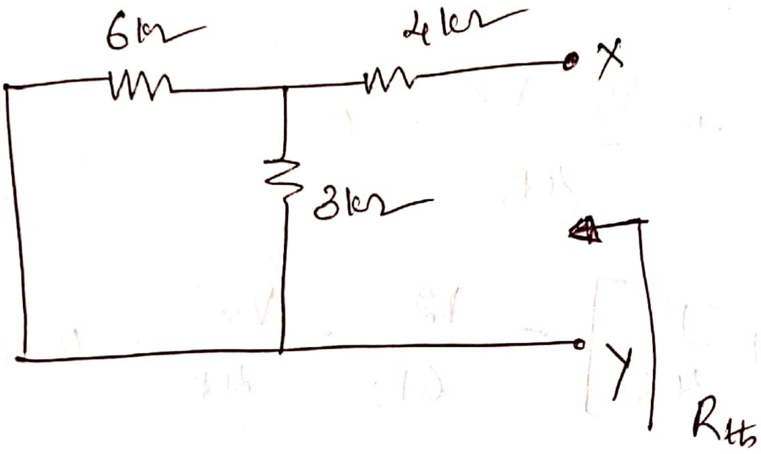
Solve (1) & (2)

$V_2 = 12$ volts $\qquad V_x = 28$ volts

And $\boxed{V_y = 0}$ $\because$ bottom node is grounded)

$V_{th} = V_{xy} = 28$ volts

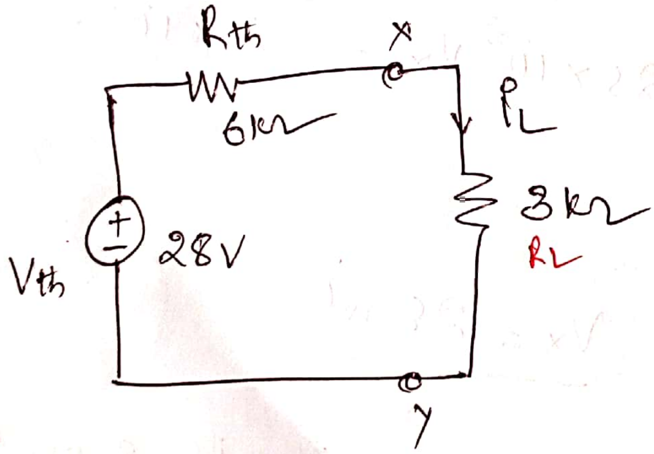
To find R_{th} :-



$$R_{th} = (6k \parallel 3k) + 4K$$

$$\boxed{R_{th} = 6\,K\Omega}$$

Thevenin's Equivalent ckt



$$i_L = \frac{V_{th}}{R_{th} + R_L}$$

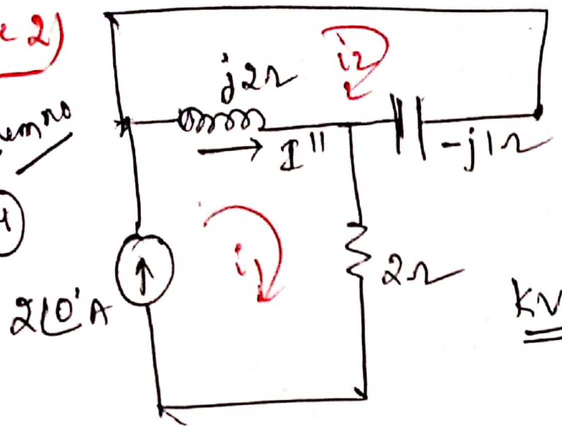
$$i_L = \frac{28}{6K + 3K}$$

$$\boxed{i_L = 3.11\,mA}$$

$\therefore \quad V_0 = I_L \times R_L = 9.33$ volts

Case 2)

Problem no (4)



$$\boxed{i_1 = 2\angle 0^\circ}$$

KVL $\quad j1(i_2) - j2(i_2 - i_1) = 0$

$$j1\,i_2 - j2\,i_2 + j2\,i_1 = 0$$

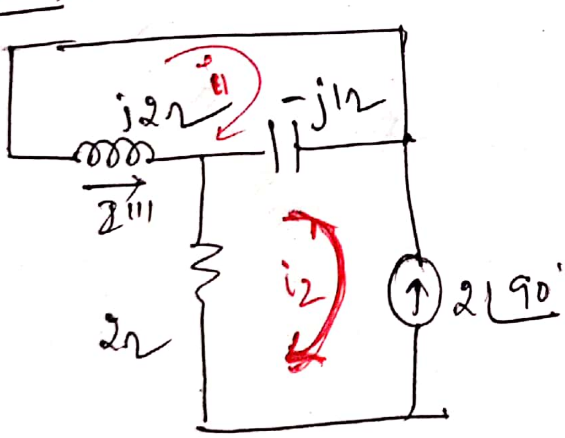
$$-\cancel{j}1\,i_2 = -\cancel{j}2\,i_1$$

$$\cancel{i_2 = 2}$$

$$i_2 = 2\times 2\angle 0^\circ = 4\angle 0^\circ$$

$$\therefore \boxed{I'' = i_1 - i_2 = 2 - 4 = -2\text{ Amp}}$$

Case iii) :-



$$\boxed{i_2 = -2\angle 90^\circ}$$

KVL : $\quad j1(i_1 - i_2) - j2\,i_1 = 0$

$$j1\,i_1 - j1\,i_2 - j2\,i_1 = 0$$

$$-j1\,i_1 = j1\,i_2$$

$$-i_1 = i_2 \qquad i_1 = -i_2 = 2\angle 90^\circ$$

$$\boxed{I''' = -i_1 = -2\angle 90^\circ}$$

$\therefore \quad I = I' + I'' + I'''$

$$I = 8\angle{-135^\circ} - 2 - 2\angle{90^\circ}$$

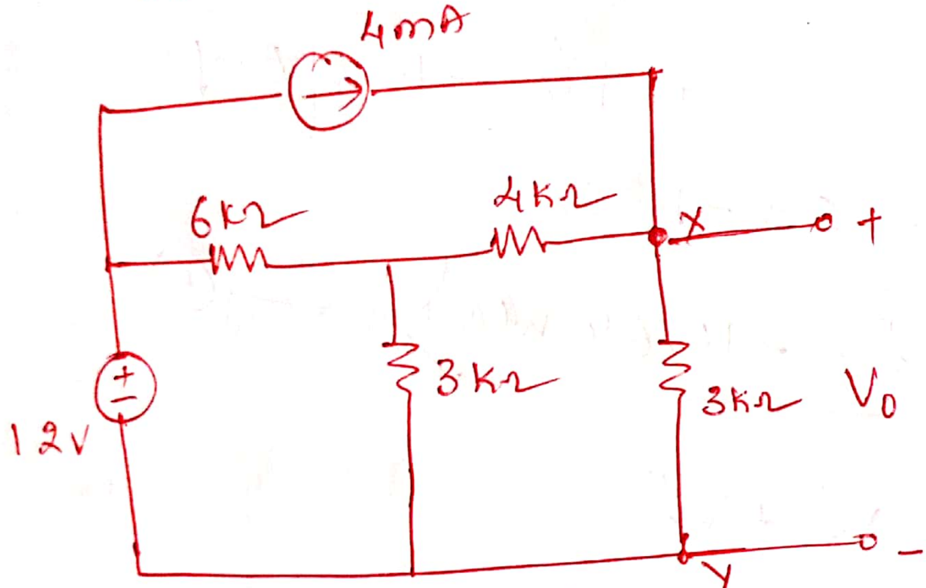
$$I = -5.65 - j5.65 - 2 - 2j$$

$$I = -7.65 - 7.65j$$

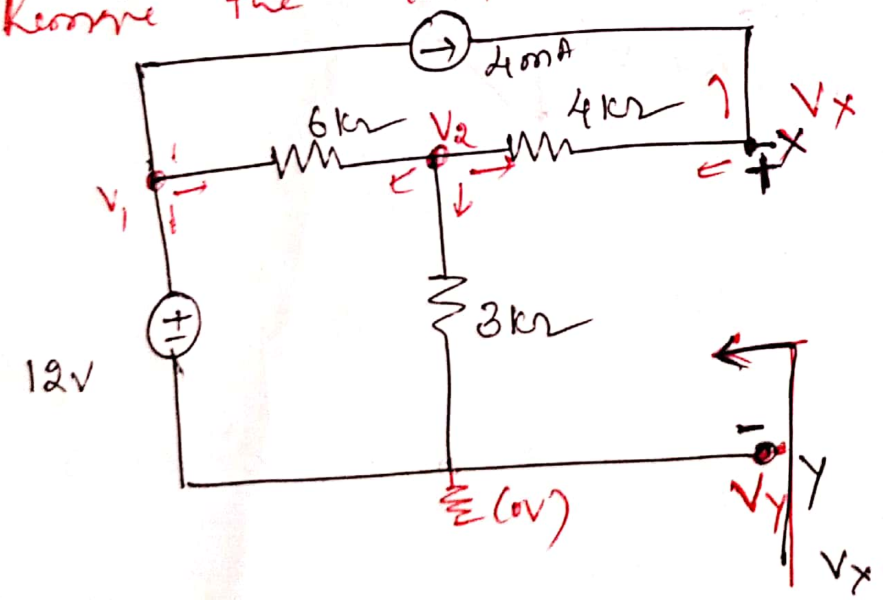
$$\boxed{I = 10.83\angle{-135^\circ} \text{ Amp}}$$

Thevenins theorem :-

1) Obtain the Thevenins equivalent of n/w shown in below fig b/n terminals X & Y. Also find V_0



Remove the load & create open ckt



V_{xy} which is nothing but V_{th}

$$V_{xy} = V_x - V_y$$

$$\boxed{V_{th} = V_{xy}}$$

KCL @ node 1

$$\boxed{V_1 = 12V}$$

KCL @ node 2

$$\frac{V_2 - V_1}{6K} + \frac{V_2}{3K} + \frac{V_2 - V_x}{4K} = 0$$

$$V_2\left[\frac{1}{6K} + \frac{1}{3K} + \frac{1}{4K}\right] - \frac{12}{6K} - \frac{V_x}{4K} = 0$$

$$0.75\times10^{-3}\,V_2 - 0.25\times10^{-3}\,V_x = 2\times10^{-3} \quad \text{——(1)}$$

KCL @ node Vx

$$-4\times10^{-3} + \frac{V_x - V_2}{4K} = 0$$

$$-0.25\times10^{-3}\,V_2 + 0.25\times10^{-3}\,V_x = 4\times10^{-3} \quad \text{——(2)}$$

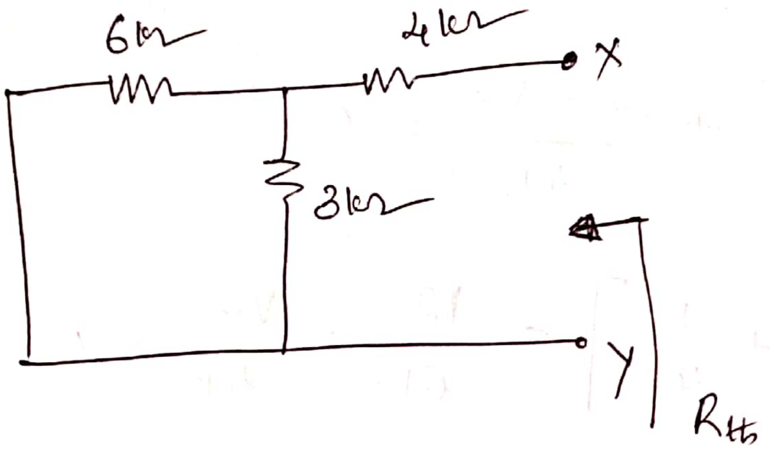
Solve (1) & (2)

$V_2 = 12$ volts $\qquad V_x = 28$ volts

And $\boxed{V_y = 0}$ $\because$ bottom node is grounded)

$$V_{th} = V_{xy} = 28 \text{ volts}$$

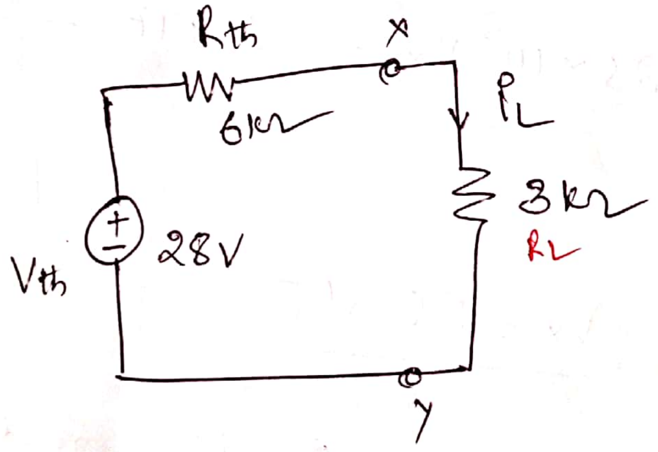
To find R_{th} :-



$$R_{th} = (6k \,||\, 3k) + 4k.$$

$$\boxed{R_{th} = 6\,k\Omega}$$

Thevenin's equivalent ckt



$$i_L = \frac{V_{th}}{R_{th} + R_L}$$

$$i_L = \frac{28}{6k + 3k}$$

$$\boxed{i_L = 3.11\,mA}$$

$$\therefore \; V_0 = I_L \times R_L = 9.33 \text{ volts}$$

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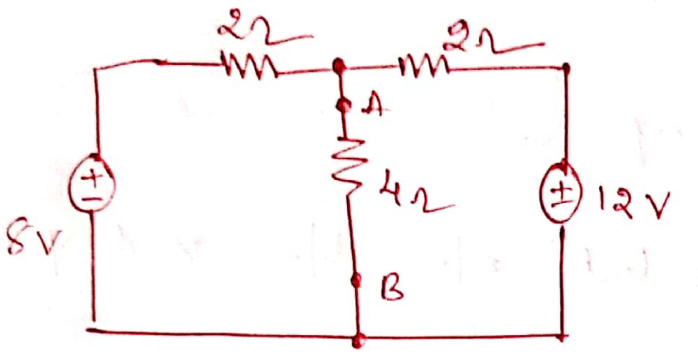
2) Find the Thevenins equivalent for the n/w at the load terminals A & B. If the load across A & B is 4Ω. Determine the load current.



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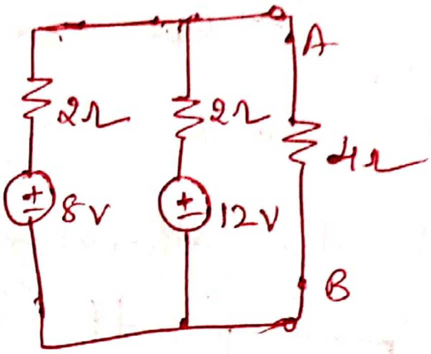
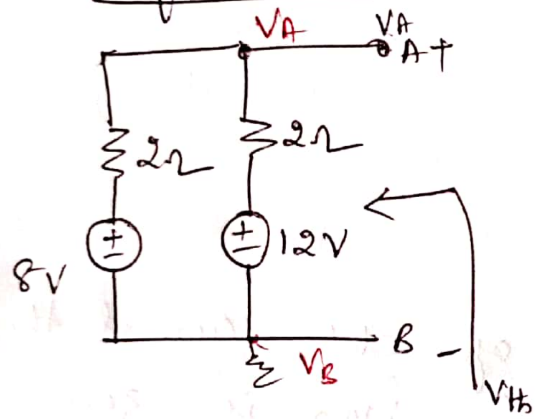


To find V_{th}



$$V_{th} = V_A - V_B$$

→ KCL @ node A

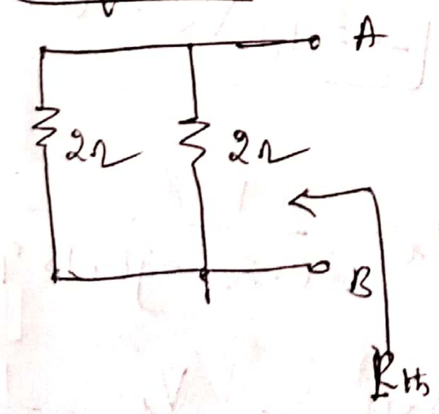
$$\frac{V_A - 8}{2} + \frac{V_A - 12}{2} = 0$$

$$0.5 V_A - 4 + 0.5 V_A - 6 = 0$$

$\boxed{V_A = 10 \text{ volts}}$ & $\boxed{V_B = 0}$

$\therefore$ $\boxed{V_{th} = 10 \text{ volts}}$

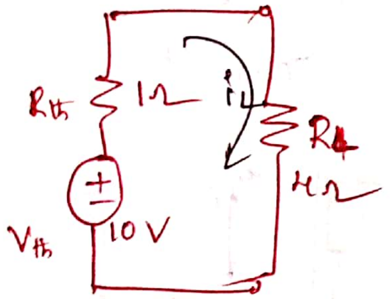
To find R_{th} :-



$$R_{th} = \frac{2 \times 2}{2+2} = 1\Omega$$

$\boxed{R_{th} = 1\Omega}$

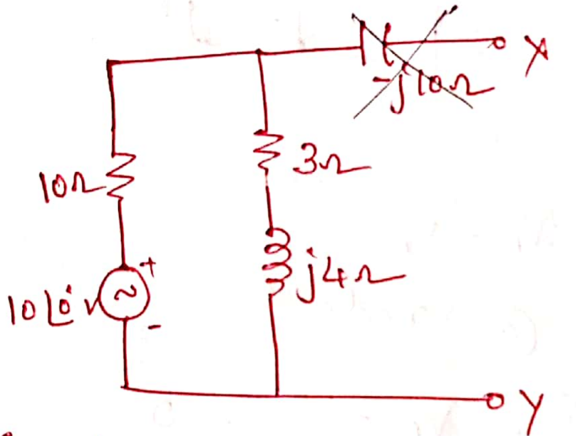
Thevenins ckt :-



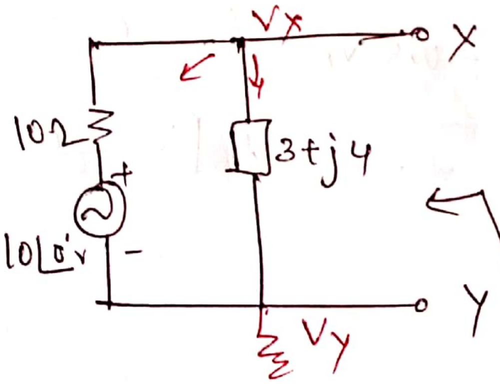
$$i_L = \frac{V_{th}}{R_{th}+R_L} = \frac{10}{4+1}$$

$$\boxed{i_L = 2\ \text{Amp}}$$

8) Obtain the thevenins equivalent n/w b/n X & Y



To find V_{th} :-



Since open ckt current through $-j10\Omega$ is zero

$$V_{th} = V_x - V_y$$

KCL

V_{th}

$$\frac{V_x - 10\angle 0^\circ}{10} + \frac{V_x}{3+j4} = 0$$

$$0.1V_x - 1\angle 0^\circ + 0.2\angle -53.13\ V_x = 0.$$

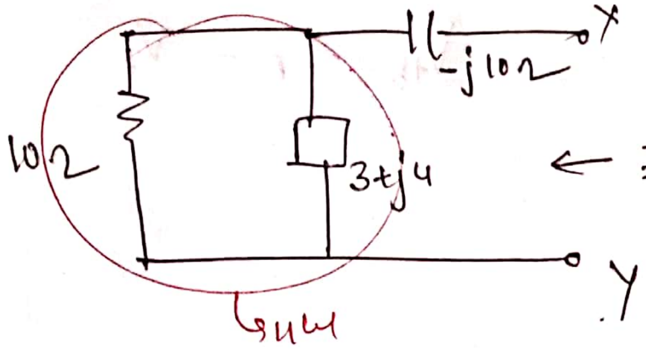
$$0.1V_x + (0.12 + j\,0.159)\,V_x = 1$$

$$(0.22 - j\,0.159)\,V_x = 1 \implies V_x = \frac{1}{0.22 - j0.159}$$

$$V_x = \frac{1}{0.27\angle -35.8^\circ}$$

$$\boxed{V_{th} = 3.676\angle 36.03\ \text{volts}}$$

To find Z_{th} :-



$\leftarrow Z_{th}$

$Z_1 = 10 \,||\, (3+j4)$

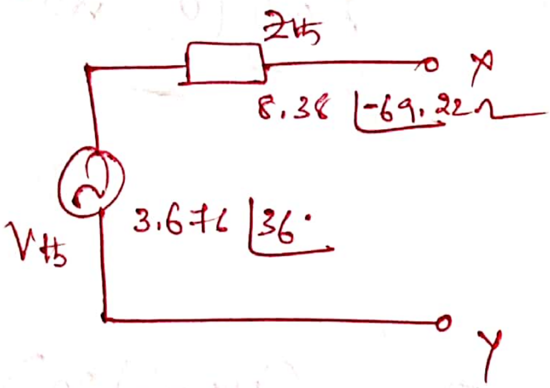
$$Z_1 = \frac{10 \times (3+j4)}{10+3+j4}$$

$$Z_1 = \frac{30+j40}{13+j4} =$$

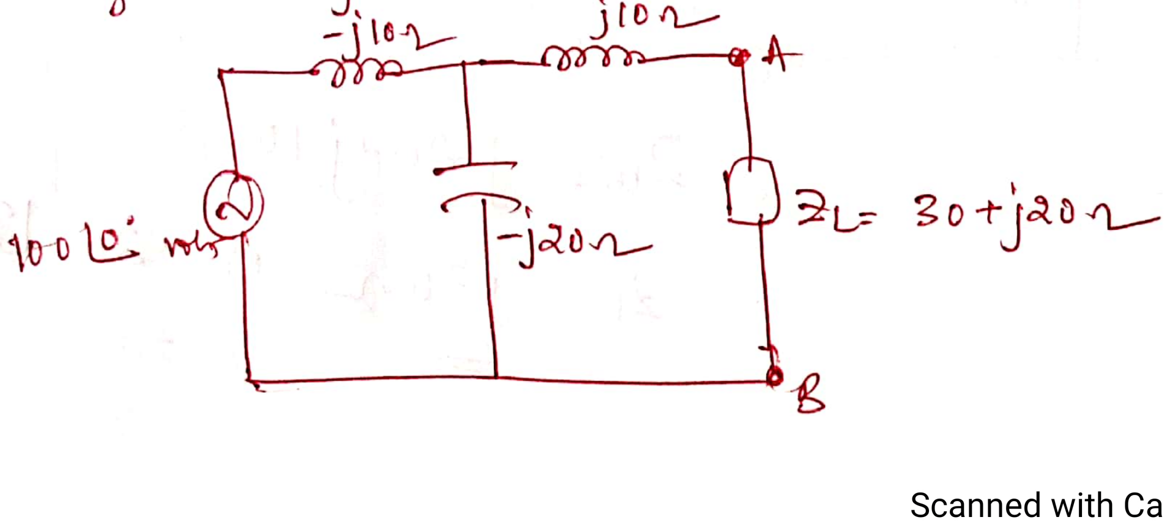
$Z_{th} = -j10 +$

$$\boxed{Z_{th} = 8.38 \angle -69.22\,\Omega}$$

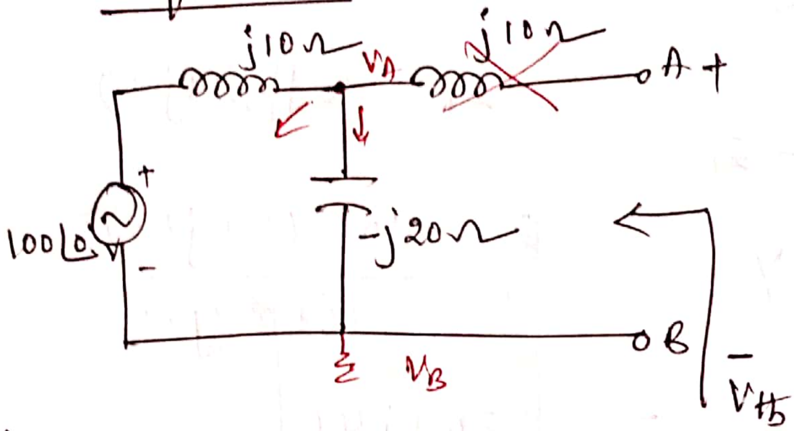
Thevenin's equivalent ckt



4) Using Thevenin's theorem find the current flowing through $30+j20\Omega$ in the ckt shown.



To find V_{th} :



$V_{th} = V_A - V_B$

KCL:
@ A

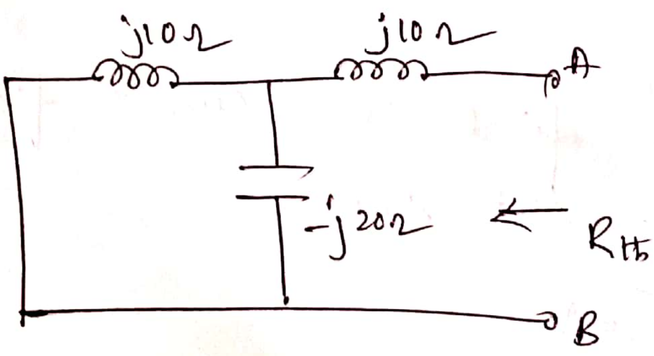
$$\frac{V_A - 100\angle 0^\circ}{j10 \rightarrow 10\angle 90^\circ} + \frac{V_A}{-j20} = 0$$

$$-j0.1V_A - 10\angle -90^\circ + j0.05V_A = 0$$

$$-j0.05V_A = 10\angle -90^\circ$$

$$V_A = \frac{10\angle -90^\circ}{0.05\angle -90^\circ} = 200\angle 0^\circ \text{ volts}.$$

To find R_{th} or Z_{th} :



$$Z_{th} = \left(\frac{j10 \times -j20}{j10 - j20}\right) + j10$$

$$Z_{th} = j20 + j10$$

$$Z_{th} = j30\,\Omega \text{ or } 30\angle 90^\circ\,\Omega$$

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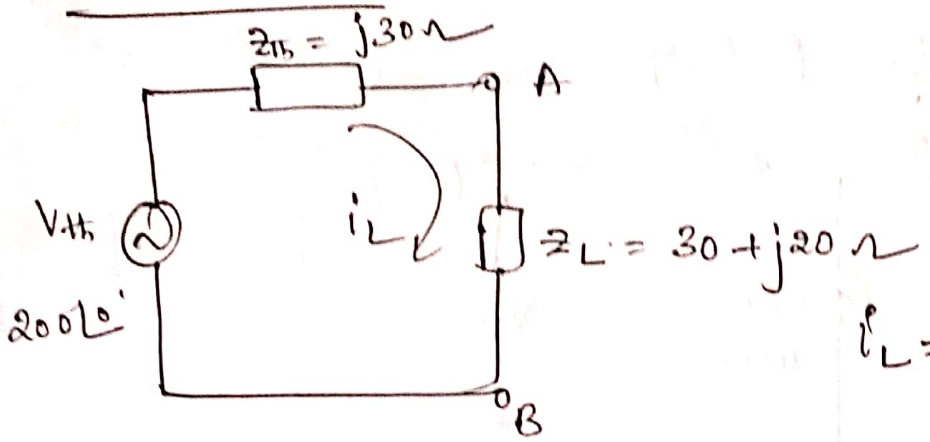
Thevenin's n/w



$$i_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$$i_L = \frac{200\angle 0^\circ}{j30 + 30 + j20}$$

$$\boxed{i_L = 3.43\angle -59^\circ \text{ Amp}}$$

Note :- If the given n/w consists of some dependent sources, then these dependent sources must be kept as it is while calculating Z_{th} & should not be shorted or open ckt'd. whether it is voltage or current source.

In such cases,

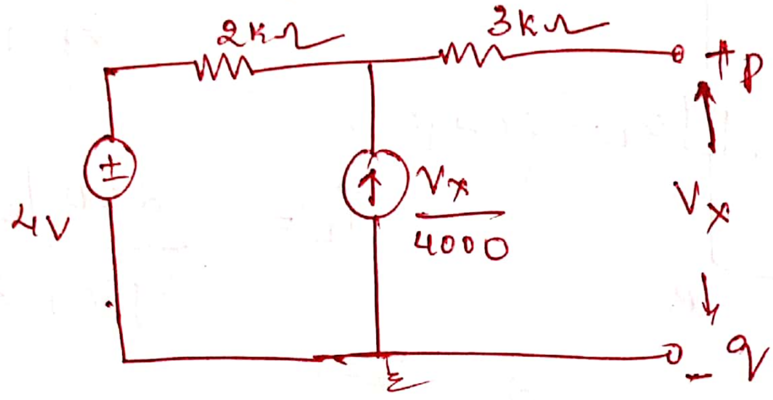
$$\boxed{Z_{th} = \frac{V_{th}}{I_{sc}}}$$

where,

$I_{sc} \rightarrow$ Short ckt current obtained by shorting the load terminals.

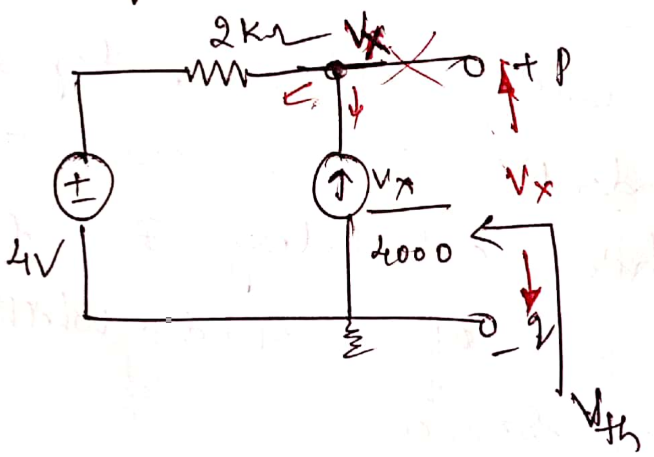
$V_{th} \rightarrow$ Thevenin's vg or open ckt vg across load terminals.

1) For the n/w shown, obtain the Thev equivalent across terminals p & q.



To find V_{th} :-

Since no current flows through 3kΩ



$V_{th} = V_x$

$V_x = V_p - V_q$ ($\because V_q = 0$)

$V_x = V_p$

KCL :-

$$\frac{V_x - 4}{2k} + \frac{V_x}{4000} = 0$$

$$\frac{V_x - 4}{2000} = \frac{V_x}{4000}$$

$$4000V_x - 16000 = 2000V_x$$

$$\boxed{V_x = 8V}$$

$$\therefore \boxed{V_{th} = 8V}$$

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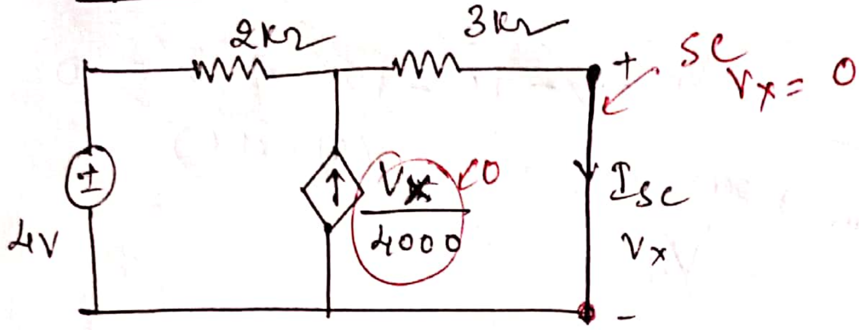
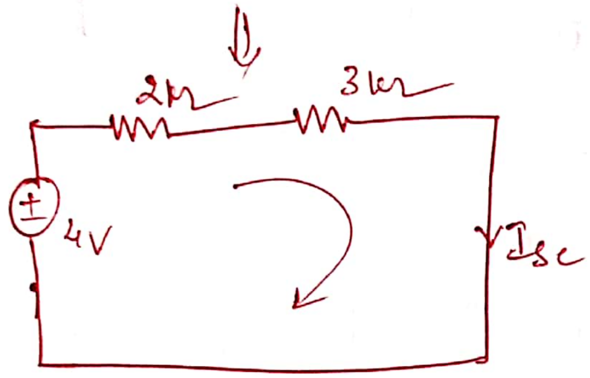
To find Z_{th} :



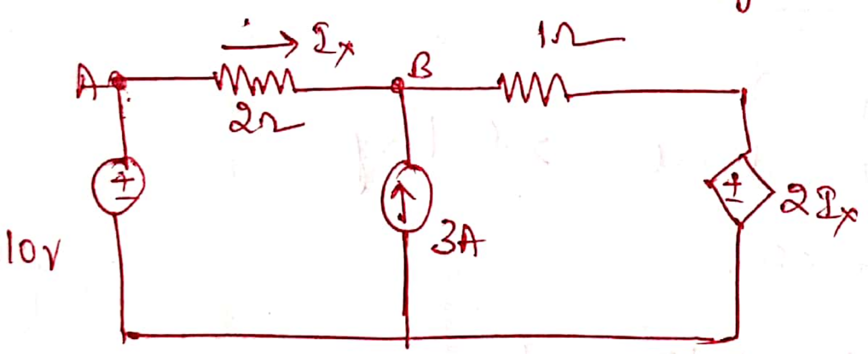


$$I_{sc} = \frac{4}{2k+3k} = 0.8\,mA$$

$$\therefore\ Z_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{0.8\,mA}$$

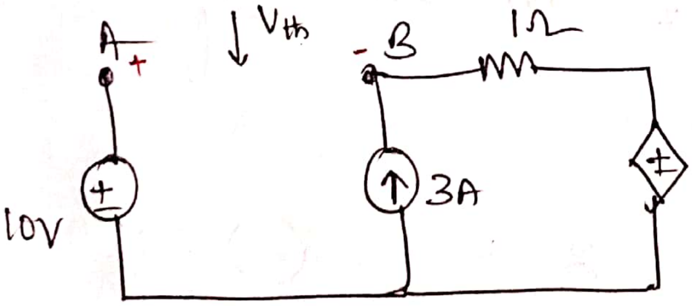
$$Z_{th} = 10\,k\Omega$$

2) Obtain the current I_x by T. theorem



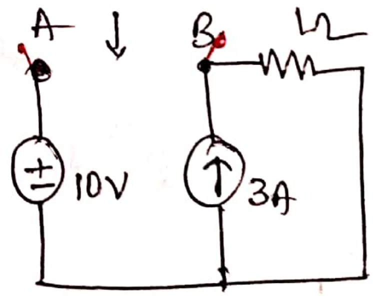
To find V_{th} :-

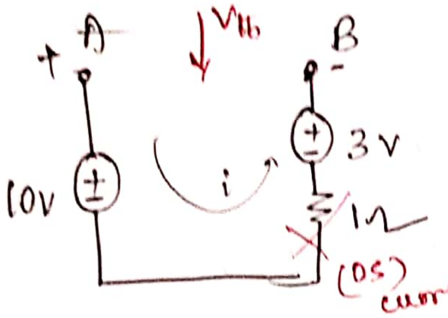
Remove 2Ω resistance. open circuit $I_x = 0$



$2I_x = 2\times 0$
$= 0V$

≅





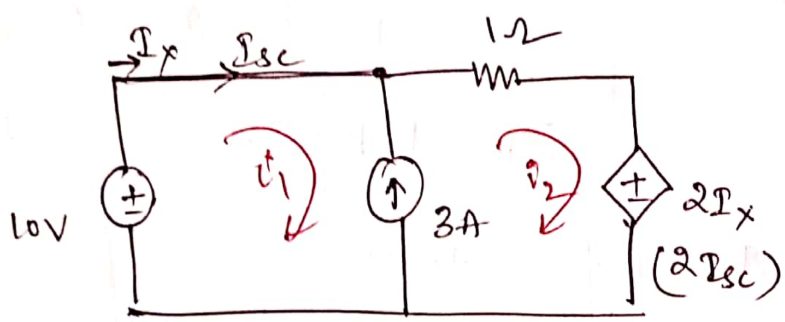
$\cancel{V_{th} = -10 - 1 \cdot i + 3 = 0}$ (open ckt)

$\cancel{V_{th} =}$

$$V_{th} - 10 + 3 = 0$$

$$\boxed{V_{th} = 7\,V}$$

To find R_{th} :-



from the fig

$I_x = I_{sc}$.

Also, $I_{sc} = I_1$

→ 3A is in b/n 1st & 2nd loop.

∴ Supermesh.

$$-i_2 - 2I_{sc} + 10 = 0$$

$$-i_2 - 2i_1 + 10 = 0.$$

or $-2i_1 - i_2 = -10$ ——— (1)

And $i_2 - i_1 = 3$

or $-i_1 + i_2 = 3$ ——— (2)

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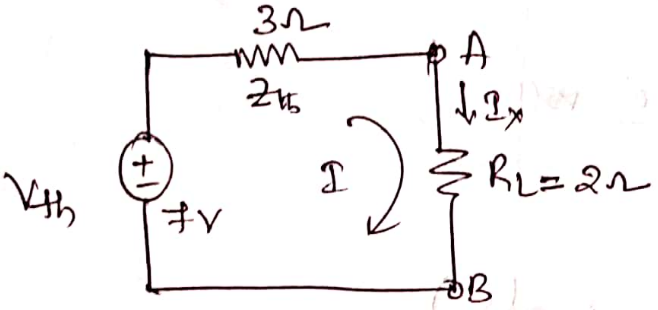
Solve ① & ②

$$\boxed{I_1 = 7/3\ A} \quad \boxed{I_2 = 16/3\ \text{Amp}}$$

i.e. $I_{sc} = I_1 = 7/3\ A$.

$$\therefore\ Z_{th} = \frac{V_{th}}{I_{sc}} = \frac{7}{7/3} = 3\Omega$$

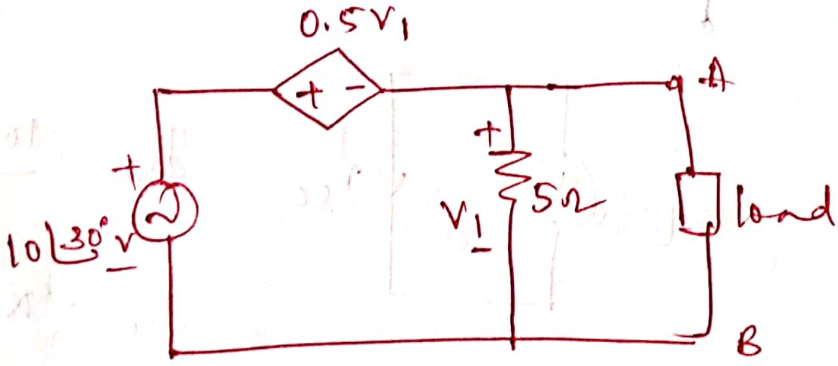


$$I = \frac{V_{th}}{Z_{th} + R_L}$$

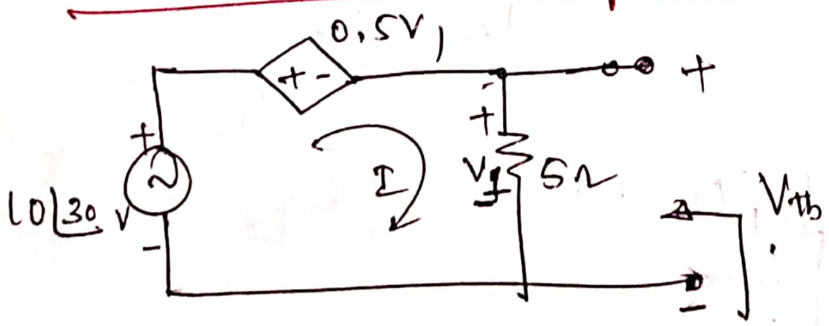
$$I = \frac{7}{3+2} = 1.4\ \text{Amp}$$

$$\Rightarrow \boxed{I_x = 1.4\ \text{Amp}}$$

3) Find the Th. equivalent ckt across load.



Remove the load to find V_{th} :-



from the fig

$$V_{th} = V_1 = 5I.$$

KVL $\quad -0.5V_1 - V_1 + 10\angle 30 = 0.$

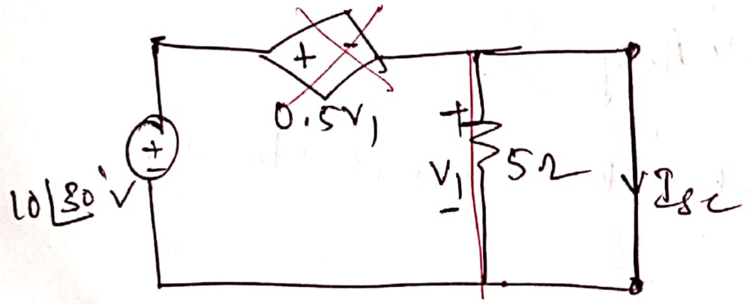
$$-1.5V_1 = -10\angle 30^\circ$$

$$V_1 = 6.67\angle 30^\circ \text{ volts}$$

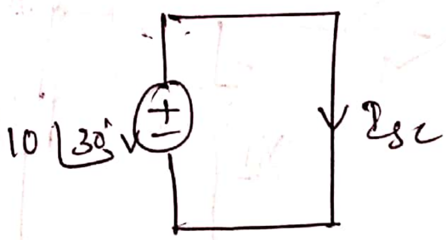
$$\therefore \boxed{V_{th} = 6.67\angle 30^\circ \text{ volts}}$$

To find R_{th}

(S.C the load terminals)



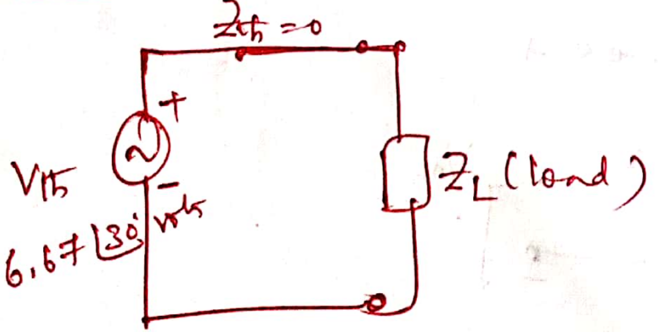
As there is a SC across 5Ω, $V_1 = 0$



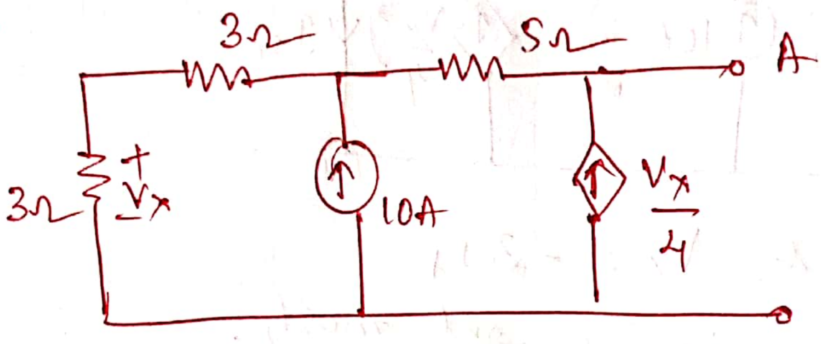
$$I_{sc} = \frac{10\angle 30^\circ}{0}$$

$$I_{sc} = \infty$$

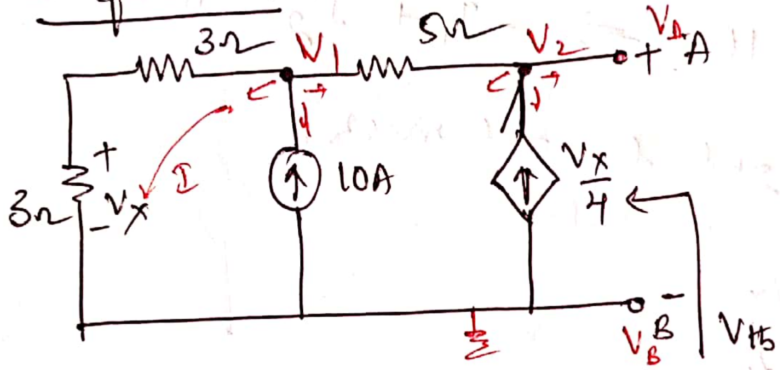
Th. Eq ckt:- $\quad \therefore Z_{th} = \frac{V_{th}}{I_{sc}} = 0\Omega$



4) Find the The eq of the n/w shown in fig



To find V_{th} :-



$V_{th} = V_A - V_B$

$\boxed{V_{th} = V_2}$

KCL @ V_1 :- $\frac{V_1}{6} - 10 + \frac{V_1 - V_2}{5} = 0$

$0.367 V_1 - 0.2 V_2 = 10$ —— ①

KCL @ V_2 $\frac{V_2 - V_1}{5} - \frac{V_x}{4} = 0$

But $V_x = 3I$ where $I = \frac{V_1}{6}$

$V_x = 3 \times V_1/6 = V_1/2 = 0.5 V_1$

$0.2 V_2 - 0.2 V_1 - \frac{0.5 V_1}{4} = 0$

$-0.325 V_1 + 0.2 V_2 = 0$ —— ②

$\boxed{V_1 = 238.1V}$ $\boxed{V_2 = 386.9 \text{ volts}}$ $\therefore \boxed{V_{th} = 386.9 \text{ volts}}$

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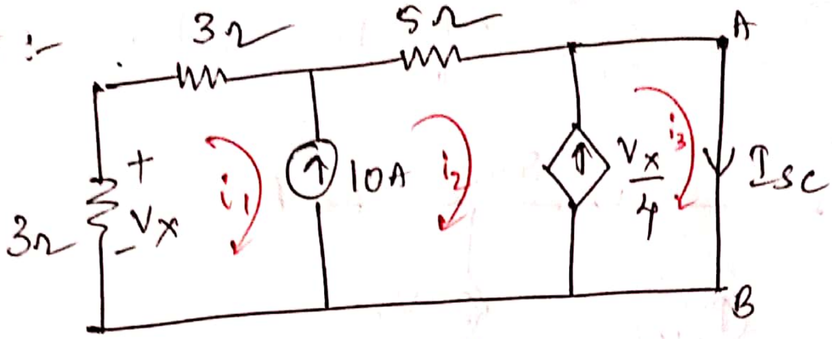
To find R_{th} :-



from the fig

$I_{sc} = i_3$ & $V_x = -3I_1$

→ 10A is b/n 1st & 2nd mesh.

$I_2 - I_1 = 10 \Rightarrow -I_1 + I_2 = 10$ —— ①

→ $\frac{V_x}{4}$ is b/n 2nd & 3rd mesh.

$i_3 - i_2 = \frac{V_x}{4}$

$i_3 - i_2 = -3I_1/4$

$0.75 i_1 - i_2 + i_3 = 0$ —— ②

∴ mesh ①, ② & ③ forms supermesh.

$-3I_1 - 5i_2 - 3i_1 = 0$

$-6i_1 - 5i_2 = 0$ —— ③

Solve ①, ② & ③. $\boxed{I_1 = -4.545A}$

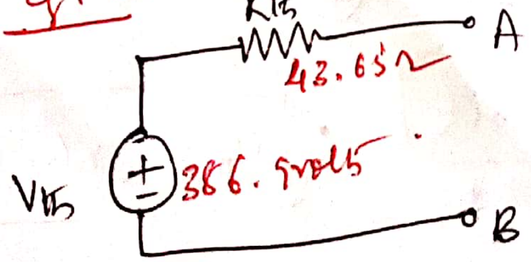
$\boxed{I_2 = 5.45}$ $\boxed{I_3 = 8.864A}$

$\boxed{I_{sc} = 8.864 \text{ Amp}}$

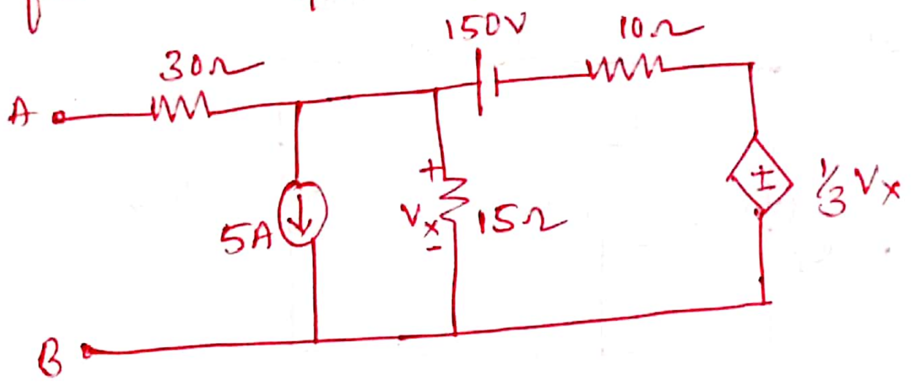
$Z_{th} = \frac{V_{th}}{I_{sc}} = \frac{386.9}{8.864}$

$\boxed{Z_{th} = 43.65Ω}$

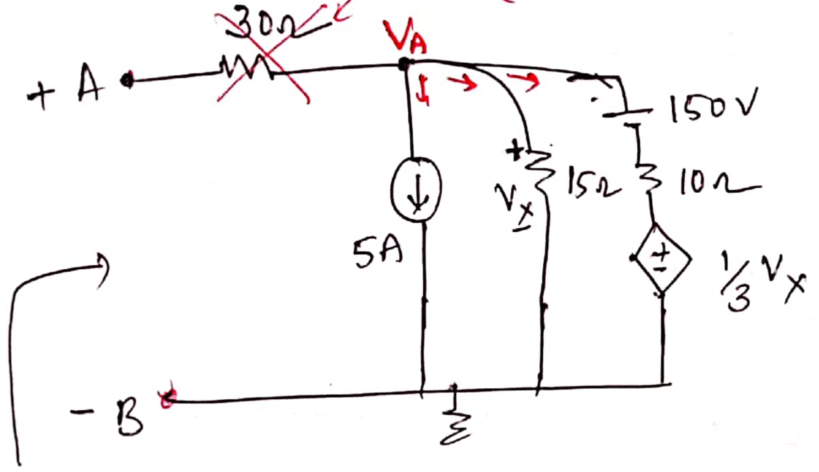
Th eq^t ckt :-



5) Calculate Thevenin's equivalent ckt across AB for the n/w shown below.



To find V_{th}:



$$V_{th} = V_A - V_B$$

KCL @ A:

$$5 + \frac{V_A}{15} + \frac{V_A - 150 - 0.33V_x}{10} = 0$$

from the fig $V_x = V_A$

$$5 + 0.066\,V_A + 0.1\,V_A - 15 - 0.033\,V_A = 0.$$

$$0.133\,V_A = 10$$

$$V_A = 75.18 \text{ volts}$$

$$\boxed{V_{th} = 75.18 \text{ volts}}$$

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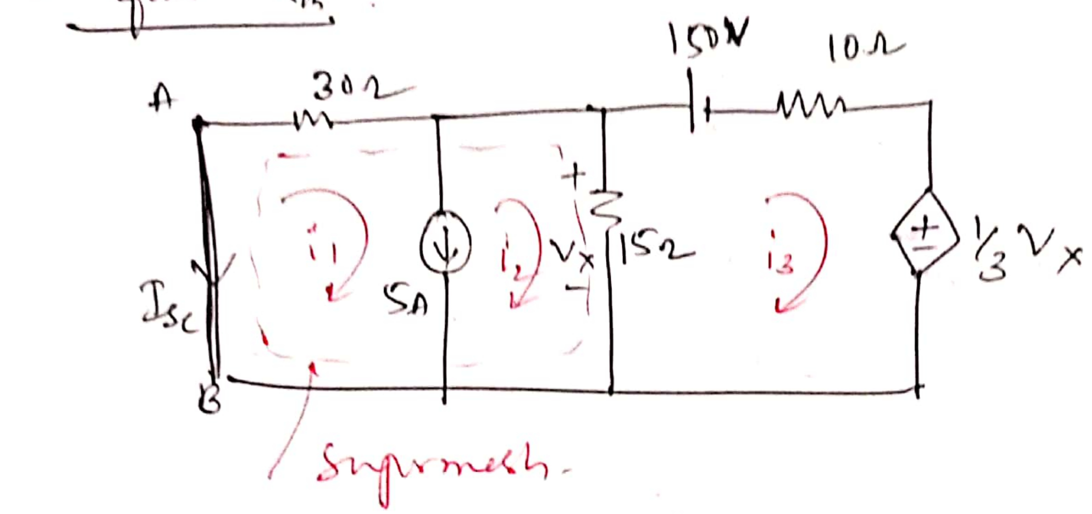
To find R_{th} :



from the supermesh

$$i_1 - i_2 = 5 \quad \text{———} \quad (1)$$

→ KVL to supermesh

$$-30i_1 - 15(i_2 - i_3) = 0$$

$$-30i_1 - 15i_2 + 15i_3 = 0 \quad \text{———} \quad (2)$$

→ KVL to 3rd loop.

$$-150 - 10i_3 - 0.33V_x - 15(i_3 - i_2) = 0$$

$$-150 - 10i_3 - 0.33V_x - 15i_3 + 15i_2 = 0$$

from the fig. $V_x = 15(i_2 - i_3)$

$$-150 - 25i_3 - \cancel{0.33} \times \cancel{15}^{\,5}(i_2 - i_3) + 15i_2 = 0.$$

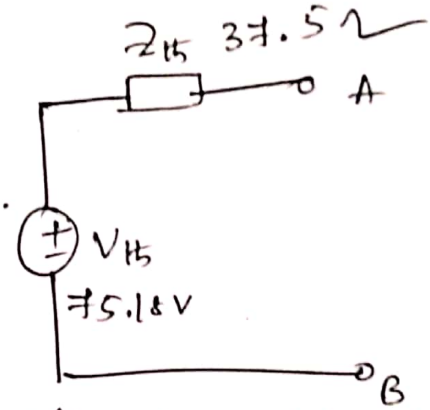
$$10i_2 - 20i_3 = 150 \quad \text{———} \quad (3)$$

$\boxed{i_1 = -2A}$ $\boxed{i_2 = -7A}$ $\boxed{i_3 = -11A}$

$\therefore \; I_{sc} = -I_1$

$$\boxed{I_{sc} = 2A}$$

$\therefore \; Z_{th} = \dfrac{V_{th}}{I_{sc}} = 37.5\,\Omega$.



Th Equivalent ckt.

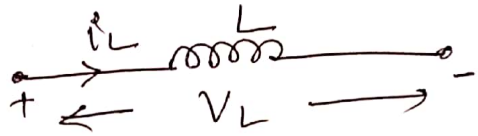
Norton's Theorem :-

Transient Behaviour & Initial Conditions

There are many reasons for studying initial & final conditions. The most important reason is that initial & final conditions evaluate the arbitrary constants in the general solution of differential equation.

* Initial and final conditions in elements :-

1) **The Inductor :-**



WKT voltage drop across inductor is $V_L = L \frac{di_L}{dt}$

for dc current, $\frac{di_L}{dt}$ becomes zero. Hence voltage across inductor is zero. Thus in steady state inductor acts as a short circuit.

Current through inductor is

$$i_L = \frac{1}{L}\int V_L \, dt$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L \, dt$$

$$i_L = \frac{1}{L}\int_{-\infty}^{0^-} V_L \, dt + \frac{1}{L}\int_{0^-}^{t} V_L \, dt$$

At $t = 0^+$

$$i_L(0^+) = i_L(0^-) + \cancelto{0}{\frac{1}{L}\int_{0^-}^{0^+} V_L \, dt}$$

$$\boxed{i_L(0^+) = i_L(0^-)}$$

Thus, current through inductor cannot change instantaneously.

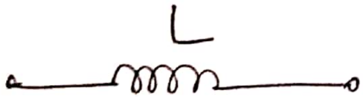
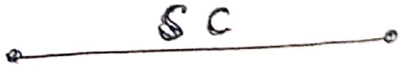
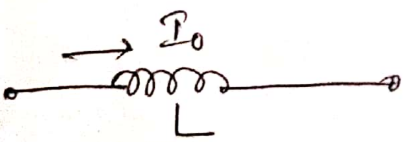
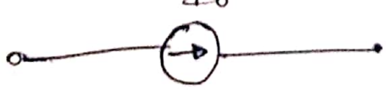
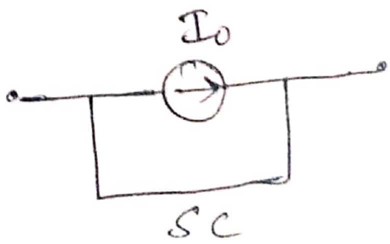
If $i_L(0^-) = 0$, then $i_L(0^+) = 0$ This means that at $t = 0^+$, inductor will act as an open circuit.

If $i_L(0^-) = I_0$, then $i_L(0^+) = I_0$, This means that @ $t = 0^+$, inductor acts as a current source I_0 Amp.

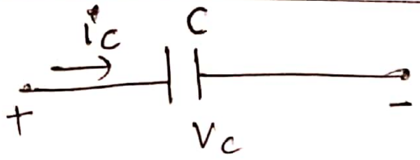
The final condition of the inductor is derived from.

$$V_L = L\frac{di_L}{dt}$$

$\frac{di_L}{dt} = 0$ under steady state condⁿ. Thus $V_L =$
& hence L acts as short circuit at $t = \infty$

Element	Equivalent ckt at $t = 0^+$	Equivalent ckt at $t = \infty$
		
		

2) <u>The Capacitor</u> :-



WKT, $i_c = C \dfrac{dV_c}{dt}$

If dc vg is applied to capr, $\dfrac{dV_c}{dt}$ becomes zero. Thus current through capacitor becomes zero i.e., $\boxed{i_c = 0}$. Thus in <u>steady state</u> (dc) capacitor acts as <u>open circuit</u>.

Now, $V_c = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i_c \, dt$.

$$V_c = \frac{1}{C}\int_{-\infty}^{0^-} i_c \, dt + \frac{1}{C}\int_{0^-}^{t} i_c \, dt$$

$$V_c = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_c \, dt .$$

@ $t = 0^+$

$$V_c(0^+) = V_c(0^-) + \cancel{\frac{1}{C}\int_{0^-}^{0^+} i_c \, dt}^{\,0}$$

$$\therefore \boxed{V_c(0^+) = V_c(0^-)}$$

Thus vg drop across <u>capr</u> cannot change instantaneously.

If $V_c(0^-) = 0$, then $V_c(0^+) = 0$. This means at $t = 0^+$, capacitor C acts as a short circuit.

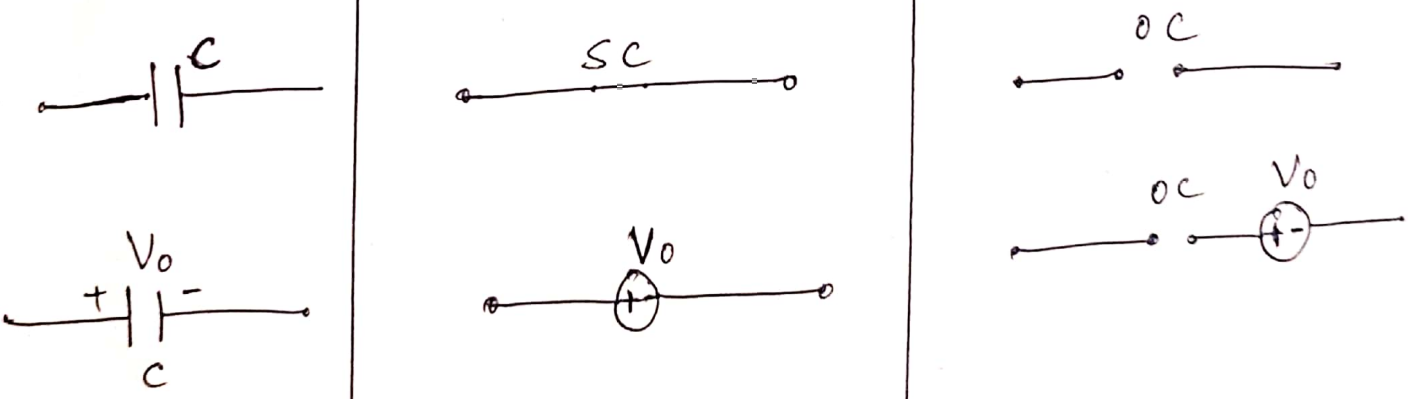
If $V_c(0^-) = V_0$ then $V_c(0^+) = V_0$, then C acts as vg source of V_0 volts.

Under steady state condition (dc) is @ $t = \infty$

$\rightarrow$ $i_c = C \frac{dV_c}{dt}$ ↙ is zero.

$\rightarrow$ $C = \frac{i_c}{0} = \infty$ Capacitor acts as open circuit @ $t = \infty$

Element	Equivalent ckt at $t = 0^+$	Equivalent circuit at $t = \infty$
C	SC	OC
V_0 + − C	V_0	OC V_0



3) Resistor :- For a resistor, the relation b/w applied vg & resulting current is given by

$$V = i \cdot R$$

Above eqn is linear & time independent.

The behaviour of R at $t = 0^+$ & also at $t = \infty$ is same.

* <u>Initial Value theorem</u> :-

If F(S) is the Laplace transform of f(t), then, initial value theorem states that,

$$\lim_{t \to 0^+} f(t) = \lim_{S \to \infty} SF(S) \quad \text{--- (1)}$$

The Laplace transform is very useful to find the initial value of the time function f(t).

<u>Proof</u> :- WKT, the Laplace transform of the real differentiation,

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = SF(S) - f(0^-) \quad \text{--- (2)}$$

Taking Limit as $S \to \infty$ on both side,

$$\lim_{S \to \infty} \mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \lim_{S \to \infty}\left[SF(S) - f(0^-)\right] \quad \text{--- (3)}$$

By defn of L.T WKT $\mathcal{L}[f(t)] = \int_{0^-}^{\infty} e^{-st} f(t)\, dt.$

$$\lim_{S \to \infty} \mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \lim_{S \to \infty} \int_{0^-}^{\infty} e^{-st} \left[\frac{d\,f(t)}{dt}\right] dt.$$

as $S \to \infty$ $e^{-st} = 0$

∴ LHS becomes zero of eqn (3)

$$\therefore \quad 0 = \lim_{s\to\infty} [sF(s) - f(0^-)]$$

$$f(0^-) = \lim_{s\to\infty} sF(s)$$

But $f(0^-) = f(0^+)$ $\quad \therefore \quad f(0^+) = \lim_{s\to\infty} sF(s)$

$$\boxed{\lim_{t\to 0^+} f(t) = f(0^+) = \lim_{s\to\infty} sF(s)}$$

Final value theorem :-

The L.T is also useful to find the final value of the time function f(t). Thus if F(s) is the L.T of f(t) then the final value theorem states that,

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)} \quad \text{——— (1)}$$

Proof :- Consider the L.T of the differentiation

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-) \quad \text{——— (2)}$$

Taking limit as $s\to 0$ on both side.

$$\lim_{s\to 0}\left[L\,\frac{df(t)}{dt}\right] = \lim_{s\to 0}[sF(s) - f(0^-)]$$

——— (3)

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Consider LHS

$$\lim_{s\to 0} \mathcal{L}\frac{df(t)}{dt} = \lim_{s\to 0} \int_{0^-}^{\infty} e^{-st} \frac{df(t)}{dt} \cdot dt$$

as $s \to 0$ $\quad e^{-st} = 1$

$$= \int_{0^-}^{\infty} \frac{df(t)}{dt} \cdot dt$$

$$= f(t)\Big|_{0^-}^{\infty}$$

$$= \lim_{t\to\infty} [f(t) - f(0^-)]$$

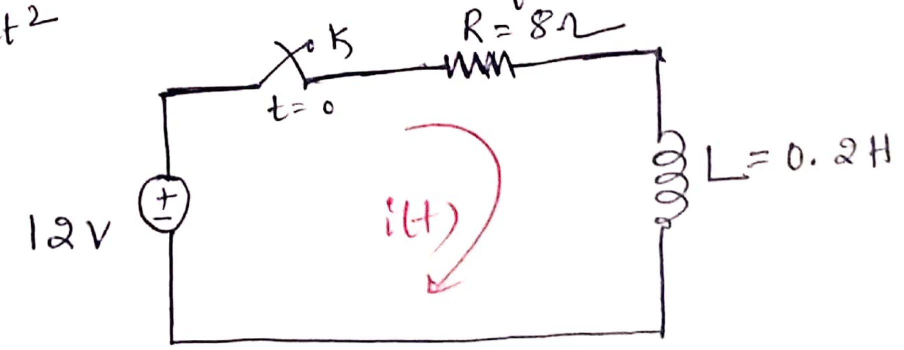
$\therefore$ eqn (3) becomes.

$$\lim_{t\to\infty} [f(t) - f(0^-)] = \lim_{s\to 0} [sF(s) - f(0^-)]$$

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

Problems :-

1) In the n/w, K is closed at $t=0$ with zero current in the inductor. Find the values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$. If $R = 8\Omega$, $L = 0.2H$.



Soln :- Switch is closed at $t=0$ means $t=0^+$

then switch opens at $t=0^-$

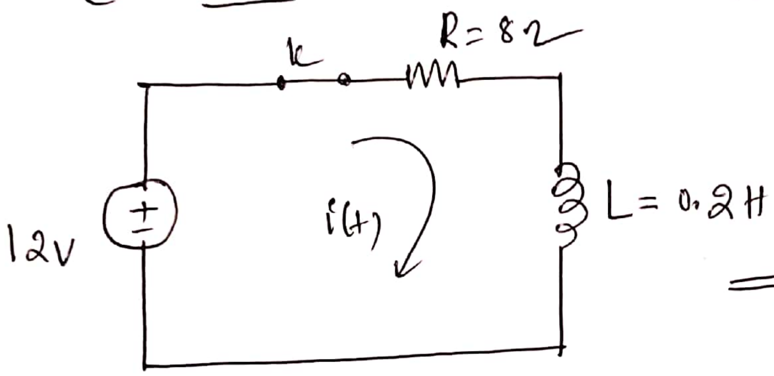
given @ $t=0$ (i.e $t=0^+$) zero current

in the inductor.

i.e, $\boxed{i(0^+) = i(0^-) = 0}$

$\left\{ \begin{array}{l} i_L(0) \\ \text{means} \\ i(0) \end{array} \right.$

@ $t=0^+$



KVL to loop

$$12 - 8i(t) - L\frac{di(t)}{dt} = 0 \quad \text{—①}$$

$$\Rightarrow 8i(t) + L\frac{di(t)}{dt} = 12$$

@ $t=0^+$ $\quad i(0^+) = 0$

$$\therefore 0.2\frac{di}{dt} = 12$$

$$\frac{di}{dt} = \frac{12}{0.2} = 60\ A/sec$$

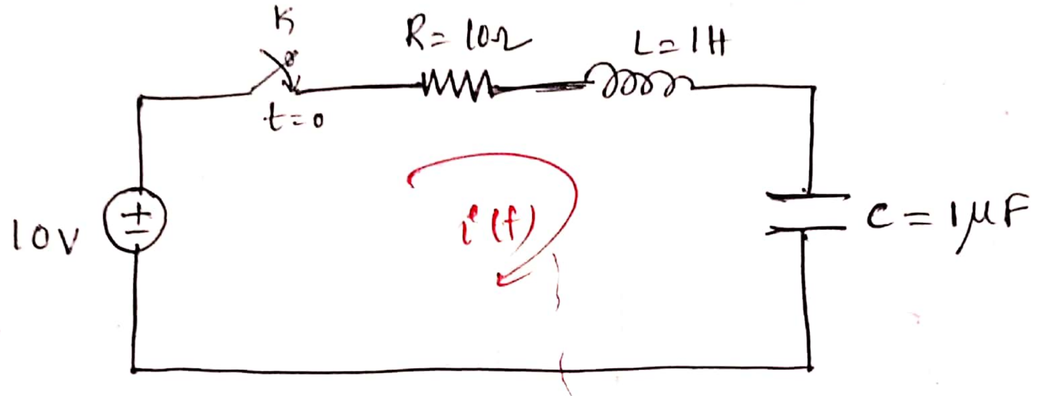
Differentiating Eqn ①

$$-8\frac{di(t)}{dt} - L\frac{d^2i(t)}{dt^2} = 0$$

$$-8\times 60 - 0.2\frac{d^2i}{dt^2} = 0$$

$$\frac{d^2i}{dt^2} = -2400\ A/sec^2$$

2) In the n/w shown, the switch is closed at $t=0$. Determine i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$.



→ In the problem it is not given. we have to indicate.

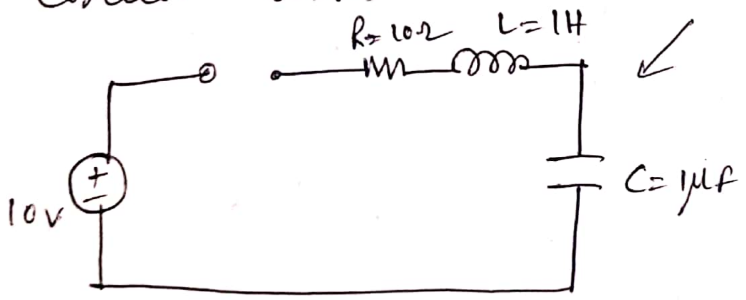
Given switch is closed at $t=0$

means $\underline{\underline{t=0^+}}$, then switch is opened

@ $\underline{\underline{t=0^-}}$.

When the switch is in opened condition,

Circuit looks like

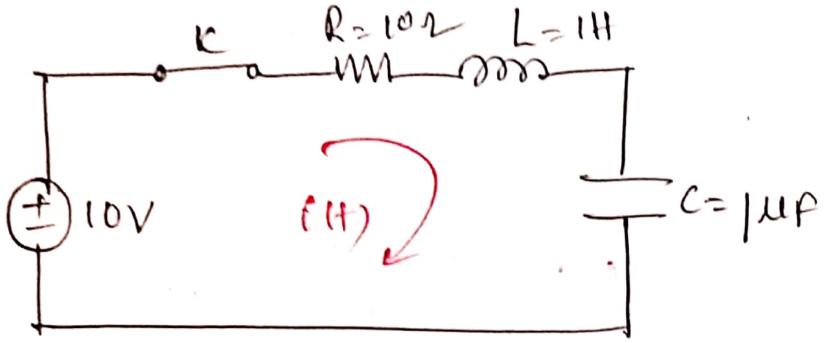


↙ Open loop ∴ no current flows

∴ $\boxed{i(0^-) = i(0^+) = 0}$

& $\boxed{V_c(0^-) = V_c(0^+) = 0}$

@ $\underline{t = 0^+}$ (switch is closed)



$V_c = \frac{1}{C}\int i(t)\,dt$

$V_L = L\frac{di}{dt}$

Apply KVL to loop.

$$10 - \underbrace{R\,i(t)}_{V_R} - \underbrace{L\frac{di}{dt}}_{V_L} - \underbrace{\frac{1}{C}\int i(t)\,dt}_{V_C} = 0 \quad \text{——} ①$$

or $$10 - R\,\cancel{i(t)}^{\to 0} - L\frac{di}{dt} - \cancel{V_c}^{\to 0} = 0$$

@ $t = 0^+$ $\quad i(0^+) = 0$ & $V_c(0^+) = 0$

$$\therefore \quad 10 - L\frac{di}{dt} = 0 \Rightarrow L\frac{di}{dt} = 10$$

$$\frac{di}{dt} = \frac{10}{L} = \frac{10}{1}.$$

$$\boxed{\frac{di}{dt} = 10\,A/sec}$$

Differentiating eqn ①

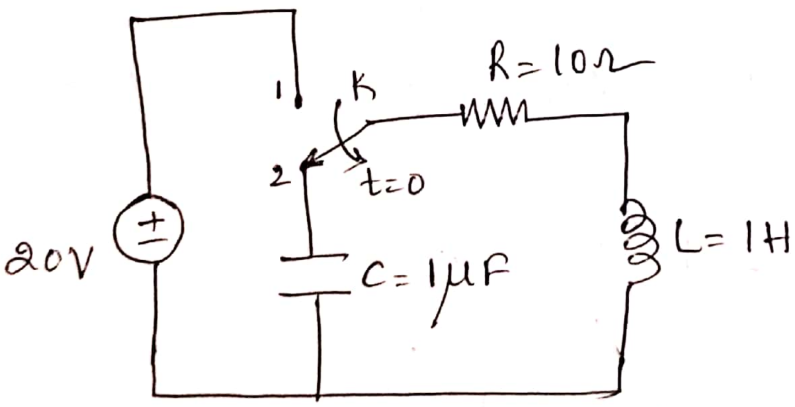
$$-R\frac{di}{dt} - L\frac{d^2i}{dt^2} - \cancel{\frac{i(t)}{C}}^{\to 0} = 0$$

@ $t = 0^+$, $i(0^+) = 0$

$$-10 \times 10 - 1\,\frac{d^2i}{dt^2} = 0$$

$$-\frac{d^2i}{dt^2} = 100 \quad \text{or} \quad \boxed{\frac{d^2i}{dt^2} = -100\ A/sec^2}$$

3) The switch K is changed from position 1 to position 2 at $t=0$. Steady state cond^n has been reached in position 1. Find the values of i, $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t = 0^+$.



Given Switch K is in position 2 at $t=0$ (i.e., $t = 0^+$)

means it is in position 1 at $t = 0^-$

* Also in position 1 steady state cond^n is reached. WKT inductor acts as short ckt in this cond^n.

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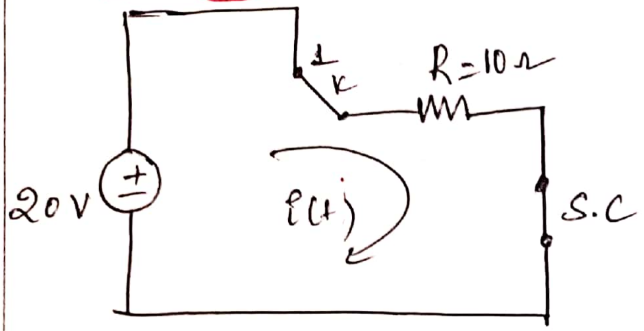
@ $(t=0^-)$



$20 - 10\, i(t) = 0$

$i(t) = \frac{20}{10} = 2\,A$

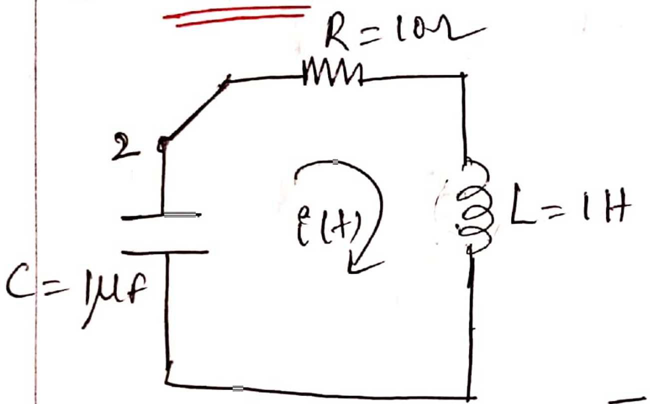
$i_L \quad \boxed{i(0^-) = 2A}$

WKT $\quad i(0^+) = i(0^-)$

$\therefore \boxed{i(0^+) = i(0^-) = 2\,Amp}$

Also $\boxed{V_c(0^-) = V_c(0^+) = 0}$ ($\because$ no vg acr capr)

@ $(t=0^+)$



KVL to loop

$$-R\, i(t) - L\frac{di}{dt} - \underbrace{\frac{1}{C}\int i(t)\,dt}_{V_c} = 0 \quad \text{—①}$$

$$-R\, i(t) - L\frac{di}{dt} - \cancel{V_c}^{\,0} = 0$$

@ $t=0^+$, $\quad i(0^+) = 2 \;\&\; V_c(0^+) = 0$

$$-10 \times 2 - 1\frac{di}{dt} = 0$$

$$\boxed{\frac{di}{dt} = -20\ A/sec}$$

Diff. Eqn ①

$$-R\frac{di}{dt} - L\frac{d^2 i}{dt^2} - \frac{i(t)}{C} = 0$$

$i(0^+) = 2$

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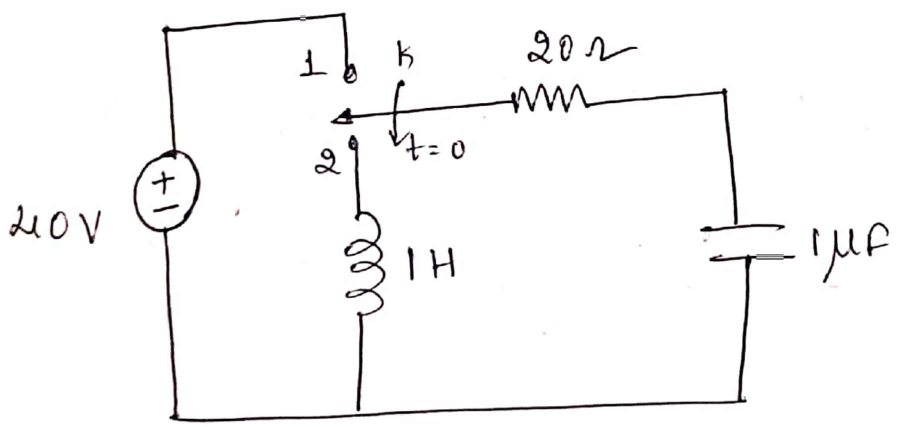
$$-10\times -20 - 1\frac{d^2i}{dt^2} - \frac{2}{1\times10^{-6}} = 0.$$

$$\frac{d^2i}{dt^2} = 200 - 2\times10^6$$

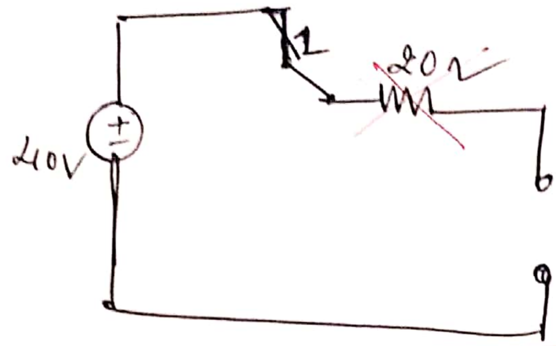
$$\boxed{\frac{d^2i}{dt^2} \cong -2\times10^6 \text{ A/sec}^2}$$

4) In the n/w, the switch is moved from position 1 to position 2 at $t=0$. The steady state reached before switching. Calculate i, $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t=0^+$.



Given switch is in position 2 at $t=0$ (i.e. $t=0^+$) means in position 1 at $t=0^-$. Also steady state reached. WKT @ steady state capacitor acts as open circuit

($\because$ oc no current flows)



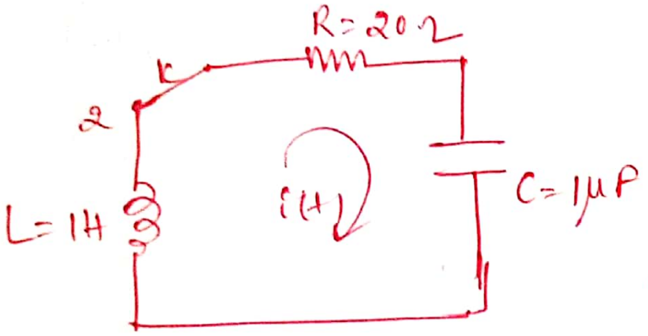
from the fig

$$\boxed{V_c = 40\,V}$$

$$\therefore \boxed{V_c(0^+) = V_c(0^-) = 40\,V}$$

Also $\boxed{i(0^+) = i(0^-) = 0}$

@ $t = 0^+$ switch in position 2



KVL to loop

$$-Ri(t) - \underbrace{\frac{1}{C}\int i(t)\,dt}_{V_c} - L\frac{di}{dt} = 0 \quad \text{——} (1)$$

$$-R\,i(t) - V_c - L\frac{di}{dt} = 0$$

@ $t = 0^+$ $i(0^+) = 0$ & $V_c(0^+) = 40V$

$$-40 - 1\frac{di}{dt} = 0 \Rightarrow \boxed{\frac{di}{dt} = -40\,A/sec}$$

Diff eqn (1)

$$-R\frac{di}{dt} - \cancel{\frac{i(t)}{C}} - L\frac{d^2i}{dt^2} = 0$$

$$-20 \times -40 - 1\frac{d^2i}{dt^2} = 0$$

$$\boxed{\frac{d^2i}{dt^2} = 800\,A/sec^2}$$

-: Resonance :-

Resonance is defined as a phenomenon in which applied voltage & resulting current are in phase.

Resonance occurs in RLC circuit. During resonance phase angle between current & voltage is zero

ie, $\phi = 0$

WKT Power factor $= \cos\phi$

$$Pf = 1$$

∴ an AC circuit is said to be in resonance when the circuit P.f is Unity.

The resonant condition in ac circuit may be achieved,

1) By varying the frequency & the supply keeping the networks elements constant.

(or)

2) By varying L or C, keeping frequency constant.

Types of Resonance :-

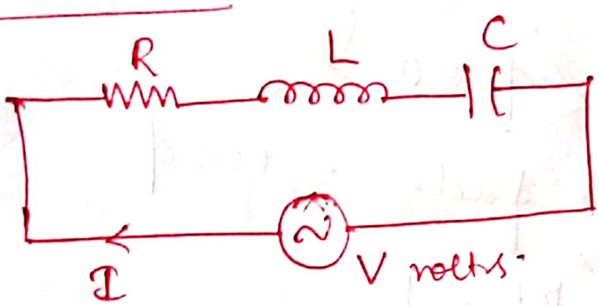
There are two types of resonance

i) Series resonance.

2) Parallel resonance.

Series resonance :-

Expression for resonant frequency in Series resonance :-



Consider a general RLC series circuit energised by a voltage source of V volts as shown in above figure.

The impedance of the circuit is given by

$$Z = R + j(X_L - X_C)$$

where $X_L = 2\pi f L$ & $X_C = \dfrac{1}{2\pi f C}$

By varying supply frequency X_L is made equal to X_C

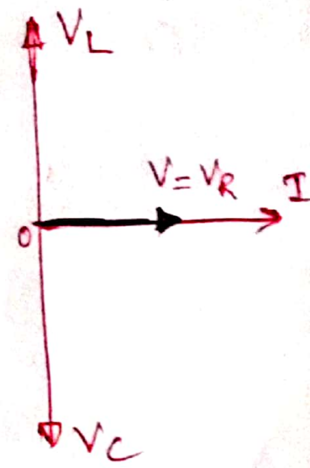
If $X_L = X_C$

Then $Z = R$

Current in phase with voltage.

$\rightarrow \phi = 0$

$\rightarrow Pf = 1$



Now the circuit is at resonance.

$\therefore$ At resonance.

$$X_L = X_C$$

$$2\pi f_o L = \frac{1}{2\pi f_o C} \quad [f_o = \text{resonant freq}].$$

$$(2\pi)^2 f_o^2 LC = 1$$

$$f_o^2 = \frac{1}{(2\pi)^2 LC}$$

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

$$2\pi f_o = \frac{1}{\sqrt{LC}} \implies \boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

Voltage magnification or figure of merit or quality factor or Q- factor :-

The ratio of voltage developed across inductor or Capacitor to the applied voltage is called voltage magnification.

i.e., $$Q = \frac{V_L \text{ or } V_C}{V}$$

$$\therefore \quad Q = \frac{V_L}{V} \quad \text{or} \quad Q = \frac{V_C}{V}$$

$$Q = \frac{I X_L}{I R} \qquad Q = \frac{I X_C}{I R}$$

$$\boxed{Q = \frac{X_L}{R}} \qquad \boxed{Q = \frac{X_C}{R}}$$

Q- factor of inductor   Q- factor of capacitor.

Expression for Q- factor in Series resonance $[Q_s]$:-

Q factor of Series resonance is nothing but the Q- factor of inductor or Q- factor of capacitor.

Let Q_s be the Q- factor in Series resonance,

$$\therefore \quad Q_s = Q$$

$$L_o = \frac{X_L}{R}$$

$X_L = 2\pi f_0 L$

$L_o = \omega_0 L$

$$Q_s = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$Q_s = \frac{1}{\cancel{\sqrt{L}}\sqrt{C}} \times \frac{\cancel{\sqrt{L}} \times \sqrt{L}}{R}$$

$$\boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Characteristics of Series resonant Circuit :-

At resonance,

1) $X_L = X_C$

2) The impedance of the circuit is minimum & is equal to the resistance of the circuit

i.e, $\boxed{Z = R}$.

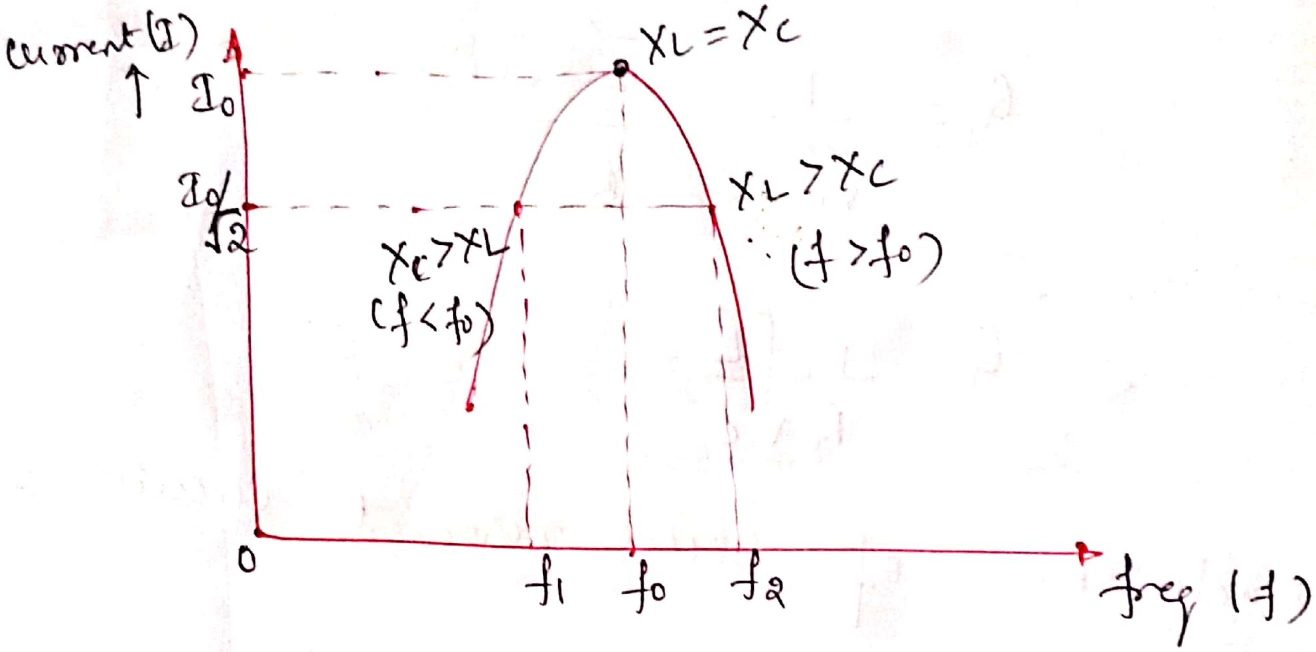
3) The current in the circuit is maximum & it is inphase with voltage.

$$\boxed{I = \frac{V}{R}}$$

4) The power factor is Unity.

i.e, $\boxed{Pf = 1}$.

Frequency resonance curve of Series resonant Circuit :-



$I_0 \rightarrow$ maximum current or current @ resonance.

$f_1 \rightarrow$ Lower cutoff frequency. } Half power
$f_2 \rightarrow$ Upper cutoff frequency. } frequency.

The power at resonance $= I_0^2 R \rightarrow$ maximum power.

If $I = \dfrac{I_0}{\sqrt{2}}$

then power $= I^2 R$

$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$= \frac{I_0^2}{2} \times R = \frac{1}{2} \times I_0^2 R$$

$$= \frac{1}{2} \times \text{Maximum power}$$

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$\therefore$ The frequencies f_1 & f_2 corresponding to $\frac{I_o}{\sqrt{2}}$ or $0.707\, I_o$ are called cut off frequencies or half power frequencies. Because the o/p power is reduced to half of the maximum power.

* **Band width :-**

The range or band of frequencies b/n f_1 & f_2 is called as Band width.

ie, $$\boxed{\text{Band width} = \Delta f = f_2 - f_1}$$

* **Quality factor :-**

Quality factor is defined as the ratio of resonant frequency to the band width.

ie, $$\boxed{Q = \frac{f_0}{B.W}}$$

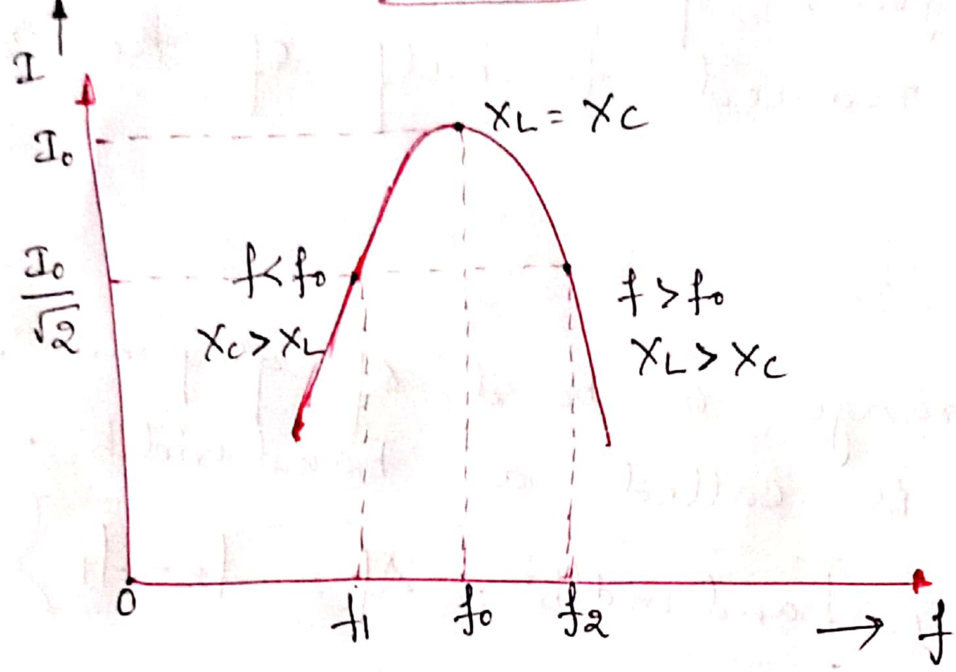
* **Selectivity :-**

It is the reciprocal of Quality factor.

ie, $$\text{Selectivity} = \frac{1}{Q}.$$

$$= \frac{1}{(f_0/B.W)} = \frac{B.W}{f_0}$$

* Show that the resonant frequency is the geometric mean of the two half power frequencies i.e, $f_0 = \sqrt{f_1 . f_2}$



Let $X_{C_1} \rightarrow$ Capacitive reactance at f_1

$X_{C_2} \rightarrow$ Capacitive reactance at f_2.

$X_{L_1} \rightarrow$ Inductive reactance at f_1

$X_{L_2} \rightarrow$ Inductive reactance at f_2.

The impedance of RLC series resonant circuit at f_1 is

$$Z_1 = \sqrt{R^2 + (X_{C_1} - X_{L_1})^2} \quad (\because @ f_1 \; X_C > X_L)$$

The impedance of RLC series resonant circuit at f_2 is

$$Z_2 = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2} \quad (\because @ f_2 \; X_L > X_C)$$

we have,

$$Z_1 = Z_2$$

$$\sqrt{R^2 + (X_{C1} - X_{L1})^2} = \sqrt{R^2 + (X_{L2} - X_{C2})^2}$$

$$\Rightarrow \cancel{R^2} + (X_{C1} - X_{L1})^2 = \cancel{R^2} + (X_{L2} - X_{C2})^2$$

$$\Rightarrow (X_{C1} - X_{L1})^{\cancel{2}} = (X_{L2} - X_{C2})^{\cancel{2}}$$

$$\Rightarrow X_{C1} - X_{L1} = X_{L2} - X_{C2}$$

$$X_{C1} + X_{C2} = X_{L2} + X_{L1} \quad \text{--- (1)}$$

But
$$\left.\begin{aligned} X_{C1} &= \frac{1}{2\pi f_1 C} = \frac{1}{\omega_1 C} \\ X_{C2} &= \frac{1}{2\pi f_2 C} = \frac{1}{\omega_2 C} \\ X_{L1} &= 2\pi f_1 L = \omega_1 L \\ X_{L2} &= 2\pi f_2 L = \omega_2 L \end{aligned}\right\} \text{--- (2)}$$

Substituting (2) in eqn (1), we get

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$\frac{1}{C}\left[\frac{\cancel{\omega_2 + \omega_1}}{\omega_1 \omega_2}\right] = L[\cancel{\omega_1 + \omega_2}]$$

$$\frac{1}{C\omega_1\omega_2} = L$$

or $$\frac{1}{LC\omega_1\omega_2} = 1 \Rightarrow \frac{1}{LC} = \omega_1\omega_2 \quad \text{--- (3)}$$

WKT $$\omega_0 = \frac{1}{\sqrt{LC}}$$

or $$\omega_0^2 = \frac{1}{LC}$$

∴ eqn (3) becomes

$$\omega_0^2 = \omega_1\omega_2$$

$$\omega_0 = \sqrt{\omega_1\omega_2}$$

$$2\pi f_0 = \sqrt{2\pi f_1 \, 2\pi f_2}$$

$$2\pi f_0 = \sqrt{(2\pi)^2 f_1 f_2} \qquad \left(\sqrt{\cancel{(2\pi)^2}}\ \sqrt{f_1 f_2}\right)$$

$$\cancel{2\pi} f_0 = \cancel{2\pi}\sqrt{f_1 f_2}$$

$$\Rightarrow \boxed{f_0 = \sqrt{f_1 f_2}}$$

Expression for bandwidth or relationship b/w bandwidth & Q- factor :

Let f_1 & f_2 be the lower & upper half power frequencies & f_0 be the resonant frequency.

At f_1, $I = \frac{V}{\sqrt{R^2 + (X_{C_1} - X_{L_1})^2}}$ $(\because @ f_1 \; X_C > X_L)$

Also @ f_1, $I = \frac{I_0}{\sqrt{2}}$ where $I_0 = \frac{V}{R}$

$$\frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + (X_{C_1} - X_{L_1})^2}}$$

$$\frac{\cancel{V}}{\sqrt{2}R} = \frac{\cancel{V}}{\sqrt{R^2 + (X_{C_1} - X_{L_1})^2}}$$

$$\sqrt{R^2 + (X_{C_1} - X_{L_1})^2} = \sqrt{2}\,R$$

Squaring on B.S

$$R^2 + (X_{C_1} - X_{L_1})^2 = 2R^2$$

$$(X_{C_1} - X_{L_1})^2 = R^2$$

$$X_{C_1} - X_{L_1} = R \quad \text{——— ①}$$

@ f_2,
$$I = \frac{V}{\sqrt{R^2 + (X_{L_2} - X_{C_2})^2}}$$

Also @ f_2, $I = \frac{I_0}{\sqrt{2}}$, where $I_0 = \frac{V}{R}$

$$\frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + (X_{L_2} - X_{C_2})^2}}$$

$$\frac{\cancel{V}}{\sqrt{2}R} = \frac{\cancel{V}}{\sqrt{R^2 + (X_{L_2} - X_{C_2})^2}}$$

$$\sqrt{R^2 + (X_{L_2} - X_{C_2})^2} = \sqrt{2}R$$

Squaring on both side

$$R^2 + (X_{L_2} - X_{C_2})^2 = 2R^2$$

$$(X_{L_2} - X_{C_2})^{\cancel{2}} = R^{\cancel{2}}$$

$$X_{L_2} - X_{C_2} = R \quad \text{——— ②}$$

① + ② gives,

$$X_{C_1} - X_{L_1} + X_{L_2} - X_{C_2} = 2R$$

$$X_{C_1} - X_{C_2} + X_{L_2} - X_{L_1} = 2R$$

$$\frac{1}{\omega_1 C} - \frac{1}{\omega_2 C} + \omega_2 L - \omega_1 L = 2R$$

$$\frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right] + L(\omega_2 - \omega_1) = 2R$$

$$(\omega_2 - \omega_1)\left[\frac{1}{C\omega_1\omega_2} + L\right] = 2R$$

$$\omega_2 - \omega_1 = \frac{2R}{\left[\frac{1}{C\omega_1\omega_2} + L\right]} \qquad \frac{\div L}{\div L}$$

$$\omega_2 - \omega_1 = \frac{\frac{2R}{L}}{1 \leftarrow \frac{1}{C\omega_1\omega_2 L} + 1}$$

$$\omega_2 - \omega_1 = \frac{2R/L}{1+1} = \frac{\frac{2R}{L}}{2/1}$$

$$\omega_2 - \omega_1 = \frac{\cancel{2}R}{L} \times \frac{1}{\cancel{2}}$$

$$\omega_2 - \omega_1 = \frac{R}{L} \times \frac{\omega_0}{\omega_0}$$

$$\omega_2 - \omega_1 = \frac{\omega_0 R}{L\omega_0}$$

$$\omega_2 - \omega_1 = \frac{\omega_0}{L\omega_0/R} = \frac{\omega_0}{\frac{X_L}{R}} \leftarrow Q$$

$$\omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$f_2 - f_1 = \frac{f_0}{Q}$$

or $$\boxed{B.W = \frac{f_0}{Q}}$$

* **Impedance of a Series resonant Circuit in terms of freq deviation (δ) :-**

Frequency deviation (δ) is defined as the ratio of the difference b/w applied frequency or operating freq & resonant frequency.

i.e, $$\delta = \frac{f - f_0}{f_0} = \frac{\omega - \omega_0}{\omega_0}$$

where $f \rightarrow$ Operating frequency.

$f_0 \rightarrow$ Resonant frequency.

The impedance of series resonance circuit is

$$Z = R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = R + \left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega C R}\right)\right]$$

$$Z = R\left[1 + j\left(\frac{\omega L}{R} \times \frac{\omega_0}{\omega_0} - \frac{1}{\omega C R} \times \frac{\omega_0}{\omega_0}\right)\right]$$

$$Z = R\left[1 + j\left(\frac{\omega_0 L}{R} \times \frac{\omega}{\omega_0} - \frac{1}{\omega_0 C R} \times \frac{\omega_0}{\omega}\right)\right]$$

$$Z = R\left[1 + j\left(Q\frac{\omega}{\omega_0} - Q\frac{\omega_0}{\omega}\right)\right]$$

$$Z = R\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \quad \text{—— (1)}$$

WKT $\quad \delta = \dfrac{\omega - \omega_0}{\omega_0}$

$$\delta = \frac{\omega}{\omega_0} - 1 \implies \frac{\omega}{\omega_0} = \delta + 1 \quad \text{—— (2)}$$

Substitute eqn (2) in (1) we get

$$Z = R\left[1 + jQ\left(\delta + 1 - \frac{1}{\delta + 1}\right)\right]$$

$$Z = R\left[1 + jQ\left(\frac{(\delta+1)^2 - 1}{\delta + 1}\right)\right]$$

$$Z = R\left[1 + jQ\left[\frac{\delta^2 + \cancel{1} + 2\delta - \cancel{1}}{\delta + 1}\right]\right]$$

$$Z = R\left[1 + jQ\left(\frac{\delta^2 + 2\delta}{\delta + 1}\right)\right]$$

$$Z = R\left[1 + jQ\,\frac{\delta(\delta + 2)}{\delta + 1}\right]$$

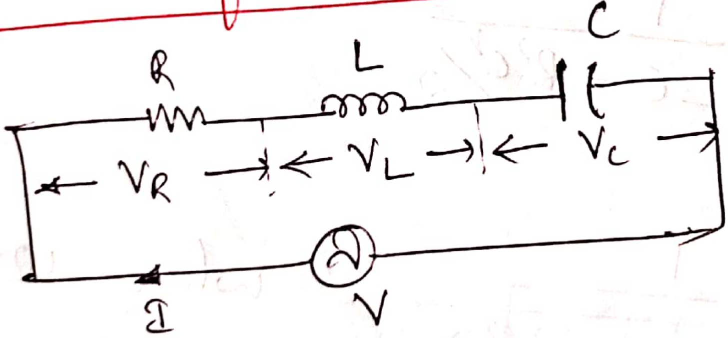
When δ is too small

$$Z = R[1 + jQ2\delta]$$

At resonance $\rightarrow$ $\delta = 0$

$$\therefore Z = R$$

* Expression for f_{Lmax} and f_{cmax} :-



Frequency at which voltage across the capacitor reaches its maximum is called f_{cmax}. V_{cmax} occurs earlier to f_0, for which $X_C > X_L$.

$$V_C = I X_C \longrightarrow V_C = \frac{V}{Z} \times \frac{1}{\omega C}$$

$$V_c = \frac{V}{\sqrt{R^2+(X_c-X_L)^2}} \times \frac{1}{\omega C}$$

Squaring on both side

$$V_c^2 = \frac{V^2}{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2} \times \frac{1}{\omega^2 C^2}$$

$$L_p = \frac{V^2}{\omega^2 C^2}\left[\frac{1}{R^2+\frac{1}{\omega^2C^2}+\omega^2L^2-2\times\frac{1}{\cancel{\omega} C}\times\cancel{\omega} L}\right]$$

$$L_p = \frac{V^2}{\omega^2C^2R^2+\omega^2C^2\left(\frac{1}{\omega^2C^2}+\omega^2L^2-2\frac{L}{C}\right)}$$

$$V_c^2 = \frac{V^2}{\omega^2C^2R^2+1+\omega^4L^2C^2-2\omega^2LC}$$

$$V_c^2 = \frac{V^2}{\omega^2C^2R^2+(\omega^2LC-1)^2}$$

V_c is max, when $\frac{dV_c^2}{d\omega}=0$

$$\frac{dV_c^2}{d\omega} = \frac{\text{Denometir}\times 0 - V^2\{2\omega C^2R^2+2(\omega^2LC-1)\times 2\omega LC\}}{\{\omega^2C^2R^2+(\omega^2LC-1)^2\}^2} = 0$$

$$= -V^2\{2\omega C^2R^2+2(\omega^2LC-1)\times 2\omega LC\} = 0$$

As $V \neq 0$ $\quad 2\omega C^2R^2+2(\omega^2LC-1)\times 2\omega LC = 0$

$$2\omega C\left[CR^2+2\omega^2L^2C-2L\right]=0$$

$$CR^2+2\omega^2L^2C-2L=0$$

$$\omega^2 = \frac{1}{LC}-\frac{R^2}{2L^2} \quad \text{or} \quad \omega=\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}$$

$$\boxed{f_{Cmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

f_{Lmax} is the freq at which V_{Lmax} occurs. V_{Lmax} occurs after f_0 for which $X_L > X_C$.

$$V_L = I X_L = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \times \omega L$$

$$V_L = \frac{V\omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V\omega L}{\sqrt{R^2 + \omega^2 L^2 + \frac{1}{\omega^2 C^2} - 2 \times \omega L \times \frac{1}{\omega C}}}$$

$$= \frac{V\omega L \times \omega C}{\sqrt{\omega^2 C^2 R^2 + \omega^4 L^2 C^2 + 1 - \frac{2L}{C}\omega^2 C^2}}$$

$$= \frac{V\omega L \times \omega C}{\sqrt{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}}$$

$$V_L = \frac{V\omega^2 LC}{\sqrt{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}}$$

Squaring on both side

$$V_L^2 = \frac{V^2 \omega^4 L^2 C^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

V_L is max $\frac{dV_L^2}{d\omega} = 0$

$$\frac{dV_L^2}{d\omega} = \frac{\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\} \times 4\omega^3 V^2 L^2 C^2 - V^2 \omega^4 L^2 C^2 \{2\omega C^2 R^2 + 2(\omega^2 LC - 1) \times 2\omega LC\}}{\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\}^2} = 0$$

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⑥

$$4\{\omega^2C^2R^2+(\omega^2LC-1)^2\}-\omega\{2\omega C^2R^2+4\omega^3L^2C^2-4\omega LC\}=0$$

$$=4\omega^2C^2R^2+4\cancel{\omega^4}L^2C^2+4-8\omega^2LC-2\omega^2C^2R^2-4\cancel{\omega^4L^2}C^2+4\omega^2LC=0$$

$$=\cancel{2\omega^2C^2R^2}-\cancel{2\omega^2C^2R^2}\quad 2\omega^2C^2R^2-4\omega^2LC+4=0$$

$$4\omega^2LC-2\omega^2C^2R^2=4$$

$$\omega^2=\frac{2}{2LC-C^2R^2}$$

$$\omega^2=\frac{1}{LC-\frac{R^2C^2}{2}}$$

$$\omega=\sqrt{\frac{1}{LC-\frac{R^2C^2}{2}}}$$

$$\boxed{f_{Lmax}=\frac{1}{2\pi}\sqrt{\frac{1}{LC-\frac{R^2C^2}{2}}}}$$

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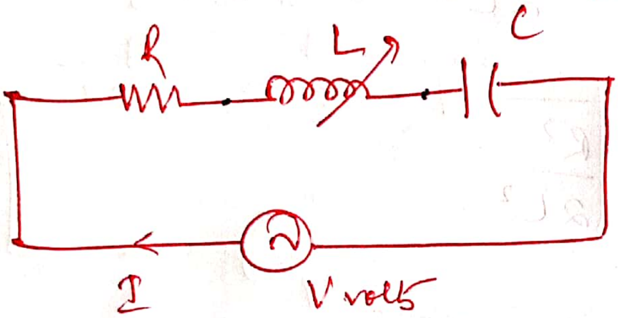
* Resonance by varying Circuit elements :-

Resonance can also be obtained by keeping f constant & by varying L & C.

The resonance is made by varying L is termed as inductive tuning.



X_C remains unchanged since both f & C are fixed.

X_L varies as L varies since $X_L = 2\pi f L$

Let L_R denotes the inductance at resonance

$X_L = 2\pi f L$

WKT $V_L = I X_L$

$$V_L = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \times X_L$$

~~V_L is max if~~ $\frac{d}{dX_L^2}$

Squaring ; $V_L^2 = \dfrac{V^2 X_L^2}{R^2 + (X_L - X_C)^2}$

V_L is max if $\dfrac{dV_L^2}{dX_L} = 0$

$$\frac{dV_L^2}{dX_L} = \frac{\left[R^2+(X_L-X_C)^2\right]V^2\times 2X_L - V^2X_L^2\left(2(X_L-X_C)\right)}{\left\{R^2+(X_L-X_C)^2\right\}^2} = 0$$

$$\Rightarrow 2V^2X_L\left[R^2+X_L^2+X_C^2-2X_LX_C\right] - V^2X_L^2(2X_L-2X_C) = 0$$

$$\Rightarrow 2X_LV^2R^2 + \cancel{2V^2X_L^3} + 2V^2X_LX_C^2 - 4V^2X_L^2X_C - \cancel{2V^2X_L^3} + 2V^2X_L^2X_C = 0$$

$$\Rightarrow V^2(2X_LR^2 + 2X_LX_C^2 - 2X_L^2X_C) = 0$$

or $\quad 2X_LR^2 + 2X_LX_C^2 - 2X_L^2X_C = 0$

$$2X_L\left[R^2+X_C^2-X_LX_C\right] = 0$$

$$R^2+X_C^2 = X_LX_C$$

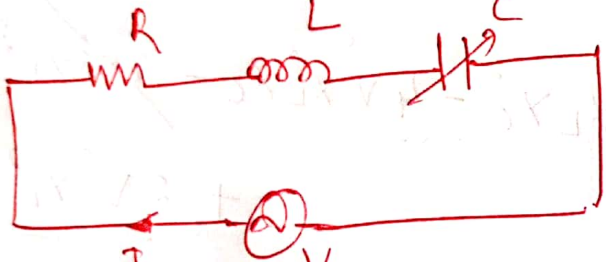
$$R^2+X_C^2 = \cancel{2\pi} fL \times \frac{1}{\cancel{2\pi} fC}$$

$$\boxed{L_R = C\left[R^2+X_C^2\right]}$$

Resonance by varying Circuit elements :-

By varying Capacitance :-

S.T the value of the capacitor for maximum voltage across it in case of capacitive tuning of series resonance is $C_R = \frac{L}{R^2 + X_L^2}$



Let C_R denotes the capacitance at resonance.

Here X_L is fixed $\because$ f & L are fixed.

WKT, $V_C = I X_C$

$$V_C = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}} X_C$$

Squaring on both side.

$$V_C^2 = \frac{V^2 X_C^2}{R^2 + (X_C - X_L)^2}$$

Voltage across the cap is maximum i.e,

V_C is max if $\frac{dV_C^2}{dX_C} = 0$

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$$\frac{dV_C^2}{dX_C} = \frac{\left[R^2 + (X_C - X_L)^2\right] \times 2V^2 X_C - V^2 X_C^2 \left[2(X_C - X_L)\right]}{\left[R^2 + (X_C - X_L)^2\right]^2} = 0$$

$$\Rightarrow \left[R^2 + X_C^2 + X_L^2 - 2X_C X_L\right] 2V^2 X_C - 2X_C^3 V^2 + 2V^2 X_C^2 X_L = 0$$

$$\Rightarrow 2V^2 X_C R^2 + 2V^2 X_C^3 + 2V^2 X_C X_L^2 - 4V^2 X_C^2 X_L - 2V^2 X_C^3 + 2V^2 X_C^2 X_L = 0$$

$$\Rightarrow 2V^2 \left[R^2 X_C + X_C X_L^2 - 2X_C^2 X_L + X_C^2 X_L\right] = 0$$

$$X_C \left[R^2 + X_L^2 - 2X_C X_L + X_C X_L\right] = 0$$

$$\Rightarrow R^2 + X_L^2 - X_C X_L = 0$$

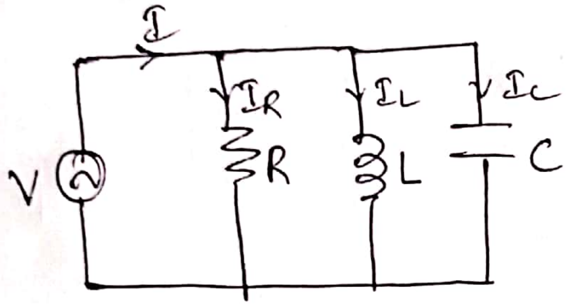
$$\Rightarrow R^2 + X_L^2 = X_C X_L$$

$$\Rightarrow R^2 + X_L^2 = \frac{1}{\cancel{2\pi} f C} \times \cancel{2\pi} f L$$

$$\Rightarrow \boxed{C_R = \frac{L}{R^2 + X_L^2}}$$

Parallel Resonance :-

1) General parallel resonance Circuit :-



WKT

$$I = I_R + I_L + I_C$$

(from KCL)

from KCL, $I = I_R + I_L + I_C$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C}$$

$$\frac{1}{Z} = \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$Y = G + jB$$

where $Y = \frac{1}{Z}$ → admittance

$G = \frac{1}{R}$ → Conductance

$B = \left(\frac{1}{X_C} - \frac{1}{X_L}\right)$ → Susceptance

At resonance the net susceptance is zero

i.e., $B = 0$

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

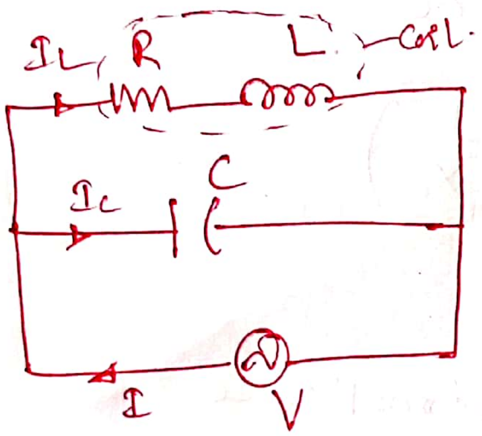
$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

*. *.
2) <u>Practical parallel resonance circuit</u>



The circuit consists of an inductive coil of resistance R Ω & inductance L Henry which is connected in ||lel with the capacitance C farad. This combo is connected across alternating supply.

$$I = I_L + I_c$$

$$\frac{V}{Z} = \frac{V}{R+jX_L} + \frac{V}{-jX_C}$$

$$\frac{1}{Z} = \frac{1}{R+jX_L} + \frac{j}{X_C}$$

$$\frac{1}{Z} = \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L} + \frac{j}{X_C}$$

$$\frac{1}{Z} = \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$\frac{1}{Z} = \frac{R}{R^2+X_L^2} - \frac{jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$\frac{1}{Z} = \frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$$

$$Y = G + jB$$

At resonance susceptance is zero. (i.e $B=0$)

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$\cancel{\omega_0} C = \frac{\cancel{\omega_0} L}{R^2 + \omega_0^2 L^2}$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{L}{C \times L^2} - \frac{R^2}{L^2}$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

@ resonance $(B=0)$

$$Y = \frac{R}{R^2 + X_L^2}$$

$$Y = \frac{R}{R^2 + \omega_0^2 L^2} \quad L/C$$

$$Y = \frac{CR}{L}$$

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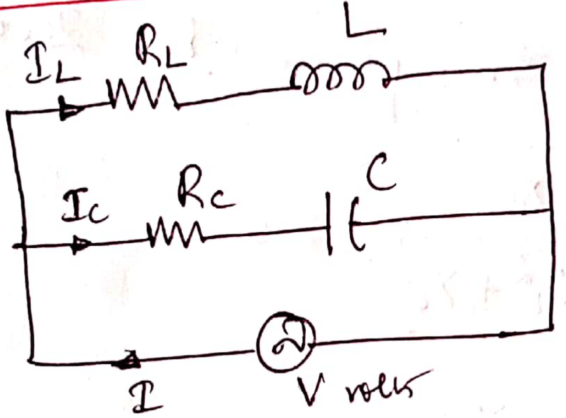
$Z = \frac{L}{RC}$ → which is called as dynamic impedance

$$\boxed{Z_d = \frac{L}{RC}}$$

3) Parallel resonance circuit when the resistance of the capacitor considered :-



$I_C = \frac{V}{Z}$

$= \frac{V}{R_C - jX_C}$

from the circuit

$$I = I_L + I_C$$

$$\frac{V}{Z} = \frac{V}{R_L + jX_L} + \frac{V}{R_C - jX_C}$$

$$\frac{\cancel{V}}{Z} = \cancel{V}\left[\frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}\right]$$

$$\frac{1}{Z} = \frac{1}{R_L + jX_L} \times \frac{R_L - jX_L}{R_L - jX_L} + \frac{1}{R_C - jX_C} \times \frac{R_C + jX_C}{R_C + jX_C}$$

$$\frac{1}{Z} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$\frac{1}{Z} = \frac{R_L}{R_L^2 + X_L^2} - \frac{jX_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + \frac{jX_C}{R_C^2 + X_C^2}$$

$$\frac{1}{Z} = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right] \quad (1)$$

$$Y = G + jB.$$

At resonance net susceptance is zero (i.e, $B = 0$)

$$\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$X_C(R_L^2 + X_L^2) = X_L(R_C^2 + X_C^2)$$

$$\frac{1}{\omega_0 C}(R_L^2 + \omega_0^2 L^2) = \omega_0 L\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right)$$

$$\frac{1}{LC}(R_L^2 + \omega_0^2 L^2) = \omega_0^2\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right)$$

$$\frac{R_L^2}{LC} + \omega_0^2 \frac{L}{C} = \omega_0^2 R_C^2 + \frac{1}{C^2}$$

$$\frac{R_L^2}{LC} - \frac{1}{C^2} = \omega_0^2 R^2 - \omega_0^2 \frac{L}{C}$$

$$\frac{R_L^2}{LC} - \frac{1}{C^2} = \omega_0^2\left(R_C^2 - \frac{L}{C}\right)$$

or $$\omega_0^2 = \frac{\frac{R_L^2}{LC} - \frac{1}{C^2}}{R_C^2 - \frac{L}{C}}$$

$$\omega_0^2 = \frac{1}{LC}\frac{\left(R_L^2 - \frac{L}{C}\right)}{\left(R_C^2 - L/C\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

Impedance at resonance :-

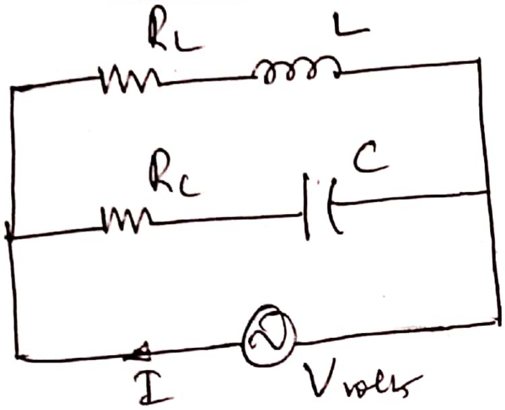
$$Y_0 = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}$$ (from ①)

$$Z_0 = \frac{1}{Y_0}$$

Current at resonance

$$I_0 = \frac{V}{Z_0} = V Y_0 = V\left[\frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2}\right]$$

* S.T the parallel ckt is resonant at all frequencies if $R_L = R_C = \sqrt{\frac{L}{C}}$



WKT,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

if $R_L = R_C$ then

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

@ resonance (B = 0)

$$\frac{X_C}{R^2+X_C^2} - \frac{X_L}{R_L^2+X_L^2} = 0$$

$$\frac{X_C}{R_C^2+X_C^2} = \frac{X_L}{R_L^2+X_L^2}$$

$$\frac{\frac{1}{\omega_0 C}}{R_C^2 + \frac{1}{\omega_0^2 C^2}} = \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2}$$

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$$\frac{\omega_0 C}{\omega_0^2 C^2 R_C^2 + 1} = \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2}$$

$$\cancel{\omega_0} C\left[R_L^2 + \omega_0^2 L^2\right] = \cancel{\omega_0} L\left[\omega_0^2 C^2 R_C^2 + 1\right]$$

$$\frac{CR_L^2}{LC} + \frac{\omega_0^2 L^2 C}{LC} = \frac{\omega_0^2 C^2 R_C^2 L}{LC} + \frac{L}{LC} \qquad \div LC$$

$$\frac{R_L^2}{L} + \omega_0^2 L = \omega_0^2 C R_C^2 + \frac{1}{C}$$

Given $R_L = R_C = \sqrt{L/C}$

$$R_L^2 = R_C^2 = L/C$$

$$\frac{\cancel{L}}{\cancel{L}C} + \omega_0^2 L = \omega_0^2 \cancel{C} \times \frac{L}{\cancel{C}} + \frac{1}{C}$$

$$\therefore \boxed{\frac{1}{C} + \omega_0^2 L = \frac{1}{C} + \omega_0^2 L}$$

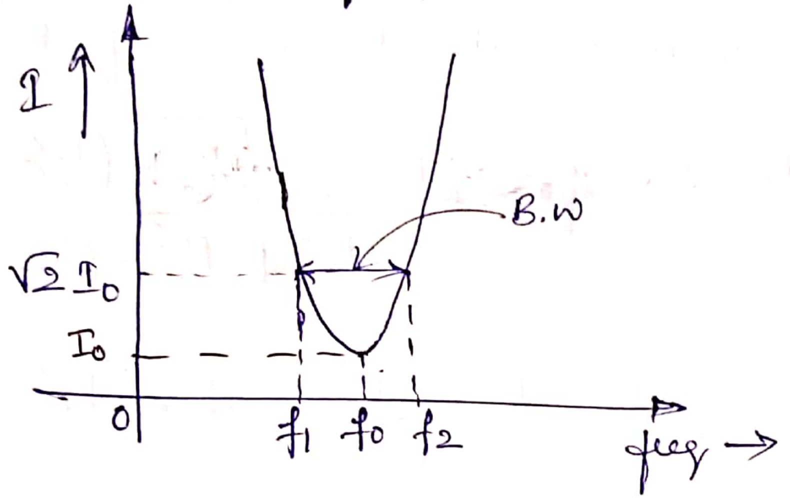
Hence whatever the value of ω_0,

RHS = LHS.

Any change in the value of ω_0 equally affects both sides.

In other words, the circuit is resonant at all frequencies.

Frequency response of parallel resonance circuit:-



Since the current at resonance is minimum the circuit is called as antiresonant circuit or rejector circuit.

Also, the impedance of the ||el resonance circuit is max @ resonance.

∴ The chars of the ||el resonant circuit are,

1) The impedance at resonance will be purely resistive & will be maximum.

2) Total current in the circuit is minimum & will be in phase with the applied vg.

3) The power factor at resonance will be unity.
4) Parallel resonance circuit is known as antiresonance ckt
5) Impedance at resonance is known as dynamic resistance (Z_d)

Comparison between series and parallel resonance

Parameter	Series Circuit	Parallel circuit
1) Impedance	$Z = R$ minimum	$Z_d = \frac{L}{CR}$ maximum
2) Power factor	unity	unity
3) Resonance freq	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
4) Current at resonance	Current is maximum at resonance, $I_0 = \frac{V}{R}$ & will be in phase with the applied vg	Current is minimum at resonance $I_0 = \frac{V}{Z_d}$ & will be in phase with the applied voltage.

Series Resonance

Note:- 1) $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Resonant frequency.

2) $Z = R$

3) $X_L = X_C$

4) $I_0 = \frac{V}{R}$

@ resonance ;

5) $Q = \frac{X_L}{R} = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}$

6) $Q = \frac{X_C}{R} = \frac{1}{\omega_0 C R}$

7) $B.W = \frac{f_0}{Q}$ & $B.W = f_2 - f_1$

8) $V_C = I X_C$ & $V_L = I X_L$

9) $f_{Lmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC - \frac{R^2C^2}{2}}}$

10) $f_{Cmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$

11) $f_0 = \sqrt{f_1 f_2}$

12) $f_1 = f_0 - \frac{R}{4\pi L}$ & $f_2 = f_0 + \frac{R}{4\pi L}$

Problems :-

1) A series RLC circuit has $R = 10\Omega$, $L = 0.1H$ & $C = 100\mu F$ is connected across 200V variable freq source. Find i) f_0 ii) Z at this freq. iii) Vg drop across L & C @ this freq. iv) Q-factor v) B.W vi) The freq @ which voltage across inductor is max. vii) the freq at which vg across capacitor is max.

Given :-

$R = 10\Omega$

$L = 0.1H$

$C = 100\mu F$

$V = 200V$

i) $f_0 = ?$

ii) Z @ f_0

iii) V_L & V_C @ f_0

iv) Q v) B.W

vi) $f_{Lmax} = ?$

vii) $f_{Cmax} = ?$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.1 \times 100 \times 10^{-6}}}$$

$$\boxed{f_0 = 50.32Hz}$$

$$\boxed{Z = R = 10\Omega}$$

$$V_L = IX_L = \frac{V}{R} \times 2\pi f_0 L$$

$$V_L = \frac{200}{10} \times 2\pi \times 50.32 \times 0.1$$

$$\boxed{V_L = 632.33V}$$

$V_C = I X_C$

$$V_C = \frac{V}{R} \times \frac{1}{2\pi f_0 C} = \frac{200}{10} \times \frac{1}{2\pi \times 50.32 \times 100 \times 10^{-6}}$$

$$\boxed{V_C = 632.57 \text{ volts}}$$

or $V_L = V_C = 632.57$ volts

$$Q = \frac{X_L}{R} = \frac{2\pi f_0 L}{R} = 3.16 \qquad \boxed{Q = 3.16}$$

$$B.W = \frac{f_0}{Q} \Rightarrow \boxed{B.W = 15.92 \text{ Hz}}$$

$$f_{Lmax} = \frac{1}{2\pi} \sqrt{\frac{1}{LC - \frac{R^2C^2}{2}}}$$

$$\boxed{f_{Lmax} = 51.63 \text{ Hz}}$$

$$f_{Cmax} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{f_{Cmax} = 49.05 \text{ Hz}}$$

2) A Series RLC circuit consists of 50Ω resistance 0.2H inductance & Capacitance of 10 μF with an apply vg of 20V. Determine i) Resonant freq. ii) Q-factor iii) Upper & lower cutoff freq & also find the B.W.

Given.

$R = 50\Omega$

$L = 0.2H$

$C = 10\mu F$

$V = 20V$

i) $f_0 = ?$

ii) $Q = ?$

iii) f_1 & $f_2 = ?$

iv) $B.W = ?$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{0.2 \times 10 \times 10^{-6}}}$$

$$\boxed{f_0 = 112.54 \text{ Hz}}$$

$$Q = \frac{X_L}{R} = \frac{2\pi f_0 L}{R}$$

$$Q = \frac{2\pi \times 112.54 \times 0.2}{50}$$

$$\boxed{Q = 2.82}$$

$$f_1 = f_0 - \frac{R}{4\pi L} = 94.39 \text{ Hz} \quad \boxed{f_1 = 94.39 \text{ Hz}}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 134.18 \text{ Hz} \quad \boxed{f_2 = 134.18 \text{ Hz}}$$

$\therefore$ $B.W = f_2 - f_1$ or $B.W = \dfrac{f_o}{Q}$

$$\boxed{B.W = 39.79 Hz}$$

$B.W = \dfrac{112.54}{2.82} = 39.9 Hz$

A Series RLC circuit Consists of $R = 10\Omega$, $L = 0.01H$ & $C = 0.01\mu F$ is Connected across a supply of 10mV. Determine.

i) f_o ii) B.W iii) Q iv) I_o v) f_1 & f_2.

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{0.01 \times 0.01 \times 10^{-6}}}$$

$$\boxed{f_o = 15.91 KHz}$$

$$Q = \frac{X_L}{R} = \frac{2\pi f_o L}{R}$$

$$\boxed{Q = 99.96}$$

$Q = \dfrac{f_o}{B.W}$ $\boxed{B.W = \dfrac{f_o}{Q} = 159.16 Hz}$

$f_1 = f_o - \dfrac{R}{4\pi L}$ $\boxed{f_1 = 15.83 KHz}$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$f_2 = 15.91\times10^3 + \frac{10}{4\pi\times 0.01}$$

$$\boxed{f_2 = 15.99\ KHz}$$

$$I_0 = \frac{V}{R} = \frac{10\times10^{-3}}{10}$$

$$\boxed{I_0 = 1m\ Amp}$$

4) A coil of resistance 20Ω & inductance $10mH$ is in series with a capacitance & is supplied with a constant voltage, variable frq source. The maximum current is 2A at 1000 Hz. Find the cut off frequencies.

Given:

$R = 20\Omega$

$L = 10\times10^{-3}H$.

$I_0 = 2A$

$f_0 = 1000\ Hz$

$f_1 = ?$

$f_2 = ?$

WKT,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0^2 = \frac{1}{4\pi^2 LC}$$

$$C = \frac{1}{4\pi^2 L f_0^2}$$

$$\boxed{C = 2.53\ \mu F}$$

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$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_1 = 1000 - \frac{20}{4\pi \times 10 \times 10^{-3}}$$

$$\boxed{f_1 = 854.02 \text{ Hz}}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$f_2 = 1000 + \frac{20}{4\pi \times 10 \times 10^{-3}}$$

$$\boxed{f_2 = 1172.33 \text{ Hz}}$$

5) An RLC series ckt draws a maximum current of 15A when connected to 230V – 50Hz supply. If the Q-factor is 5, find the parameters of the circuit.

Given.

$I_0 = 15A$

$f_0 = 50 \text{ Hz}$

$V = 230V$

$Q = 5$

$R = ?$ $L = ?$

$C = ?$

WKT, $I_0 = \frac{V}{R}$

$$R = \frac{V}{I_0} = 15.33\,\Omega$$

$$\boxed{R = 15.33\,\Omega}$$

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$\rightarrow Q = \frac{X_L}{R}$

$X_L = Q R$

$2\pi f_o L = Q . R. \longrightarrow L = \frac{Q \times R}{2\pi f_o} = \frac{5 \times 15.33}{2\pi \times 50}$

$$\boxed{L = 0.24 H}$$

$\rightarrow Q = \frac{X_C}{R}$

or $X_C = Q R$

$\frac{1}{2\pi f_o C} = Q R \implies C = \frac{1}{2\pi f_o Q R}$

$$\boxed{C = 41.52 \mu F}$$

6) An RLC series circuit has $R = 50\Omega$, $L = 0.5H$ & $C = 20\mu F$. If a constant voltage of 200V at variable freq is applied. Find the frequency at which resonance occurs. Also find the max vg drop across L & C.

Given:

$R = 50\Omega$

$L = 0.5 H$

$C = 20 \mu F$

WKT.

$f_o = \frac{1}{2\pi \sqrt{LC}}$

$f_o = \frac{1}{2\pi \sqrt{0.5 \times 20 \times 10^{-6}}}$

$$\boxed{f_0 = 50.33\ Hz}$$

To find V_{Lmax} :-

$$f_{Lmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC - \frac{R^2C^2}{2}}}$$

$$f_{Lmax} = 51.63\ Hz.$$

$$V_{Lmax} = I X_L$$

$$= \frac{V}{Z} X_L$$

$$= \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \times X_L$$

Now X_L & X_C @ f_{Lmax}

$X_L = 2\pi f_{Lmax} L$

$= 2\pi \times 51.63 \times 0.5$

$= 162.2\ \Omega$

$$X_C = \frac{1}{2\pi f_{Lmax} C}$$

$X_C = 154.13\ \Omega$

$$\therefore V_{Lmax} = \frac{200}{\sqrt{50^2 + (162.2 - 154.13)^2}} \times 162.2$$

$$\boxed{V_{Lmax} = 640.54\ \text{volts}}$$

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To find V_{cmax} :-

$$f_{cmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$f_{cmax} = 49.69 \text{ Hz}$$

X_L & X_C @ f_{cmax}

$$X_L = 2\pi f_{cmax} L \qquad X_C = \frac{1}{2\pi f_{cmax} C}$$

$$X_L = 156.12\,\Omega \qquad X_C = 160.13\,\Omega$$

$$V_{cmax} = I X_C$$

$$= \frac{V}{Z} X_C$$

$$= \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}} X_C$$

$$= \frac{200}{\sqrt{R^2 + (160.13 - 156.12)^2}} \times 160.12$$

$$\boxed{V_{cmax} = 638.46 \text{ volts}}$$

Extra

$$I_0 = \frac{V}{R} = \frac{200}{50} = 4 \text{ Amp.}$$

$V_L = I_0 X_L$

$= I_0 \times 2\pi f_0 L$

$= 4 \times 2\pi \times 50.33 \times 0.5$

$\boxed{V_L = 632.46 \text{ volts}}$

$V_C = I_0 X_C$

$= I_0 \times \frac{1}{2\pi f_0 C}$

$\boxed{V_C = 632.46 \text{ volts}}$

$\boxed{X_L = X_C}$ @ resonance.

$X_L = X_C = 158.115\ \Omega$

7) A voltage of $100\sqrt{2} \sin \omega t$ is applied to an RLC series circuit. At resonance freq, the vg across the inductor is 500 V, the B.W is 50 Hz. The impedance at resonance is 75Ω, find f_0 & constants of the circuit.

Given :-

$V = 100\sqrt{2} \sin \omega l$

At f_0, $V_L = 500$ volts.

$B.W = 50$ Hz.

$Z = R = 75\Omega$

$f_0 = ?$

R, L & C = ?

$V = \underbrace{100\sqrt{2}}_{V_m} \sin \omega t$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 100 \text{ volts}$$

$\therefore \boxed{V = V_{rms} = 100 \text{ volts}}$

$$Q = \frac{V_L}{V} = \frac{500}{100}$$

$$\boxed{Q = 5}$$

WKT $Q = \frac{f_0}{B.W}$

$$f_0 = 5 \times 50$$

$$\boxed{f_0 = 250\ Hz}$$

At resonance $Z = R = 75\,\Omega$

$$\boxed{R = 75\,\Omega}$$

$$Q = \frac{X_L}{R} \qquad Q = \frac{X_C}{R}$$

$$X_L = QR \qquad \frac{1}{2\pi f_0 C} = QR$$

$$2\pi f_0 L = QR \qquad C = \frac{1}{2\pi f_0 QR}$$

$$L = \frac{QR}{2\pi f_0}$$

$$L = \frac{5 \times 75}{2\pi \times 250} \qquad C = \frac{1}{2\pi \times 250 \times 5 \times 75}$$

$$\boxed{L = 0.23\ H} \qquad \boxed{C = 1.69\ \mu F}$$

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8) A voltage of $E = 100 \sin \omega t$ is applied to an RLC series circuit at resonant freq, the vg across the capacitor found to be 400V. The B.W is 75 Hz, the impedance at resonance is 100Ω. find the resonant freq & the constants of the circuit.

Given :-

$E = 100 \sin \omega t$

$V_C = 400\,V$

$B.W = 75\,Hz$

$Z = R = 100\,\Omega$

$f_0 = ?$

$R, L, C = ?$

Given, $E = 100 \sin \omega t$

$E_m = V_m = 100$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$V_{rms} = 70.71$ volts

$$Q = \frac{V_C}{V} = \frac{400}{70.71} = 5.65.$$

WKT $$Q = \frac{f_0}{B.W}$$

$f_0 = 5.65 \times 75 = 423.75\,Hz$

$$\boxed{f_0 = 423.75\,Hz}$$

$$Q = \frac{X_L}{R}$$

$$X_L = QR$$

$$2\pi f_0 L = QR$$

$$L = \frac{5.65 \times 100}{2\pi \times 423.75}$$

$$\boxed{L = 0.21\ H}$$

$$Q = \frac{X_C}{R}$$

$$QR = \frac{1}{2\pi f_0 C}$$

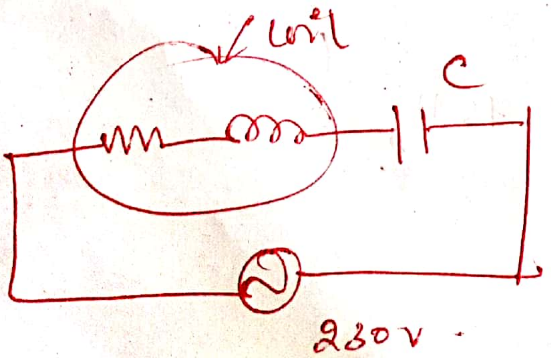
$$C = \frac{1}{2\pi f_0 QR}$$

$$C = \frac{1}{2\pi \times 423.75 \times 5.65 \times 100}$$

$$\boxed{Z = R = 100\,\Omega}$$

$$\boxed{C = 0.66\,\mu F}$$

9) A coil of resistance 20Ω & inductance 1 H is connected in series with a capacitor. The resonant freq is 100 rad/sec. If the supply is 230V - 50 Hz, find the
1) Line current 2) P.f 3) voltage across the coil & cap.



$R = 20\Omega$

$L = 1H$

$\omega_0 = 100 rad/sec$

$V = 230V, 50Hz$

$I = ?$

$P.f = ?$

$V_{coil} = ?$

To find line current,

$$I = \frac{V}{Z}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I = \frac{230}{\sqrt{20^2 + (314.15 - 31.83)^2}} = 0.812A$$

WKT, $X_L = 2\pi f L$

$X_L = 2\pi \times 50 \times 1$

$$\boxed{X_L = 314.15\Omega}$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2\pi \times 50 \times 100\mu}$$

$$\boxed{X_C = 31.83\Omega}$$

WKT, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0^2 = \frac{1}{LC}$$

$$C = \frac{1}{L\omega_0^2} = 100\mu F$$

$\therefore$ $$\boxed{I = 0.812 Amp}$$

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$$\text{Power factor} = \frac{R}{Z}$$

$$= \frac{20}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{20}{\sqrt{20^2+(314.15-31.83)^2}}$$

$$\boxed{P.f = 0.07}$$

Vg across the coil.

$$V_{coil} = I_{coil} \times Z_{coil}$$

$$Z_{coil} = \sqrt{R^2+X_L^2}$$

$$= \sqrt{20^2+(314.15)^2}$$

$$= 314.79\,\Omega$$

$$V_{coil} = 0.812 \times 314.79$$

$$\boxed{V_{coil} = 255.6 \text{ volts}}$$

$$V_c = I X_c$$

$$= 0.812 \times 31.83$$

$$\boxed{V_c = 25.84 \text{ volts}}$$

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10) A 220 V, 100 Hz AC source supplies a series RLC circuit with a capr & a coil. If the coil has 50 mΩ resistance & 5 mH inductance, find at a resonant frq of 100 Hz what is the value of capacitor. Also calculate the Q-factor & half power frequencies of the circuit.

Given.

$V = 220$ volts

$f = 100$ Hz

$R = 50\,m\Omega$

$L = 5\,mH$

At $f_0 = 100$ Hz.

$C = ?$

$Q = ?$

f_1 & $f_2 = ?$

WKT

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Squaring

$$C = \frac{1}{4\pi^2 L f_0^2}$$

$$C = \frac{1}{4\pi^2 \times 5\times10^{-3} \times (100)^2}$$

$$\boxed{C = 506.6\,\mu F}$$

$$Q = \frac{X_L}{R}$$

$$Q = \frac{2\pi f_0 L}{R}$$

$$\boxed{Q = 62.83}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_1 = 100 - \frac{50\times10^{-3}}{4\pi \times 5\times10^{-3}}$$

$$\boxed{f_1 = 99.20\text{ Hz}}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$f_2 = 100 + \frac{50\times10^{-3}}{4\pi\times5\times10^{-3}}$$

$$\boxed{f_2 = 100.8\text{ Hz}}$$

11) A Series RLC circuit has a resistance of 10Ω an inductance of 0.3H & a capacitance of $100\mu F$. The applied vg is 230V. Find i) f_0 ii) Q iii) lower & upper cut off fs iv) B.W v) Current at resonance vi) Currents at f_1 & f_2 vii) vg across inductance at resonance.

given

$R = 10\Omega$

$L = 0.3H$

$C = 100\mu F$

$V = 230V$

$f_0 =$

$Q =$

$f_1 =$

$f_2 =$

$B.W =$

$I_0 =$

Current at f_1 & f_2

@ f_1 $I = \frac{I_0}{\sqrt{2}} =$

$V_L = I_0 X_L$

$L_0 = I_0 \times 2\pi f_0 L$

$V_L =$

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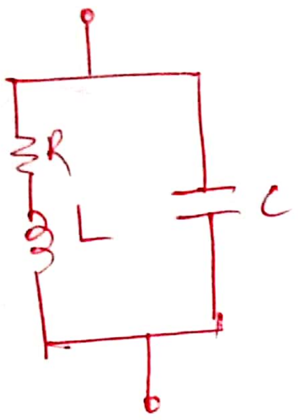
1) If $R = 25\Omega$, $L = 0.5H$ & $C = 5\mu F$, find the ω_0, f_0, Q & B.W for the circuit shown below.



WKT,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.5\times5\mu} - \frac{25^2}{(0.5)^2}}$$

$$\boxed{f_0 = 100.58\ Hz}$$

$\omega_0 = 2\pi f_0$

$\hookrightarrow = 2\pi \times 100.58$

$$\boxed{\omega_0 = 631.96\ rad/sec}$$

$$Q = \frac{\omega_0 L}{R}$$

$$Q = \frac{631.96\times 0.5}{25}$$

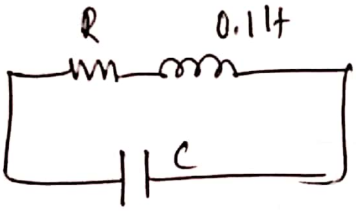
$$\boxed{Q = 12.64}$$

$$B.W = \frac{f_0}{Q}$$

$$B.W = \frac{100.58}{12.64}$$

$$\boxed{B.W = 50\ rad/sec}$$

2) In the ckt given, an inductance of 0.1H having a Q-factor of 5 is in ||lel with a capr. Determine the value of capacitor & coil resistance at resonant frq of 500 rad/sec.



$$Q = \frac{X_L}{R}$$

$R = ?$

$C = ?$

$W_0 = 500$ rad/sec

$L = 0.1H$

$Q = 5$

$$R = \frac{X_L}{Q} = \frac{W_0 L}{Q}$$

$$R = \frac{500 \times 0.1}{5}$$

$$\boxed{R = 10\Omega}$$

WKT,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$W_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\frac{1}{LC} = W_0^2 + \frac{R^2}{L^2}$$

$$\frac{1}{C} = L\left[W_0^2 + \frac{R^2}{L^2}\right] = 0.1\left[500^2 + \frac{10^2}{0.1^2}\right]$$

$$\boxed{C = 38.46\,\mu F}$$

3) A coil of 20Ω resistance has an inductance of $0.2H$ & is connected in $\parallel^{el}$ with $100\mu F$ capr. Cal the freq at which the circuit will act as a non-inductive resistance & also find the value of non-inductive resistance.

↳ pure resistance. (dynamic resistance)

$R = 20\Omega$

$L = 0.2H$

$C = 100\mu F$

$f_0 = ?$

$Z_d = ?$

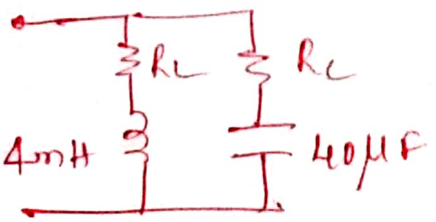
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_0 = 31.83\ Hz}$$

$$\& \ Z_d = \frac{L}{RC} = \frac{0.2}{20 \times 100 \times 10^{-6}}$$

$$\boxed{Z_d = 100\Omega}$$

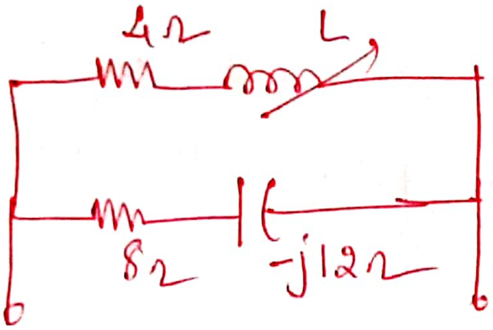
4) Determine R_L & R_C for which the circuit shown below resonates at all frequencies



WKT, $R_L = R_C = \sqrt{\frac{L}{C}}$

$$\boxed{R_L = R_C = 10\Omega}$$

5) Find the value of L for which the ckt given in below figure resonates at $\omega = 5000$ rad/sec.



WKT,

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = \frac{1}{4+jX_L} \times \frac{4-jX_L}{4-jX_L} + \frac{1}{8-j12} \times \frac{8+j12}{8+j12}$$

$$Y = \frac{4-jX_L}{4^2+X_L^2} + \frac{8+j12}{64+12^2}$$

$$Y = \frac{4}{16+X_L^2} + \frac{8}{208} - j\underbrace{\frac{X_L}{16+X_L^2} + j\frac{12}{208}}$$

@ resonance (B=0)

$$\frac{12}{208} = \frac{X_L}{16+X_L^2}$$

$$12(16+X_L^2) = 208X_L$$

$$192 + 12X_L^2 = 208X_L$$

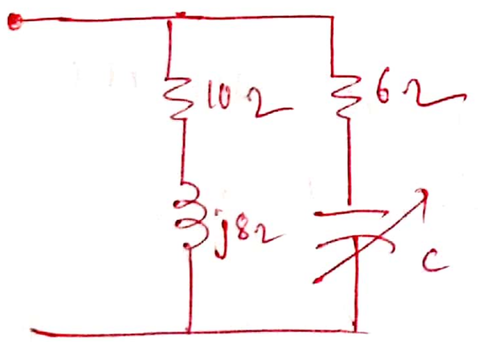
144
64
208

$$12X_L^2 - 208X_L + 192 = 0$$

$X_L = 16.36\,\Omega$ or $X_L = 0.978\,\Omega$

$\omega_0 L = 16.36$	$\omega_0 L = 0.978$
$L = \dfrac{16.36}{5000}$	$L = \dfrac{0.978}{5000}$
$L = 3.72\ mH$	$L = 0.196\ mH$

6) Find the value of C for which the circuit given in fig resonants at 750 Hz.



$$Y = \frac{1}{10+j8} + \frac{1}{6-jX_C}$$

$$Y = \frac{1}{10+j8} \times \frac{10-j8}{10-j8} + \frac{1}{6-jX_C} \times \frac{6+jX_C}{6+jX_C}$$

$f = 750\,Hz$

$$Y = \frac{10-j8}{100+64} + \frac{6+jX_C}{36+X_C^2}$$

$$Y = \frac{10}{164} - \frac{j8}{164} + \frac{6}{36+X_C^2} + \frac{jX_C}{36+X_C^2}$$

$Y = G + jB$ @ resonance $B = 0$.

$$\frac{X_c}{36 + X_c^2} = \frac{8}{164}$$

$$164 X_c = 288 + 8 X_c^2$$

$$8 X_c^2 - 164 X_c + 288 = 0$$

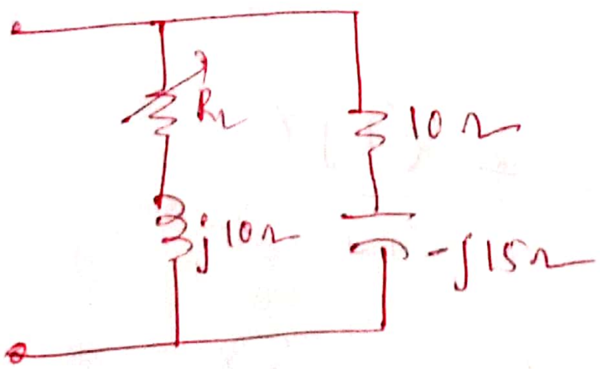
$X_c = 18.56\,\Omega$ or $X_c = 1.94\,\Omega$

$$\frac{1}{\omega_0 C} = 18.56\,\Omega \qquad \frac{1}{2\pi f_0 C} = 1.94$$

$$C = \frac{1}{2\pi f_0 \times 18.56} \qquad C = \frac{1}{2\pi \times 750 \times 1.94}$$

$$\boxed{C = 11.43\,\mu F} \qquad \boxed{C = 109.38\,\mu F}$$

7) find the value of R_L for which, the ckt shown in below fig is ~~Res~~ resonant.



$$Y = \frac{1}{R_L + j10} + \frac{1}{10 - j15}$$

$$Y = \frac{R_L - j10}{R_L^2 + 10^2} + \frac{10 + j15}{10^2 + 15^2}$$

$$Y = \frac{R_L}{R_L^2 + 100} + \frac{10}{100 + 225} + j\underbrace{\left[\frac{15}{225} - \frac{10}{R_L^2 + 100}\right]}_{\text{reactance}}$$

$B = 0$

$$\Rightarrow \frac{15}{225} = \frac{10}{R_L^2 + 100}$$

$$15\left[R_L^2 + 100\right] = 2250$$

$$15R_L^2 + 1500 = 3250$$

$$15R_L^2 = 3250 - 1500$$

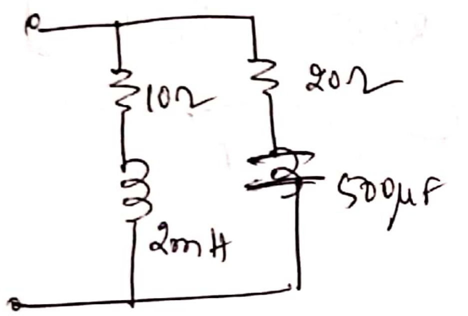
$$15R_L^2 = 1750$$

$$R_L^2 = \frac{1750}{15}$$

$$R_L^2 = 116.67$$

$$\boxed{R_L = 10.8\,\Omega}$$

8) An inductive coil of resistance 10Ω & inductance $20mH$ is connected in $\parallel^{el}$ with another branch consisting of a resistance of 20Ω in series with a capacitance of $500\mu F$. Find the resonant frq & the corresponding current, when the applied vg is 230V.



$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

$R_L = 10\Omega$

$R_C = 20\Omega$

$L = 2mH$

$C = 500\mu F$

$f_0 = ?$

$I_0 = ?$

$V = 230V$

$$f_0 = \frac{1}{2\pi\sqrt{2\times10^{-3}\times 500\times10^{-6}}}\sqrt{\frac{100 - \frac{2\times10^{-3}}{500\times10^{-6}}}{400 - \frac{2\times10^{-3}}{500\times10^{-6}}}}$$

$$f_0 = 78.36\ Hz$$

$$I_0 = VY_0$$

$$I_0 = V\left[\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right]$$

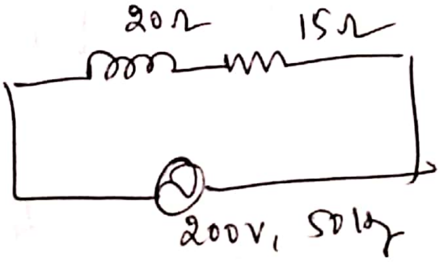
$X_L = 2\pi f_0 L$ & $X_C = \frac{1}{2\pi f_0 C}$

$X_L = 0.98\ \Omega$ $X_C = 4.06\ \Omega$

$$I_0 = 230\left[\frac{10}{100+0.98^2} + \frac{20}{400+4.06^2}\right]$$

$$\boxed{I_0 = 33.82 \text{ Amp}}$$

9) A circuit has inductive reactance of 20Ω at 50Hz in series with a resistance of 15Ω for an applied vg of 200V at 50Hz. Calculate
1) Phase angle b/n current & voltage
2) the current.
3) The value of shunting capacitance to bring the ckt to resonance & the current at resonance



$X_L = 20\Omega$

$f = 50Hz$

$R = 15\Omega$

$V = 200V$

1) $\phi = ?$

2) $I = ?$

3) $\left.\begin{array}{l} C = ? \\ I_0 = ? \end{array}\right\} @ f_0$

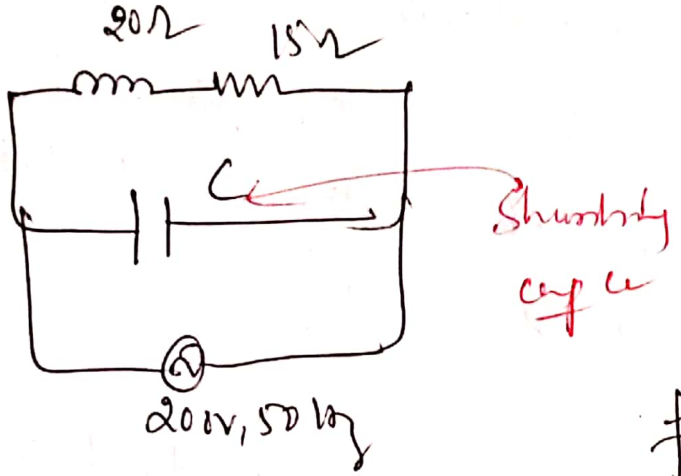
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\boxed{\phi = 53.13^\circ}$$

$$I = \frac{V}{Z} = \frac{V}{R + jX_L}$$

$$I = \frac{200}{15 - j20} = 4.8 - j6.4$$

$$\boxed{I = 8 \angle -53.13^\circ \text{ amp}}$$



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0^2 = \frac{1}{4\pi^2}\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]$$

$$4\pi f_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$X_L = 20$

$2\pi f_0 L = 20$

$$L = \frac{20}{2\pi \times 50}$$

$L = 0.0637 H$

$$\frac{1}{LC} = 4\pi f_0^2 + \frac{R^2}{L^2}$$

$$\frac{1}{C} = L\left[4\pi f_0^2 + \frac{R^2}{L^2}\right]$$

$$\frac{1}{C} = 0.0637\left[4\pi^2 \times (50)^2 + \frac{15^2}{(0.0637)^2}\right]$$

$$\frac{1}{C} = 6322.26$$

Scanned with CamScanner

or $\boxed{C = 158.17\ \mu F}$

$I_0 = \frac{V}{Z_d}$ where $Z_d = \frac{L}{RC}$

$Z_d = \frac{0.0637}{15 \times 158.17 \times 10^{-6}}$

$I_0 = \frac{200}{26.55}$

$Z_d = 26.55\ \Omega$

$\boxed{I_0 = 7.533\ \text{amp}}$

10) A parallel resonant ckt has a capacitance of 100pF in one branch & an inductance of 100μH with a resistance of 10Ω in the other branch, the line voltage is 100V. Find i) f_0 2) I_L & I_C 3) I & Z at resonance

$R = 10\ \Omega$

$L = 100\ \mu H$

$C = 100\ pf$

$V = 100V$

$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$\boxed{f_0 = 1.59\ MHz}$

Scanned with CamScanner

$$I_L = \frac{V}{Z_L} = \frac{V}{R + jX_L}$$

$$X_L = 2\pi f_0 L$$

$$X_L = 999.02\,\Omega$$

$$= \frac{100}{10 + j999.02}$$

$$\boxed{I_L = 0.1 \angle -89.42 \text{ Amp}}$$

$$I_c = \frac{V}{Z_c} = \frac{V}{-jX_c}$$

$$X_c = \frac{1}{2\pi f_0 C}$$

$$I_c = \frac{100}{-j1\times10^3}$$

$$X_c = 1k\Omega$$

$$I_c = \frac{100}{1\times10^3 \angle -90}$$

$$\boxed{I_c = 0.1 \angle 90^\circ \text{ amp}}$$

$$Z_d = \frac{L}{RC} = 100k\Omega$$

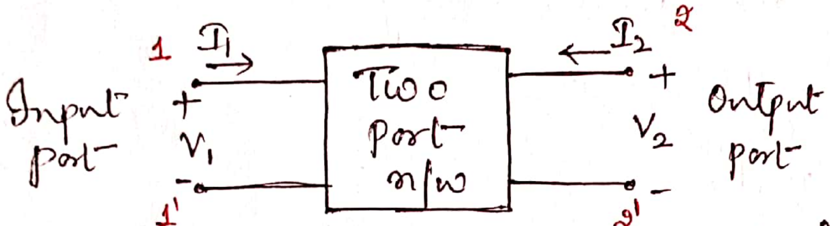
$$I_0 = \frac{V}{Z_d} = 1m \text{ Amp.}$$

Module-5 Two-Port Network Parameters

Representation of a two-port network:-

A two-port network is a four terminal network. Two input terminals & two output terminals. V_1 & I_1 are the variables at input port & V_2 & I_2 are the variables @ output port. Out of 4 variables V_1, I_1, V_2 & I_2, two of them chosen as independent variables & the remaining two as dependent variables.

11' → i/p port.
22' → o/p port.



Two-port networks are important in modeling electronic devices & system components.

For eg: In electronics, two port n/ws are employed to model transistors & op-amps.

Other examples of electrical components modeled by two ports are transformers & transmission lines.

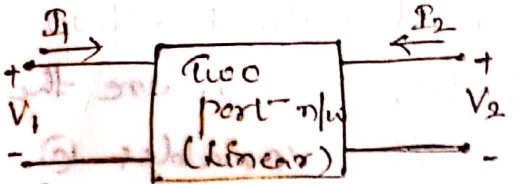
The parameters of a two-port n/w completely describes the physical behavior of any electronic devices.

Thus knowing the parameters of a two-port n/w permitts us to describe its operation when it is connected to a larger network.

Two port n/w parameters are classified as

1) Impedence parameters or Z-parameters. (open-ckt)
2) Admittence parameters or Y-parameters (Short-ckt)
3) Hybrid parameters or h-parameters
4) Transmission parameters or ABCD parameters or T-parameters.

⇒ Impedance Parameters (or Z-parameters or Open ckt impedance parameters)



The n/w shown in figure is assumed to be linear & no independent sources. Then using superposition theorem, we can write the i/p & output voltages as the sum of two components, one due to I_1 & other due to I_2.

Z-parameters eqns are described by,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (2)}$$

In matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The Z-parameters are defined as follows,

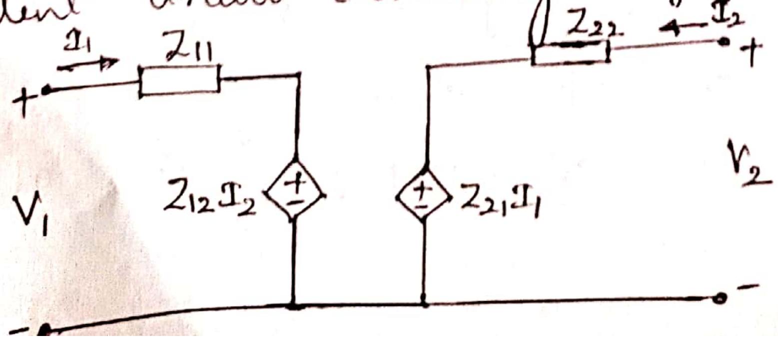
$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ open circuit input impedance parameter.

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ Open circuit reverse transfer impedance.

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ open circuit forward transfer impedance.

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ open circuit output impedance parameter

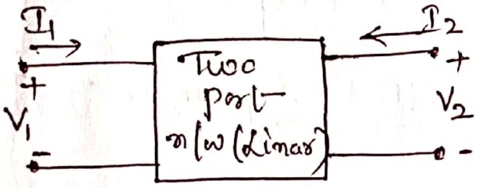
Equivalent circuit describing Z-parameters are,



If $\boxed{Z_{12} = Z_{21}}$ i.e, the transfer impedances are equal then such a n/w is called the "RECIPROCAL NETWORK"

If $\boxed{Z_{11} = Z_{22}}$ then such a n/w is called a Symmetrical n/w.

⇒ Admittence Parameters or Y-Parameters (or Short-circuit admittence parameters)



The n/w shown in figure is assumed to be linear & no independent sources. Then using Superposition theorem, we can write the i/p & output currents as the sum of two components, one due to V_1 & other due to V_2.

Y-Parameters eqns are described by,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— (2)}$$

In matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Y-Parameters are defined as follows,

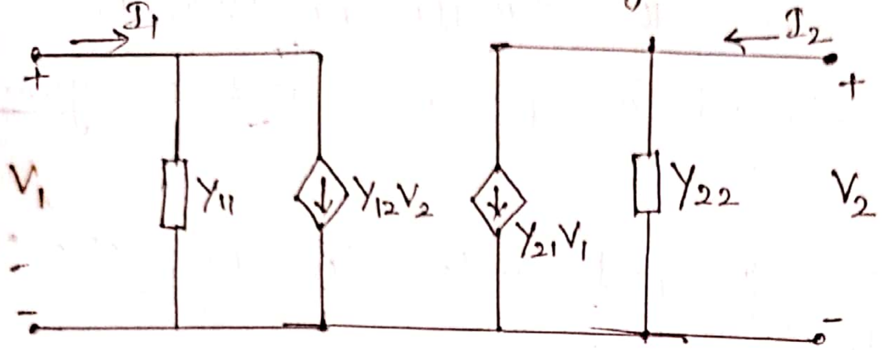
$Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$ Short circuit input admittence parameters.

$Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0}$ Short circuit reverse transfer admittence parameters.

$Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$ Short circuit forward transfer admittence.

$Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0}$ Short circuit Output admittence parameter.

Equivalent Circuit describing Y-Parameters are,



If $\boxed{Y_{12} = Y_{21}}$ then such a n/w is called a "reciprocal n/w"

& If $\boxed{Y_{11} = Y_{22}}$ then such a network is called "symmetrical n/w"

⇒ Hybrid Parameters or h-parameters :-

Hybrid parameters are very useful in constructing models for transistors. h-parameters completely describes the internal behaviour of transistors (BJT's).

h-parameters eqns are described by,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——} (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——} (2)$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h-parameters are defined as follows,

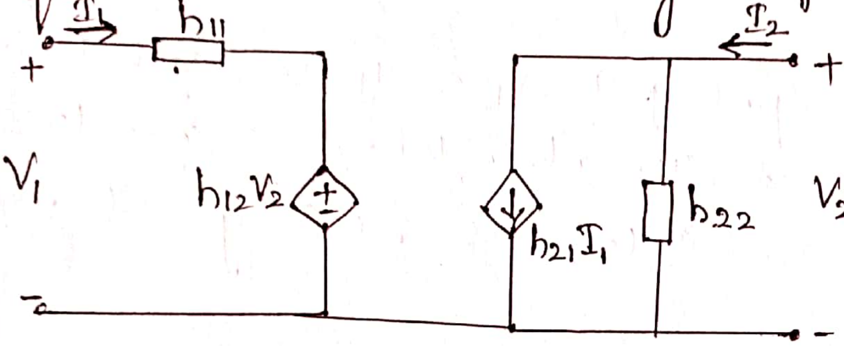
$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Short-circuit input impedance parameter.

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ Known as reverse-Voltage gain.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Known as forward-current gain.

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ known as Open-Circuit Output admittance parameter.

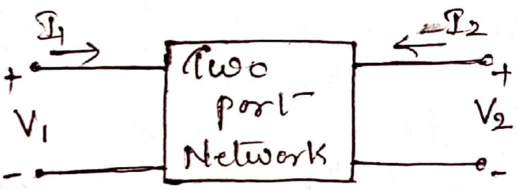
Equivalent circuit describing h-parameters,



If $\boxed{h_{12} = -h_{21}}$ then the circuit is said to be reciprocal n/w

& if $\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$ or $\boxed{\Delta h = 1}$ then the n/w is said to be Symmetrical n/w.

⇒ Transmission parameters or ABCD parameters or T-parameters.



The T-parameters eqns are described by,

$V_1 = AV_2 - BI_2$ —— ①

$I_1 = CV_2 - DI_2$ —— ②

In matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

T-parameters are mainly used in analysis of Transmission lines & Cascaded networks. & hence they are called as transmission parameters.

T-parameters are defined as,

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ reverse voltage gain with o/p port open circuited

$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0}$ reverse transfer impedance

$\therefore C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ reverse transfer admittance.

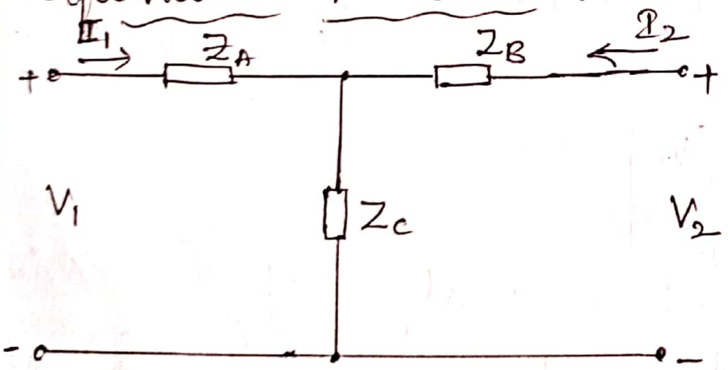
$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$ reverse current gain with o/p port short ckt'd.

Quantities at the input port V_1 & I_1 are called as sending end voltages & currents. Where as quantities at the output port are called as receiving end voltages & currents.

T and π network representation of a two port network

Practically the transmission lines, filters & attenuators are represented in the form of equivalent T or π networks. Thus if the Y-parameters are known, an equivalent π n/w can be easily constructed & if the Z-parameters are known an equivalent T n/w can be made.

Equivalent T-network:



It is required to determine Z_A, Z_B & Z_C in terms of Z_{11}, Z_{12}, Z_{21} & Z_{22}.

W.K.T $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$$\therefore \boxed{Z_{11} = Z_A + Z_C}$$

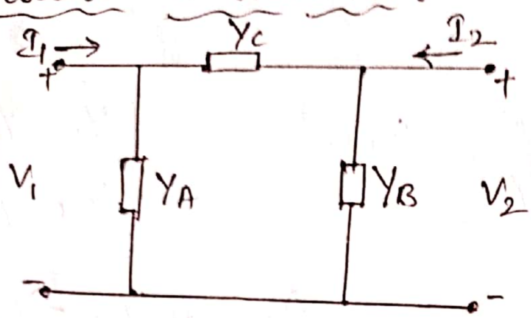
Also $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ (when $I_1=0$ current through Z_A is zero, ∴ it can be neglected)

$$\boxed{Z_{22} = Z_B + Z_C}$$

& $\boxed{Z_{12} = Z_{21} = Z_C}$

If Z-parameters of the n/w are known, the equivalent T n/w can be found out using the above relations.

Equivalent π network :-



It is required to find Y_A, Y_B & Y_C in terms of Y-parameters.

W.K.T $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

When $V_2=0$, i.e, o/p port shorted then Y_B becomes zero,

$$\therefore \boxed{Y_{11} = Y_A + Y_C}$$

IIIly $\boxed{Y_{22} = Y_B + Y_C}$

Also, $Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$, $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

$$\boxed{Y_{12} = Y_{21} = -Y_C}$$

Thus knowing Y-parameters of a 2-port n/w, elements of π n/w can be found out using the relations derived above.

Relations between two parameters :-

1) Z-parameters in terms of Y-parameters

The equations describing Z-parameters are

$V_1 = Z_{11} I_1 + Z_{12} I_2$ ——— ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ ——— ②

The equations describing Y-parameters are

$I_1 = Y_{11} V_1 + Y_{12} V_2$ ——— ③

$I_2 = Y_{21} V_1 + Y_{22} V_2$ ——— ④

Y-parameters in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By using Crammer's rule,

$$V_1 = \frac{\Delta_1}{\Delta Y} \quad , \quad V_2 = \frac{\Delta_2}{\Delta Y} \quad \text{where } \Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\Delta Y}$$

$$V_1 = \frac{Y_{22} I_1 - Y_{12} I_2}{\Delta Y} = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2$$

$$V_1 = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2 \quad \text{——— (5)}$$

Comparing equations (5) & (1)

$$\boxed{Z_{11} = \frac{Y_{22}}{\Delta Y}} \qquad \boxed{Z_{12} = \frac{-Y_{12}}{\Delta Y}}$$

Also, $V_2 = \dfrac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y}$

$$V_2 = \frac{Y_{11} I_2 - Y_{21} I_1}{\Delta Y}$$

$$V_2 = \frac{Y_{11}}{\Delta Y} I_2 - \frac{Y_{21}}{\Delta Y} I_1 \quad \text{rearranging the terms}$$

$$V_2 = \frac{-Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 \quad \text{——— (6)}$$

Comparing eqn ② & ⑥

$$\boxed{Z_{21} = -\frac{Y_{21}}{\Delta Y}} \qquad \boxed{Z_{22} = \frac{Y_{11}}{\Delta Y}}$$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{+Y_{11}}{\Delta Y} \end{bmatrix}$$

2) Z-parameters in terms of h-parameters :

The equations describing Z-parameters are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{——} ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{——} ②$$

The equations describing h-parameters are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——} ③$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——} ④$$

From eqn ④ $\quad I_2 = h_{21} I_1 + h_{22} V_2$

$$h_{22} V_2 = I_2 - h_{21} I_1$$

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}} I_1$$

or $\quad V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{——} ⑤$

Comparing ② & ⑤

$$\boxed{Z_{21} = \frac{-h_{21}}{h_{22}}} \qquad \boxed{Z_{22} = \frac{1}{h_{22}}}$$

Substitute eqn ⑤ in eqn ③, we get.

$$V_1 = h_{11} I_1 + h_{12} \left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right]$$

$$V_1 = h_{11} I_1 - \frac{h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{(h_{11} h_{22} - h_{12} h_{21})}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{——} ⑥$$

where $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Comparing ① & ⑥, we get-

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}}}, \quad \boxed{Z_{12} = \frac{h_{12}}{h_{22}}}$$

Z — Y, h, T

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

3) Z- parameters in terms of T- parameters.

Z-parameters equations are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{——} ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{——} ②$$

T- parameters eqns are.

$$V_1 = A V_2 - B I_2 \quad \text{——} ③$$

$$I_1 = C V_2 - D I_2 \quad \text{——} ④$$

from eqn ④, $I_1 = C V_2 - D I_2$

$$C V_2 = I_1 + D I_2$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{——} ⑤$$

Comparing this with eqn ②.

$$\boxed{Z_{21} = \frac{1}{C}} \quad \boxed{Z_{22} = \frac{D}{C}}$$

From eqn, ③.

$$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \frac{AD}{C} I_2 - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \frac{(AD - BC)}{C} I_2 \quad \text{—— (6)}$$

Comparing (6) with eqn (1), we get,

$$\boxed{Z_{11} = \frac{A}{C}} \qquad \boxed{Z_{12} = \frac{AD - BC}{C}}$$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{AD-BC}{C} \\ \frac{1}{C} & D/C \end{bmatrix}$$

4) Y-parameters in terms of Z-parameters:

The equations describing Y-parameters are,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— (2)}$$

The equations describing Z-parameters are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (4)}$$

In matrix form, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

But $I_1 = \frac{\Delta_1}{\Delta Z}$, $I_2 = \frac{\Delta_2}{\Delta Z}$ $\qquad \Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\Delta Z}$$

$$I_1 = \frac{V_1 Z_{22} - V_2 Z_{12}}{\Delta Z} = \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12}}{\Delta Z} V_2 \quad \text{—— (5)}$$

Comparing ⑤ with eqn ①, we get

$$\boxed{Y_{11} = \frac{Z_{22}}{\Delta Z}} \qquad \boxed{Y_{12} = \frac{-Z_{12}}{\Delta Z}}$$

Similarly, $I_2 = \dfrac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z}$

$$I_2 = \frac{Z_{11} V_2 - Z_{21} V_1}{\Delta Z}$$

$$I_2 = \frac{Z_{11}}{\Delta Z} V_2 - \frac{Z_{21}}{\Delta Z} V_1 \quad \text{rearranging the terms}$$

or $I_2 = \dfrac{-Z_{21}}{\Delta Z} V_1 + \dfrac{Z_{11}}{\Delta Z} V_2$ —— ⑥

Comparing ⑥ with eqn ②, we get.

$$\boxed{Y_{21} = \frac{-Z_{21}}{\Delta Z}} \qquad \boxed{Y_{22} = \frac{Z_{11}}{\Delta Z}}$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

5) Y- parameters in terms of h-parameters :

Y- parameters eqns are

$\therefore I_1 = Y_{11} V_1 + Y_{12} V_2$ —— ①

$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— ②

h-parameters eqns are,

$V_1 = h_{11} I_1 + h_{12} V_2$ —— ③

$I_2 = h_{21} I_1 + h_{22} V_2$ —— ④

From ③, $V_1 = h_{11} I_1 + h_{12} V_2$

$$I_1 = \frac{V_1 - h_{12} V_2}{h_{11}} = \frac{1}{h_{11}} V_1 - \frac{h_{12} V_2}{h_{11}} \quad \text{—— ⑤}$$

Comparing (5) & (1).

$$\boxed{Y_{11} = \frac{1}{h_{11}}} \qquad \boxed{Y_{12} = \frac{-h_{12}}{h_{11}}}$$

Substitute (5) in eqn (4)

$$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 - \frac{h_{21}h_{12}}{h_{11}}V_2 + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + V_2\left[h_{22} - \frac{h_{21}h_{12}}{h_{11}}\right] \longrightarrow \underbrace{h_{11}h_{22} - h_{21}h_{12}}_{\Delta h}$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + \frac{\Delta h}{h_{11}}V_2 \quad \text{——(6)}$$

Comparing (6) & (2), we get.

$$\boxed{Y_{21} = \frac{h_{21}}{h_{11}}} \qquad \boxed{Y_{22} = \frac{\Delta h}{h_{11}}}$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$$

Y — Z, h, T

6) Y- Parameters in terms of T- Parameters :

Y- parameters eqns are,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{——(1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{——(2)}$$

T- parameters eqns are

$$V_1 = AV_2 - BI_2 \quad \text{——(3)}$$

$$I_1 = CV_2 - DI_2 \quad \text{——(4)}$$

From (3) $V_1 = AV_2 - BI_2$

or $I_2 = \dfrac{AV_2 - V_1}{B} = -\dfrac{1}{B}V_1 + \dfrac{A}{B}V_2$ ——(5)

Comparing ② & ⑤

$$\boxed{Y_{21} = \frac{-1}{B}} \qquad \boxed{Y_{22} = \frac{A}{B}}$$

from equation ④,

$$I_1 = CV_2 - D\left[\frac{-1}{B}V_1 + \frac{A}{B}V_2\right]$$

$$I_1 = CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2$$

$$I_1 = \left(C - \frac{AD}{B}\right)V_2 + \frac{D}{B}V_1$$

or $$I_1 = \frac{D}{B}V_1 + \frac{BC-AD}{B}V_2 \quad \text{——} ⑥$$

Comparing ① & ⑥.

$$\boxed{Y_{11} = \frac{D}{B}} \qquad \boxed{Y_{12} = \frac{BC-AD}{B}}$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & \frac{BC-AD}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

7) h-parameters in terms of Z-parameters :-

h-parameters eqns are

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{——} ①$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{——} ②$$

Z-parameters eqns are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{——} ③$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{——} ④$$

From eqn ④

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$I_2 = \frac{V_2 - Z_{21}I_1}{Z_{22}} = \frac{1}{Z_{22}}V_2 - \frac{Z_{21}}{Z_{22}}I_1$$

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \quad \text{—— (5)}$$

Comparing eqn (2) & (5)

$$\boxed{h_{21} = -\frac{Z_{21}}{Z_{22}}} \qquad \boxed{h_{22} = \frac{1}{Z_{22}}}$$

From (3), $V_1 = Z_{11} I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right]$

$$V_1 = Z_{11} I_1 + \frac{Z_{12}}{Z_{22}} V_2 - \frac{Z_{12} Z_{21}}{Z_{22}} I_1$$

$$V_1 = \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad \text{—— (6)}$$ where $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

Comparing (1) & (6), we get,

$$\boxed{h_{11} = \frac{\Delta Z}{Z_{22}}}, \qquad \boxed{h_{12} = \frac{Z_{12}}{Z_{22}}}$$

$$\therefore \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

h — Z, Y, T

8) h-parameters in terms of Y-parameters

h-parameters equations are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—— (2)}$$

Y-parameters eqns are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— (4)}$$

From (3) $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad \text{—— (5)}$$

Compare (5) with (1), we get.

$$\boxed{h_{11} = \frac{1}{Y_{11}}} \qquad \boxed{h_{12} = \frac{-Y_{12}}{Y_{11}}}$$

From (4),

$$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 - \frac{Y_{21} Y_{12}}{Y_{11}} V_2 + Y_{22} V_2.$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + V_2\left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}}\right] \rightarrow \Delta h \quad \text{—— (6)}$$

Compare (6) with (2), weget,

$$\boxed{h_{21} = \frac{Y_{21}}{Y_{11}}} \qquad \boxed{h_{22} = \frac{\Delta Y}{Y_{11}}}$$

$$\therefore \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$$

9) h-parameters in terms of T-parameters :

h-parameters eqns are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—— (2)}$$

T-parameters eqns are,

$$V_1 = AV_2 - BI_2 \quad \text{—— (3)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—— (4)}$$

From (4), $I_2 = \dfrac{CV_2 - I_1}{D}$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1 \Rightarrow I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \quad \text{—— (5)}$$

Comparing ⑤ with eqn ②

$$\boxed{h_{21} = \frac{-1}{D}} \qquad \boxed{h_{22} = \frac{C}{D}}$$

From eqn (3),

$$V_1 = AV_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right]$$

$$V_1 = AV_2 + \frac{B}{D} I_1 - \frac{BC}{D} V_2 = \frac{(AD - BC)}{D} V_2 + \frac{B}{D} I_1$$

$$\Rightarrow V_1 = \frac{B}{D} I_1 + \frac{(AD - BC)}{D} V_2 \quad \text{—— ⑥}$$

Comparing ⑥ with ①

$$\boxed{h_{11} = \frac{B}{D}} \qquad \boxed{h_{12} = \frac{AD - BC}{D}}$$

$$\therefore \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

10) T-parameters in terms of Z-parameters

T-parameters eqns are

$$V_1 = AV_2 - BI_2 \quad \text{—— ①}$$

$$I_1 = CV_2 - DI_2 \quad \text{—— ②}$$

Z-parameters eqns are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— ③}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— ④}$$

$$T \begin{cases} Z \\ Y \\ h \end{cases}$$

From (4),

$$I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}}$$

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{—— ⑤}$$

from ② & ⑤

$$\boxed{C = \frac{1}{Z_{21}}} \qquad \boxed{D = \frac{Z_{22}}{Z_{21}}}$$

From (3), $V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \underbrace{\frac{Z_{11} Z_{22}}{Z_{21}} I_2 + Z_{12} I_2}$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 + I_2\left[\frac{Z_{12} Z_{21} - Z_{11} Z_{22}}{Z_{21}}\right]$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{(\overbrace{Z_{11} Z_{22} - Z_{12} Z_{21}}^{\Delta Z})}{Z_{21}} I_2$$

$$\therefore \quad V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2 \quad \text{—— (6)}$$

Comparing (6) & (1) we get.

$$\boxed{A = \frac{Z_{11}}{Z_{21}}} \qquad \boxed{B = \frac{\Delta Z}{Z_{21}}}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

iii) T- parameters in terms Y- parameters.

T- parameters eqns are

$V_1 = A V_2 - B I_2$ —— (1)

$I_1 = C V_2 - D I_2$ —— (2)

Y - parameters eqns are,

$I_1 = Y_{11} V_1 + Y_{12} V_2$ —— (3)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— (4)

from (4)

$$V_1 = \frac{I_2 - Y_{22} V_2}{Y_{21}}$$

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \quad \text{—— (5)}$$

Comparing ① & ⑤, we get

$$\boxed{A = \frac{-Y_{22}}{Y_{21}}} \qquad \boxed{B = -\frac{1}{Y_{21}}}$$

from (3) $I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2$

$$I_1 = -\frac{Y_{11} Y_{22}}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 + Y_{12} V_2$$

$$I_1 = -\left(\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{21}}\right) V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

or $I_1 = \frac{-\Delta Y}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2$ ——— ⑥

Comparing ⑥ & ② we get,

$$\boxed{C = \frac{-\Delta Y}{Y_{21}}} \qquad \boxed{D = \frac{-Y_{11}}{Y_{21}}}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

12) T-parameters in terms of h-parameters :-

T-parameters eqns are

$V_1 = AV_2 - BI_2$ ——— ①

$I_1 = CV_2 - DI_2$ ——— ②

h-parameters eqns are

$V_1 = h_{11} I_1 + h_{12} V_2$ ——— ③

$I_2 = h_{21} I_1 + h_{22} V_2$ ——— ④

from eqn (4) $I_1 = \frac{I_2 - h_{22} V_2}{h_{21}}$

$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \quad ——— ⑤$$

Comparing (5) with (2)

$$\boxed{C = \frac{-h_{22}}{h_{21}}} \qquad \boxed{D = \frac{-1}{h_{21}}}$$

From (3), $V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right] + h_{12} V_2$

$$V_1 = -\frac{h_{11}h_{22}}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2 + h_{12} V_2$$

$$V_1 = -\left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}\right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$\left\{ \begin{aligned} &V_2\left(h_{12} - \frac{h_{11}h_{22}}{h_{21}}\right) \\ &V_2\left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right] \\ &-\left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}\right) \end{aligned} \right.$$

or $V_1 = -\frac{\Delta h}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2$ —— (6)

Comparing (6) with (1)

$$\boxed{A = \frac{-\Delta h}{h_{21}}} \qquad \boxed{B = \frac{-h_{11}}{h_{21}}}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$$

Symmetry & Reciprocity of two-port n/ws.

"Symmetry & reciprocity are the two imp characteristics of two-port n/ws. A network is said to be symmetrical if the it exhibits the same characteristics when its ports are interchanged."

For Z-parameters, if the impedance measured at one point is equal to the impedance measured @ the other port with remaining port open ckt'd, the n/w is said to be symmetrical.

$$\boxed{Z_{11} = Z_{22}}$$

"A n/w is said to be reciprocal, if it exhibits the property that the ratio of source to response @ both ports are same."

For Z-parameters, if $\boxed{Z_{12} = Z_{21}}$, the networks is said to be reciprocal.

2) Y-Parameters :-

Condition of symmetry for y-parameters is $\boxed{Y_{11} = Y_{22}}$

& Condition of reciprocity is $\boxed{Y_{12} = Y_{21}}$.

3) h-parameters :-

For h-parameters, condition of symmetry is $\Delta h = 1$

i.e., $\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$

& the condition of reciprocity is $\boxed{h_{12} = -h_{21}}$

4) ABCD or T-parameters :

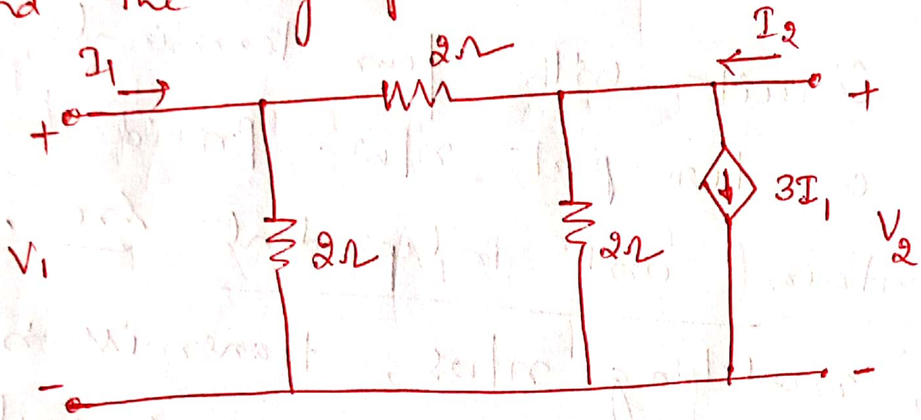
T-n/w is said to be symmetrical if $\boxed{A = D}$ & it is said to be reciprocal if $\boxed{AD - BC = 1}$.

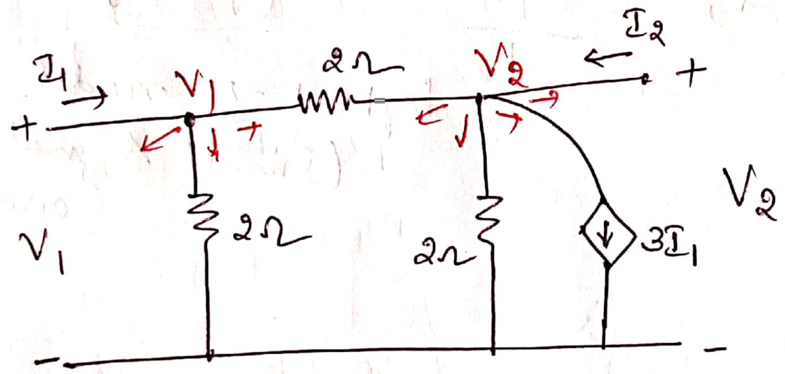
	Parameters	Condition of symmetry	Cond. of reciprocity
a)	Z	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
b)	Y	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
c)	h	$\Delta h = 1$	$h_{12} = -h_{21}$
d)	T	$A = D$	$AD - BC = 1$

Problems on Two-port network :-

I) Problems on dependent source [Always use KCL method]

1) Find the y- parameters.





Apply KCL @ node 1

$$-I_1 + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0$$

$$-I_1 + \frac{V_1}{2} + \frac{V_1}{2} - \frac{V_2}{2} = 0$$

$$I_1 = V_1 - \frac{V_2}{2} \Rightarrow I_1 = V_1 - 0.5V_2 \quad \text{—(1)}$$

Scanned with CamScanner

Comparing with std eqn

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$\boxed{Y_{11} = 1 \mho} \qquad \boxed{Y_{12} = -0.5 \mho}$$

Apply KCL @ node 2.

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2} + 3I_1 - I_2 = 0$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{2} + 3(I_1) - I_2 = 0$$

↳ eqn ①

$$V_2 - 0.5 V_1 + 3(V_1 - 0.5 V_2) - I_2 = 0$$

$$V_2 - 0.5 V_1 + 3V_1 - 1.5 V_2 - I_2 = 0$$

$$I_2 = 2.5 V_1 - 0.5 V_2 \quad \text{——} ②$$

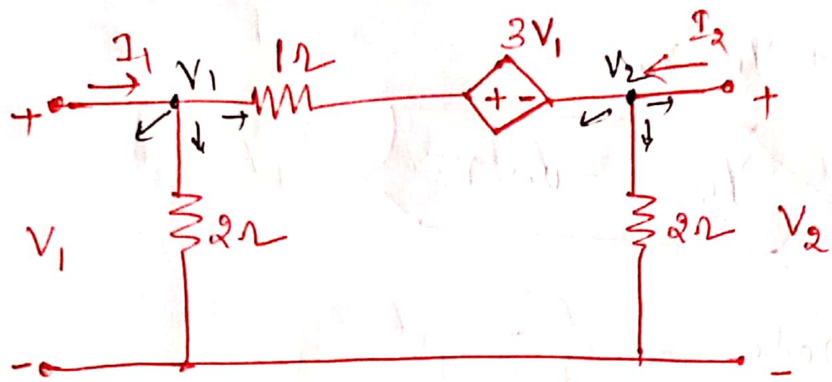
Comparing with std eqn

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\boxed{Y_{21} = 2.5 \mho} \qquad \boxed{Y_{22} = -0.5 \mho}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ 2.5 & -0.5 \end{bmatrix} \mho$$

2) Find Y- parameters



Apply KCL @ node 1.

$$-I_1 + \frac{V_1}{2} + \left(\frac{V_1 - 3V_1 - V_2}{1}\right) = 0$$

$$-I_1 + 0.5\,V_2 + V_1 - 3V_1 - V_2 = 0$$

$$I_1 = -1.5\,V_1 - V_2 \quad \text{—— (1)}$$

Comparing with Std eqn. $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$\boxed{Y_{11} = -1.5\,\mho} \qquad \boxed{Y_{12} = -1\,\mho}$$

Apply KCL @ node 2

$$\frac{V_2}{2} + \frac{V_2 + 3V_1 - V_1}{1} - I_2 = 0$$

$$0.5V_2 + V_2 + 3V_1 - V_1 - I_2 = 0$$

$$I_2 = 2V_1 + 1.5\,V_2$$

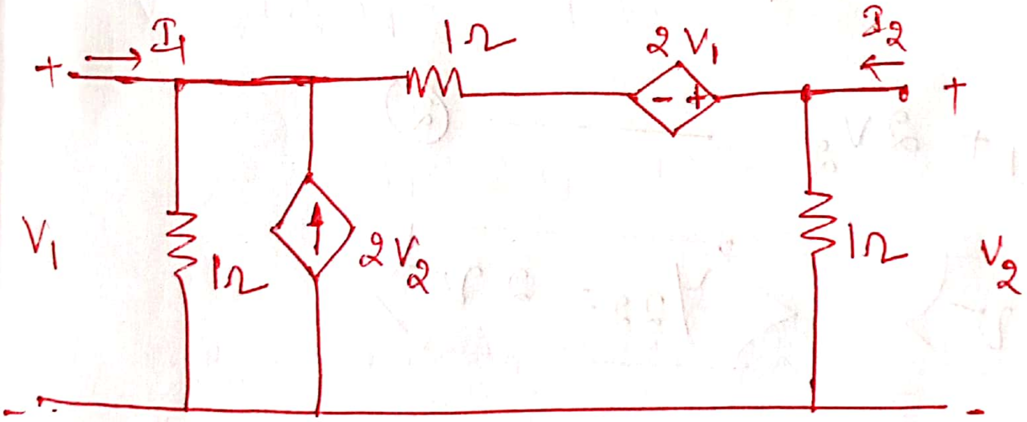
Company with std eqn

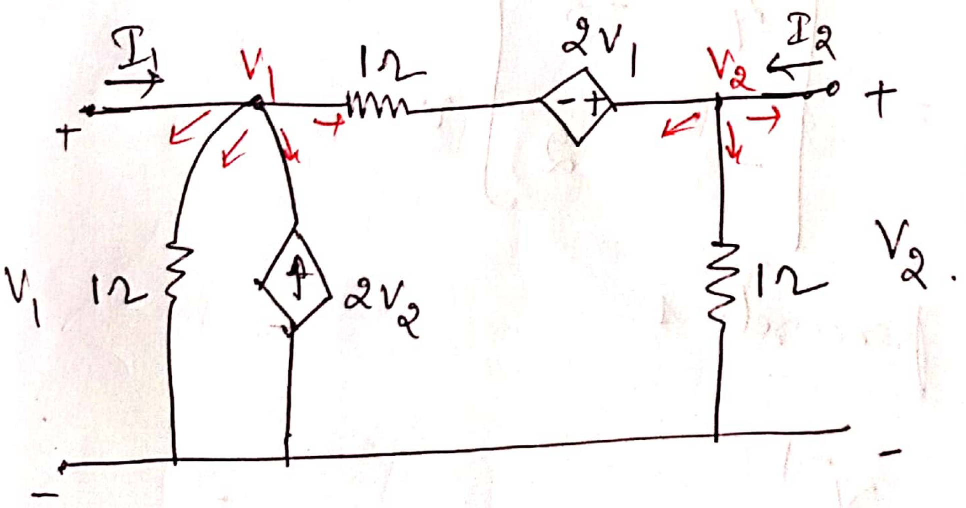
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$\boxed{Y_{21} = 2\,\mho}$ $\boxed{Y_{22} = 1.5\,\mho}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -1.5 & -1 \\ 2 & 1.5 \end{bmatrix} \mho$$

) Find 'Y' & Z parameters :





$$-I_1 + \frac{V_1}{1} - 2V_2 + \frac{V_1 + 2V_1 - V_2}{1} = 0$$

$$-I_1 + V_1 - 2V_2 + V_1 + 2V_1 - V_2 = 0$$

$$I_1 = 4V_1 - 3V_2 \quad \text{——— (1)}$$

$\boxed{Y_{11} = 4\,℧}$ $\boxed{Y_{12} = -3\,℧}$

Apply KCL @ node 2.

$$\frac{V_2 - 2V_1 - V_1}{1} + \frac{V_2}{1} - I_2 = 0$$

$$I_2 = -3V_1 + 2V_2 \quad \text{——— (2)}$$

$\boxed{Y_{21} = -3\,℧}$ $\boxed{Y_{22} = 2\,℧}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} ℧$$

to find Z- parameters

$$Z = Y^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

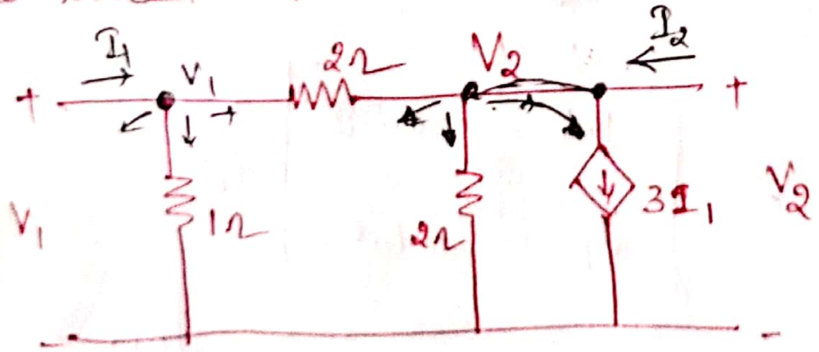
$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta Y = 4 \times 2 - (-3)(-3)$$

$$\Delta Y = 8 - 9 = -1$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix} \Omega$$

4) Find Y & Z parameters for the n/w which contains current controlled src.



KCL @ node 1

$$-I_1 + \frac{V_1}{1} + \frac{V_1 - V_2}{2} = 0$$

Scanned with CamScanner

$$I_1 = 1.5V_1 - 0.5V_2 \quad \text{—— (1)}$$

on comparing. $\boxed{Y_{11} = 1.5\,℧}$ $\boxed{Y_{12} = -0.5\,℧}$

KCL @ node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2} + 3I_1 - I_2 = 0$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{2} + 3I_1 - I_2 = 0$$

$$I_2 = 0.5V_2 - 0.5V_1 + 0.5V_2 + 3I_1$$

$$I_2 = -0.5V_1 + V_2 + 3\underbrace{[1.5V_1 - 0.5V_2]}_{\text{from eqn (1)}}$$

$$I_2 = 4V_1 - 0.5V_2 \quad \text{—— (2)}$$

on comparing

$\boxed{Y_{21} = 4\,℧}$ $\boxed{Y_{22} = -0.5\,℧}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5℧ \\ 4 & -0.5 \end{bmatrix} ℧$$

$$Z = Y^{-1} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$$\frac{-0.5}{1.25} \quad \frac{+0.5}{1.25}$$

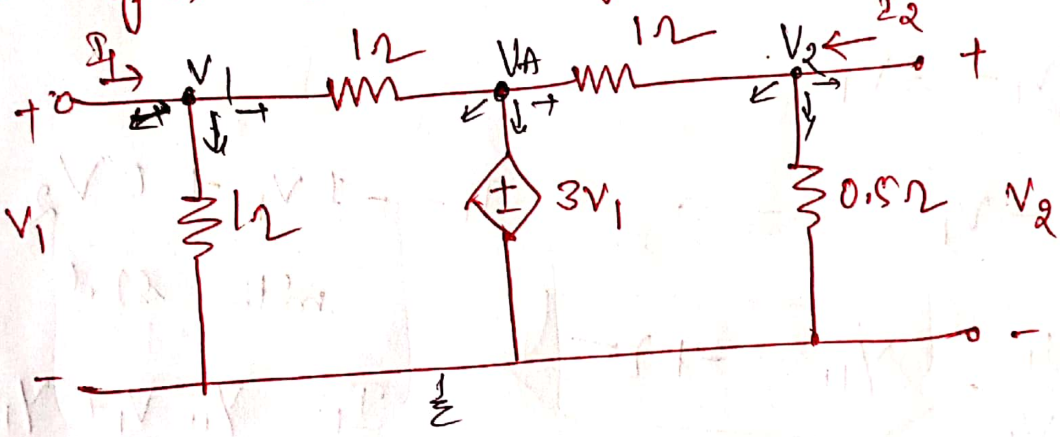
$$\frac{-4}{1.25} \quad \frac{1.5}{1.25}$$

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta Y = 1.25 \ \mho$$

$$Z = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \Omega$$

5) Find open circuit impedance parameters



KCL @ node 1

$$-I_1 + \frac{V_1}{1} + \frac{V_1 - V_A}{1} = 0$$

$$I_1 = 2V_1 - V_A \quad \text{——— (1)}$$

@ node A

$$V_A = 3V_1 \quad \text{——— (2)}$$

@ node 2

$$\frac{V_2 - V_A}{1} + \frac{V_2}{0.5} - I_2 = 0$$

$$V_2 - 3V_1 + 2V_2 = I_2$$

or $I_2 = -3V_1 + 3V_2$ ——— (3)

on comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$\boxed{Y_{21} = -3\,\mho} \qquad \boxed{Y_{22} = 3\,\mho}$$

Substitute V_A in eqn (1)

$$I_1 = 2V_1 - 3V_1$$

$I_1 = -V_1 \Longrightarrow I_1 = -1V_1 + 0V_2$

with std eqn

$I_1 = Y_{11}V_1 + Y_{12}V_2$.

$$\Rightarrow \frac{I_1}{V_1} = \boxed{Y_{11} = -1\,\mho}$$

$$\boxed{Y_{12} = 0\,\mho}$$

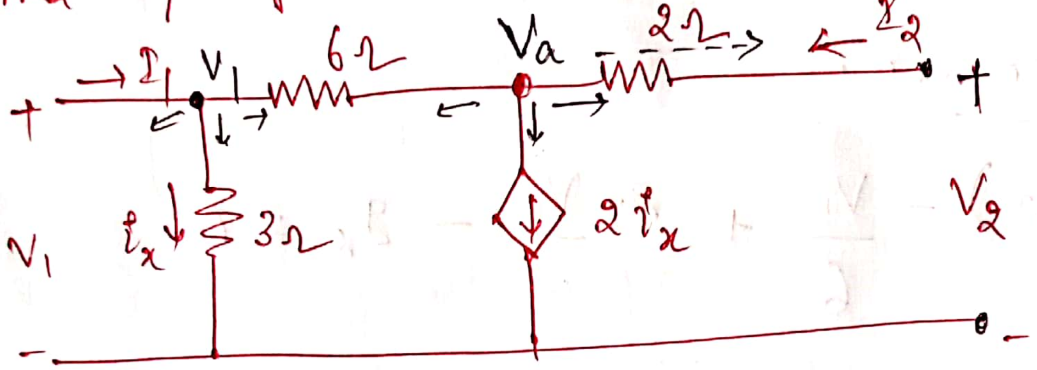
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 3 \end{bmatrix}\mho$$

$$Z = Y^{-1} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{22}}{\Delta Y} \end{bmatrix} = \begin{bmatrix} 3/-3 & 0 \\ \frac{+3}{-3} & \frac{-1}{-3} \end{bmatrix}$$

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = -3$$

$$Z = \begin{bmatrix} -1 & 0 \\ -1 & 1/3 \end{bmatrix} \Omega$$

6) find Y parameters



KCL @ node 1

$$-I_1 + \frac{V_1}{3} + \frac{V_1 - V_a}{6} = 0 \quad \text{--- (1)}$$

KCL @ node a

$$\frac{V_a - V_1}{6} + 2i_x - I_2 = 0 \quad \text{--- (2)}$$

from the figure,

$$i_x = \frac{V_1}{3}$$

& $$I_2 = \frac{V_2 - V_a}{2}$$

$$2I_2 = V_2 - V_a \implies V_a = V_2 - 2I_2$$

eqn (1) becomes

$$-I_1 + \frac{V_1}{3} + \frac{V_1}{6} - \frac{1}{6}\left[V_2 - 2I_2\right]$$

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$$I_1 = \frac{V_1}{3} + \frac{V_1}{6} - \frac{V_2}{6} + \frac{2I_2}{6}$$

$$I_1 - 0.33 I_2 = 0.5V_1 - 0.166\,V_2 \quad \text{---} \ (*)$$

eqn ② becomes.

$$\frac{(V_2 - 2I_2)}{6} - \frac{V_1}{6} + 2\times\frac{V_1}{3} - I_2 = 0$$

$$\frac{V_2}{6} + \frac{I_2}{3} - \underbrace{\frac{V_1}{6} + \frac{2V_1}{3}} - I_2 = 0$$

or $\quad 0.5V_1 + 0.166\,V_2 = I_2 + I_2/3 = 1.33\,I_2$

or $\quad I_2 = 0.375\,V_1 + 0.125\,V_2 \quad \text{---} \ ③$

on comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$.

$$\boxed{Y_{21} = 0.375\ \mho} \qquad \boxed{Y_{22} = 0.125\ \mho}$$

from (*)

$$I_1 = 0.5V_1 - 0.166\,V_2 + 0.33\underbrace{(0.375V_1 + 0.125V_2)}$$

$$I_1 = 0.5V_1 - 0.166\,V_2 + 0.1237V_1 + 0.041\,V_2$$

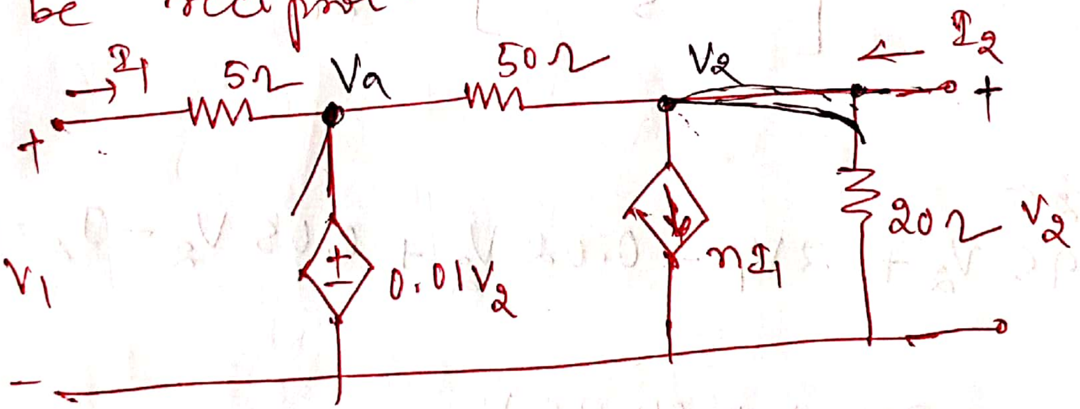
$$I_1 = 0.625 V_1 - 0.125 V_2.$$

On comparing with $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$\boxed{Y_{11} = 0.625\ \mho} \quad \boxed{Y_{12} = -0.125\ \mho}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix} \mho$$

7) Find Y_{12} & Y_{21} for the n/w for $n=10$. what is the value of 'n' for the n/w to be reciprocal.



from the figure

$$\boxed{V_a = 0.01 V_2} \quad \text{——} ①$$

KCL @ node 2.

$$\frac{V_2 - V_a}{50} + nI_1 + \frac{V_2}{20} - I_2 = 0 \quad \text{——} ②$$

$$\frac{V_2}{50} - \frac{0.01 \overset{V_a}{V_2}}{50} + 10 I_1 + \frac{V_2}{20} - I_2 = 0$$

$$0.0198 V_2 + 10 I_1 + \frac{V_2}{20} - I_2 = 0$$

from the fig

$$I_1 = \frac{V_1 - V_a}{5}$$

$$I_1 = \frac{V_1 - 0.01 V_2}{5} \quad \text{——— (3)}$$

$$0.0198 V_2 + 10 \left[\frac{V_1 - 0.01 V_2}{5} \right] + \frac{V_2}{20} - I_2 = 0$$

$$0.0198 V_2 + 2V_1 - 0.02 V_2 + 0.05 V_2 - I_2 = 0$$

$$I_2 = 2V_1 + 0.0498 V_2$$

Comparing with std eqn.

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\boxed{Y_{21} = 2 \mho} \quad \boxed{Y_{22} = 0.0498 \mho}$$

$5 I_1 = V_1 - 0.01 V_2$

$I_1 = 0.2 V_1 - 0.002 V_2$

Comparing with $I_1 = Y_{11} V_1 + Y_{12} V_2$

$\boxed{Y_{11} = 0.2 \mho}$ $\boxed{Y_{12} = -0.002 \mho}$

Conditions for reciprocal

$Y_{12} = Y_{21}$

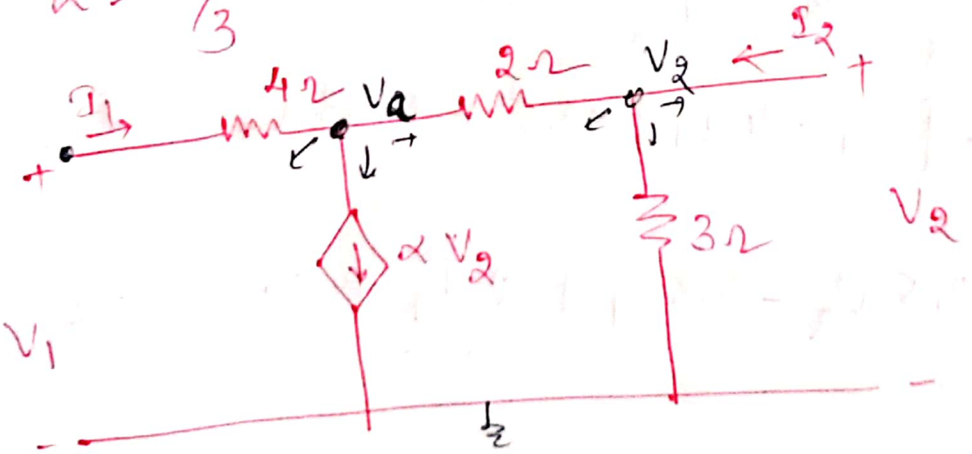
$-0.002 = 2$

$\boxed{n = 0.001}$ or $\boxed{n = 1000}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.2 & -0.002 \\ 2 & 0.0498 \end{bmatrix}$$

** 8) Find the Z-parameters for the n/w take

$\alpha = 4/3$



KCL @ node a

$$-I_1 + \alpha V_2 + \frac{V_a - V_2}{2} = 0$$

$$I_1 = \frac{4}{3} V_2 + \frac{V_a}{2} - \frac{V_2}{2} \quad \text{—— ①}$$

KCL @ node 2 :

$$\frac{V_2 - V_a}{2} + \frac{V_2}{3} - I_2 = 0$$

$$\frac{V_2}{2} - \frac{V_a}{2} + \frac{V_2}{3} - I_2 = 0 \quad \text{—— ②}$$

from the $I_1 = \frac{V_1 - V_a}{4}$

$$4I_1 = V_1 - V_a$$

$$V_a = V_1 - 4I_1$$

Eqn ② becomes

$$0.5\,V_2 - [V_1 - 4I_1] \times 0.5 + 0.33\,V_2 = I_2$$

or $$I_2 = 0.5V_2 - 0.5V_1 + 2I_1 + 0.33\,V_2 \quad \text{—— ②*}$$

Eqn ① becomes.

$$I_1 = 1.33 V_2 + \frac{(V_1 - 4 I_1)}{2} - 0.5 V_2$$

$$I_1 = 1.33 V_2 + 0.5 V_1 - 2 I_1 - 0.5 V_2$$

$$3 I_1 = 0.5 V_1 + 0.83 V_2$$

or $I_1 = 0.166 V_1 + 0.2766 V_2$ —— (3)

Comparing with $I_1 = Y_{11} V_1 + Y_{12} V_2$

$\boxed{Y_{11} = 0.166\ \mho}$ $\boxed{Y_{12} = 0.2766\ \mho}$

Substituting (3) in (2)*

$$I_2 = 0.5 V_2 - 0.5 V_1 + 2 [0.166 V_1 + 0.2766 V_2] + 0.33 V_2.$$

$$I_2 = -0.168 V_1 + 1.386 V_2$$

Comparing with $I_2 = Y_{21} V_1 + Y_{22} V_2$

$\boxed{Y_{21} = -0.168\ \mho}$ $\boxed{Y_{22} = 1.386\ \mho}$

$$Z = Y^{-1} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

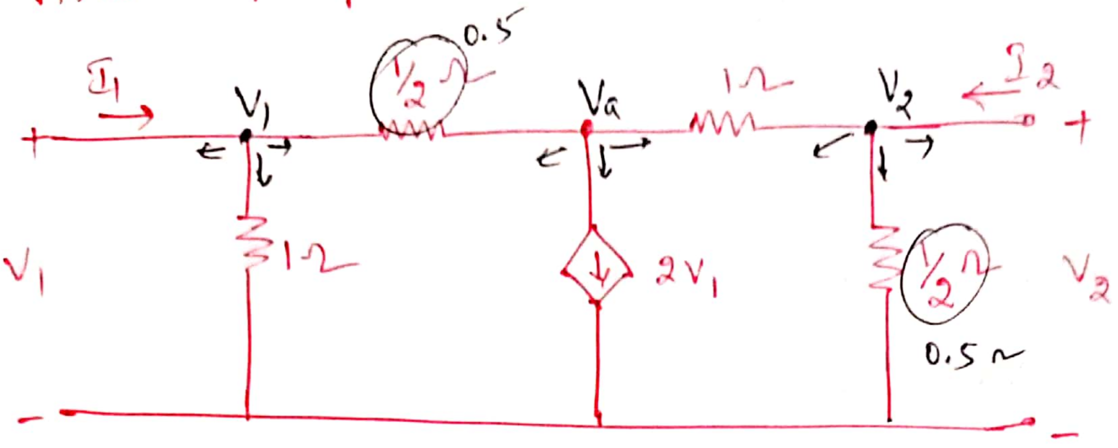
$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta Y = 0.276$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1.386}{0.276} & \frac{-0.2766}{0.276} \\ \frac{-(0.168)}{0.276} & \frac{0.166}{0.276} \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 5.02 & -1 \\ 0.608 & 0.6 \end{bmatrix} \Omega$$

Q) find Y paramelers



KCL @ node 1

$$-I_1 + \frac{V_1}{1} + \frac{V_1 - V_a}{0.5} = 0$$

$$-I_1 + V_1 + \frac{V_1}{0.5} - \frac{V_a}{0.5} = 0 \quad \text{——} \ (1)$$

@ node a:

$$\frac{V_a - V_1}{0.5} + 2V_1 + \frac{V_a - V_2}{1} = 0$$

$$\frac{V_a}{0.5} - \cancel{\frac{V_1}{0.5}}^{\,2V_1} + \cancel{2V_1} + V_a - V_2 = 0$$

$$2V_a \cancel{+ 2V_1} + V_a - V_2 = 0 \quad \text{——} \ (2)$$

@ node 2

$$\frac{V_2 - V_a}{1} + \frac{V_2}{0.5} - I_2 = 0$$

$V_2 - V_a + 2V_2 - I_2 = 0$ —— (3)

from (2)

$3V_a = V_2$

$$\boxed{V_a = \frac{V_2}{3}}$$

$\therefore$ eqn (1) becomes.

$$-I_1 + V_1 + \frac{V_1}{0.5} - \frac{V_2}{3 \times 0.5} = 0$$

or $I_1 = 3V_1 - 0.666\, V_2$

$\therefore$ $\boxed{Y_{11} = 3\,\mho}$ $\boxed{Y_{12} = -0.66\,\mho}$

eqn (3) becomes,
$$V_2 - \frac{V_2}{3} + \frac{V_2}{0.5} = I_2$$

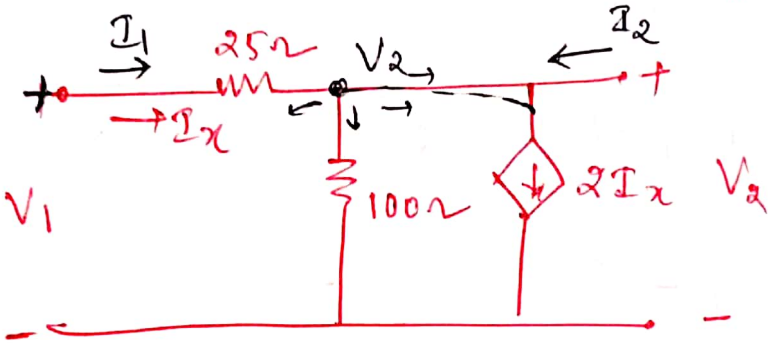
or $I_2 = 2.667\, V_2$

$\therefore$ $I_2 = 0\,V_1 + 2.667\,V_2$

$\boxed{Y_{21} = 0\,\mho}$ $\boxed{Y_{22} = 2.667\,\mho}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3 & -0.66 \\ 0 & 2.667 \end{bmatrix} \mho.$$

10) Find the Y Parameters of the n/w shown



→ KCL @ node 2 :

$$-I_1 + \frac{V_2}{100} + 2I_x - I_2 = 0$$

from the fig.

$$\boxed{I_x = I_1} \quad \text{——— (1)}$$

$$-I_1 + \frac{V_2}{100} + 2I_1 - I_2 = 0 \quad \text{——— (2)}$$

from the figure,

$$I_1 = \frac{V_1 - V_2}{25}$$

$$I_1 = 0.04\,V_1 - 0.04\,V_2 \quad \text{——— (3)}$$

Comparing with std eqn

$$\boxed{Y_{11} = 0.04\,\mho} \quad \boxed{Y_{12} = -0.04\,\mho}$$

from eq (2),

$$I_1 + 0.01 V_2 - I_2 = 0$$

$$I_2 = I_1 + 0.01 V_2$$

Substituting (3) in the above eqn

$$I_2 = [0.04 V_1 - 0.04 V_2] + 0.01 V_2$$

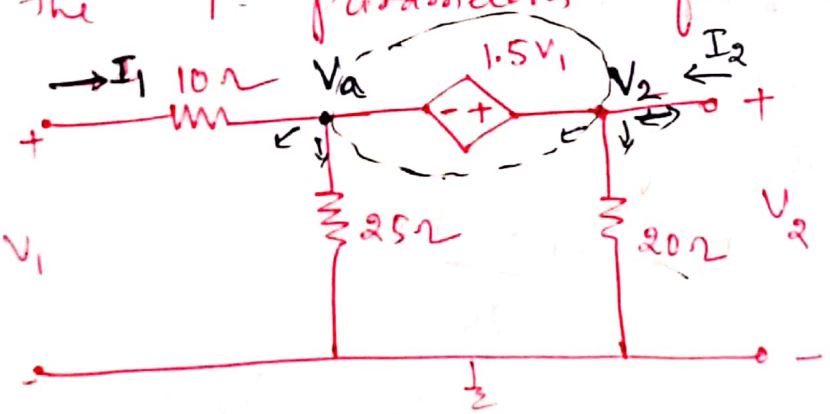
$$I_2 = 0.04 V_1 - 0.03 V_2 \quad \text{—— (4)}$$

comparing with Std eqn

$$\boxed{Y_{21} = 0.04 \, \mho} \quad \boxed{Y_{22} = -0.33 \, \mho}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.04 & -0.04 \\ 0.04 & -0.03 \end{bmatrix} \mho$$

11) Find the T-parameters for the n/w.



Since $1.5 V_1$ is b/n Va & V_2 it leads to super node.

$$V_2 - V_a = 1.5 V_1 \quad \text{—— (1)}$$

Applying KCL to super node.

$$-I_1 + \frac{V_a}{25} + \frac{V_2}{20} - I_2 = 0 \qquad \text{—②}$$

from the fig.

$$I_1 = \frac{V_1 - V_a}{10}$$

$$10 I_1 = V_1 - V_a$$

$$\boxed{V_a = V_1 - 10 I_1}$$

Eqn ① becomes.

$$V_2 - [V_1 - 10 I_1] - 1.5 V_1 = 0$$

$$V_2 - V_1 + 10 I_1 - 1.5 V_1 = 0$$

$$10 I_1 = 2.5 V_1 - V_2$$

$$\text{or } I_1 = 0.25 V_1 - 0.1 V_2$$

Comparing with std eqn.

$$\boxed{Y_{11} = 0.25\ \mho} \qquad \boxed{Y_{12} = -0.1\ \mho}$$

Eqn ② is Now.

$$I_2 = -[0.25 V_1 - 0.1 V_2] + 0.04 [V_1 - 10 I_1]$$

$$+ 0.05 V_2$$

$$I_2 = -0.25 V_1 + 0.1 V_2 + 0.04 V_1 - 0.4 I_1 + 0.05 V_2$$

$$I_2 = -0.25V_1 + 0.1V_2 + 0.04V_1 - 0.4[0.25V_1 - 0.1V_2] + 0.05V_2$$

$$I_2 = -0.21V_1 + 0.15V_2 - 0.1V_1 + 0.04V_2$$

$$\boxed{I_2 = -0.31V_1 + 0.19V_2}$$

Comparing with std eqn.

$$\boxed{Y_{21} = -0.31\,\mho} \qquad \boxed{Y_{22} = 0.19\,\mho}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.25 & -0.1 \\ -0.31 & 0.19 \end{bmatrix} \mho$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.633 & 3.23 \\ 0.053 & 0.05 \end{bmatrix}$$

$V_1 = AV_2 - BI_2$ — 1

$I_1 = CV_2 - DI_2$ — 2

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — 3

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — 4

$$Y_{21}V_1 = I_2 - Y_{22}V_2$$

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2 \quad — 5$$

$$A = \frac{-Y_{22}}{Y_{21}} = -\frac{0.19}{-0.31} = 0.633$$

$$B = \frac{-1}{Y_{21}} = \frac{-1}{-0.31} = 3.23$$

eqn (3) becomes.

$$I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2 .$$

$$I_1 = -\frac{Y_{11} Y_{22}}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 + Y_{12} V_2$$

$$I_1 = \frac{(-Y_{11} Y_{22} + Y_{21} Y_{12})}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = \underbrace{\frac{-\Delta Y}{Y_{21}}}_{C} V_2 + \underbrace{\frac{Y_{11}}{Y_{21}}}_{D} I_2$$

$$C = \frac{-\Delta Y}{Y_{21}} \qquad D = -\frac{Y_{11}}{Y_{21}} = -\frac{0.25}{-0.31} = 0.05$$

$$\Delta Y = 0.25 \times 0.19 - [(-0.1)(-0.31)] = 0.0165$$

$$C = 0.053$$

12) The eqns that describes behaviour of n/w are

$11 I_1 + 4 I_2 = 5V_1$

$4 I_1 + 6 I_2 = 5V_2$. Find Y parameters.

Given $5V_1 = 11 I_1 + 4 I_2$

$$V_1 = \frac{11}{5} I_1 + \frac{4}{5} I_2$$

$$V_1 = 2.2 I_1 + 0.8 I_2 \quad \text{——— (1)}$$

$$5V_2 = 4 I_1 + 6 I_2$$

$$V_2 = \frac{4}{5} I_1 + \frac{6}{5} I_2$$

$$V_2 = 0.8 I_1 + 1.2 I_2 \quad \text{——— (2)}$$

WKT $V_1 = Z_{11} I_1 + Z_{12} I_2$ ——— (3)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ ——— (4)

From (1) & (3)

$\boxed{Z_{11} = 2.2\Omega}$ & $\boxed{Z_{12} = 0.8\Omega}$

From (2) & (4)

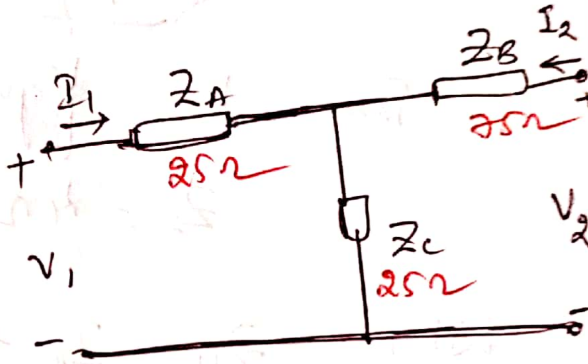
$\boxed{Z_{21} = 0.8\Omega}$ & $\boxed{Z_{22} = 1.2\Omega}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.2 & 0.8 \\ 0.8 & 1.2 \end{bmatrix}$$

13) The impedance parameters of the T n/w are given by $\begin{bmatrix} 50 & 25 \\ 25 & 100 \end{bmatrix}$. Find the parameters of the T- n/w.

Given $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 50 & 25 \\ 25 & 100 \end{bmatrix}$

WKT $Z_{11} = Z_A + Z_C$



$Z_{22} = Z_B + Z_C$

& $Z_{12} = Z_{21} = Z_C$

$\therefore \boxed{Z_C = 25\Omega}$

$\therefore Z_A = Z_{11} - Z_C$

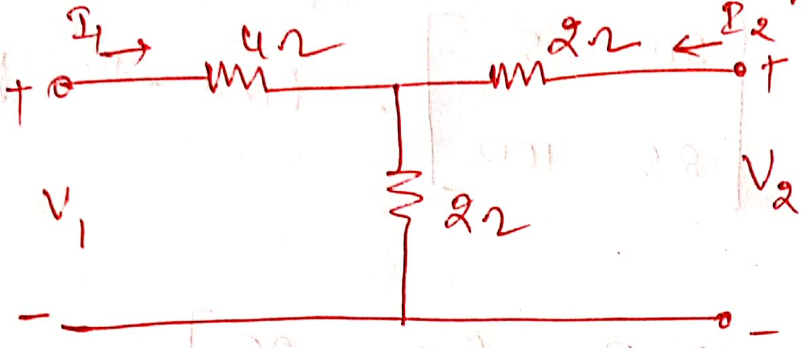
$Z_A = 50 - 25$

$\boxed{Z_A = 25\Omega}$

& $Z_B = Z_{22} - Z_C$

$Z_B = 100 - 25$ $\boxed{Z_B = 75\Omega}$

14) Determine Y parameters of the T-n/w



For T-n/w first find Z & then

$$\boxed{Y = Z^{-1}}.$$

from the figure

$Z_A = 4\Omega$, $Z_B = 2\Omega$, $Z_C = 2\Omega$

WKT $Z_{11} = Z_A + Z_C$ $\quad Z_{12} = Z_{21} = Z_C$

$$\boxed{Z_{11} = 6\Omega} \qquad \boxed{Z_{12} = Z_{21} = 2\Omega}$$

& $Z_{22} = Z_B + Z_C$

$$\boxed{Z_{22} = 4\Omega}$$

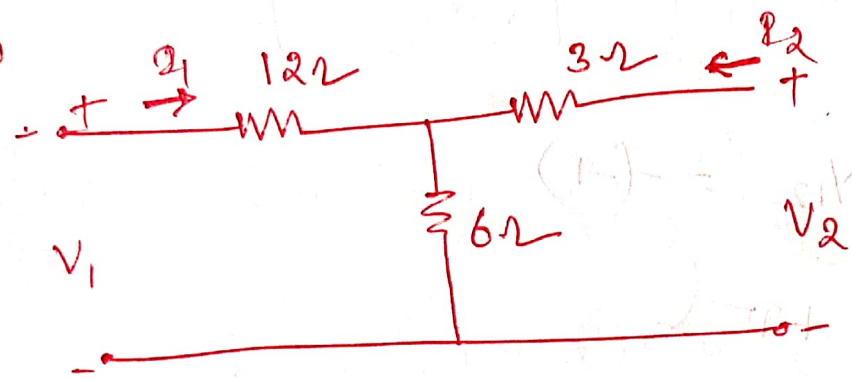
$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \Omega$$

$$Y = Z^{-1} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$\Delta Z = 24 - 4 = 20$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.3 \end{bmatrix} \mho$$

15) Find the Z- parameters of the n/w shown below



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix} \Omega.$$

16) The port current of a two-port n/w are given by

$$I_1 = 2.5 V_1 - V_2$$

$$I_2 = -V_1 + 5V_2$$

Find equivalent π n/w.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 2.5 & -1 \\ -1 & 5 \end{bmatrix} \mho$$

The admittances of π n/w are, Y_A, Y_B & Y_C

$$Y_{11} = Y_A + Y_C$$

$$Y_{22} = Y_B + Y_C$$

& $Y_{12} = Y_{21} = -Y_C$

$$\cancel{Y_A = Y_{11} + Y_{12}}$$

$$\cancel{= 25 - 1 = 1.5\mho}$$

$$\cancel{Y_A = 1.5\mho}$$

$$\cancel{Y_B = Y}$$

$$Y_C = -Y_{12} = -(-1)$$

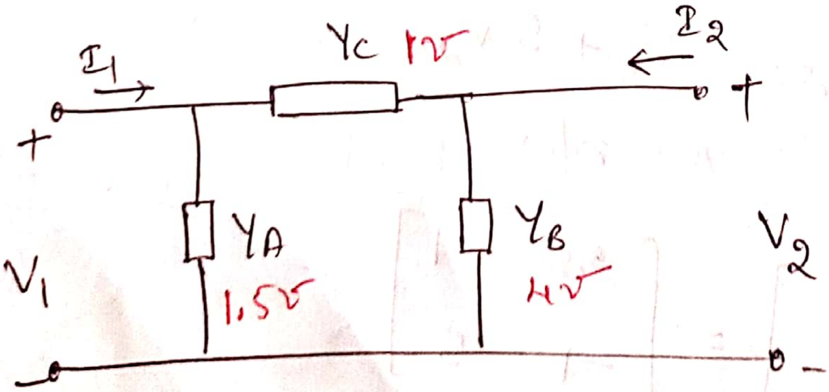
$$\boxed{Y_C = 1\mho}$$

$$Y_A = Y_{11} - Y_C$$

$$= 2.5 - 1 = 1.5\mho$$

$$Y_B = Y_{22} - Y_C$$

$$Y_B = 5 - 1 = 4\mho$$



17) The Z- parameters of a two port n/w are

$Z_{11} = 20\Omega$, $Z_{12} = 10\Omega$, $Z_{21} = 10\Omega$ & $Z_{22} = 10\Omega$.

find its Y & ABCD parameters.

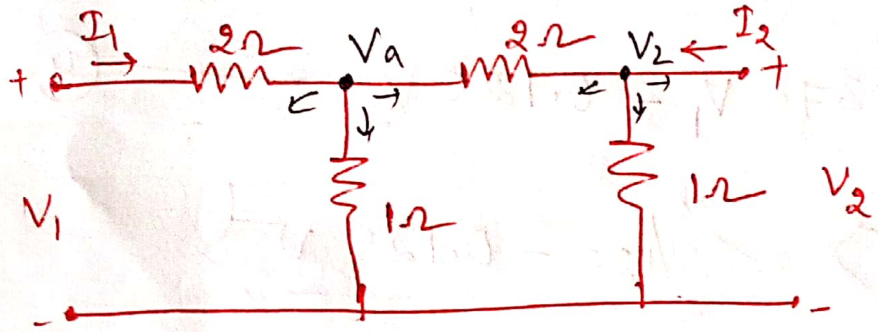
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

18) Find Z-parameters of the n/w shown.

June/July – 19



→ Apply KCL @ node a

$$\frac{V_a}{1} + \frac{V_a - V_2}{2} - I_1 = 0$$

$$I_1 = 1.5V_a - 0.5V_2 \quad \text{——— (1)}$$

→ KCL @ node 2

$$\frac{V_2 - V_a}{2} + \frac{V_2}{1} - I_2 = 0$$

$$I_2 = 1.5V_2 - 0.5V_a \quad \text{——— (2)}$$

from the figure.

$$I_1 = \frac{V_1 - V_a}{2} \Rightarrow 2I_1 = V_1 - V_a$$

$$\text{of } V_a = V_1 - 2I_1 \quad \text{——— (3)}$$

Substitute (3) in (1)

$$I_1 = 1.5\left[V_1 - 2I_1\right] - 0.5V_2$$

$$I_1 = 1.5V_1 - 3I_1 - 0.5V_2$$

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$$4I_1 = 1.5V_1 - 0.5V_2$$

$$I_1 = 0.375\,V_1 - 0.125\,V_2 \quad \text{--- (4)}$$

$$\boxed{Y_{11} = 0.375\ \mho} \qquad \boxed{Y_{12} = -0.125\ \mho}$$

Substitute (3) in (2)

$$I_2 = 1.5V_2 - 0.5\left[V_1 - 2I_1\right]$$

$$I_2 = 1.5V_2 - 0.5V_1 + I_1$$

$$I_2 = 1.5V_2 - 0.5V_1 + 0.375V_1 - 0.125V_2$$

$$I_2 = -0.125V_1 + 1.375V_2$$

$$\boxed{Y_{21} = -0.125\ \mho} \qquad \boxed{Y_{22} = 1.375\ \mho}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.375 & -0.125 \\ -0.125 & 1.375 \end{bmatrix} \mho$$

$$Z = Y^{-1} = \begin{bmatrix} \dfrac{Y_{22}}{\Delta Y} & -\dfrac{Y_{12}}{\Delta Y} \\ -\dfrac{Y_{21}}{\Delta Y} & \dfrac{Y_{11}}{\Delta Y} \end{bmatrix}$$

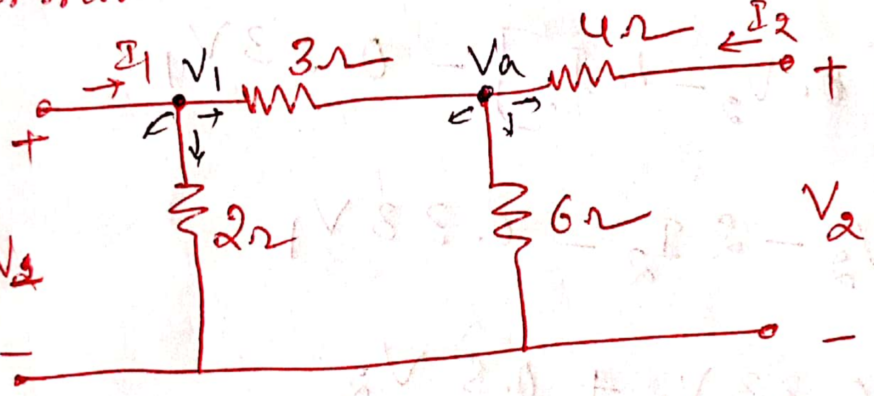
$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$= 0.375 \times 1.375 - (-0.125)(-0.125)$$

$$= 0.51 - 0.015 = \cancel{0.2225}\ 0.5$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \Omega$$

19) Determine Y & Z parameters:



KCL @ node V_1

$$-I_1 + \frac{V_1}{2} + \frac{V_1 - V_a}{3} = 0$$

$$0.5 V_1 + 0.33 V_1 - 0.33 V_a - I_1 = 0$$

$$\text{or } I_1 = 0.83 V_1 - 0.33 V_a \quad \text{——— (1)}$$

KCL @ node V_a

$$\frac{V_a - V_1}{3} + \frac{V_a}{6} - I_2 = 0$$

or $I_2 = 0.5V_a - 0.33V_1$ —— (2)

from the fig.

$$I_2 = \frac{V_2 - V_a}{4}$$

$4I_2 = V_2 - V_a$

or $V_a = V_2 - 4I_2$ —— (3)

Substitute (3) in (2)

$I_2 = 0.5[V_2 - 4I_2] - 0.33V_1$

$I_2 = 0.5V_2 - 2I_2 - 0.33V_1$

$3I_2 = -0.33V_1 + 0.5V_2$

$I_2 = -0.11V_1 + 0.166V_2$ —— *

Comparing with Std eqn $Y_{21} = -0.11\,\mho$

& $Y_{22} = 0.166\,\mho$

Now (3) in (1) becomes.

$I_1 = 0.83V_1 - 0.33[V_2 - 4I_2]$

$$I_1 = 0.83\,V_1 - 0.33\,V_2 + 1.32\,I_2$$

$$I_1 = 0.83\,V_1 - 0.33\,V_2 + 1.32\Big[\underbrace{-0.11V_1 + 0.166\,V_2}_{I_2 \text{ eqn } (*)}\Big]$$

$$I_1 = 0.83\,V_1 - 0.33\,V_2 - 0.145\,V_1 + 0.219\,V_2$$

$$I_1 = 0.685\,V_1 - 0.111\,V_2$$

Comparing with Std eqn $I_1 = Y_{11}V_1 + Y_{12}V_2$

$\boxed{Y_{11} = 0.685\,\mho}$ $\boxed{Y_{12} = -0.111\,\mho}$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.685 & -0.111 \\ -0.111 & 0.166 \end{bmatrix} \mho$$

$$Z = Y^{-1} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

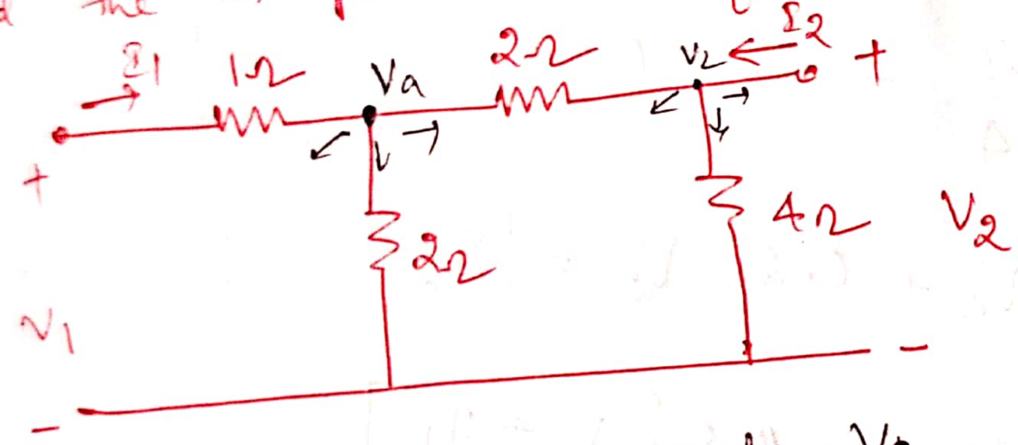
$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = 0.113 - [0.0123]$$

$$\underline{\underline{\Delta Y = 0.1}}$$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 1.66\,\Omega & 1.1 \\ 1.1 & 6.8 \end{bmatrix} \Omega$$

2) Find the h-parameters of the ckt shown Nov-2020 (ACV)

KCL method



Apply KCL at node V_a

$$-\cancel{I_1 + \dots} \; \frac{V_a - V_2}{2} + \frac{V_a}{2} - I_1 = 0$$

$$I_1 = \frac{V_a - V_2}{2} \quad \text{——— } (1)$$

from the fig $I_1 = \dfrac{V_1 - V_a}{1}$

$$V_a = V_1 - I_1 \quad \text{—— } (2)$$

substitute ② in ①

$$I_1 = V_1 - I_1 - \frac{V_2}{2}$$

$$2I_1 = V_1 - V_2/2$$

$$\cancel{I_1 = 0.5 V_1 - 0.8 V_2}$$

$$I_1 = 0.5 V_1 - 0.25 V_2 \quad \text{——— ③}$$

$$\boxed{Y_{11} = 0.5\ \mho} \quad \boxed{Y_{12} = -0.25\ \mho}$$

KCL @ node 2

$$\frac{V_2 - V_a}{2} + \frac{V_2}{4} - I_2 = 0$$

$$0.5 V_2 - 0.5 V_a + 0.25 V_2 = I_2$$

or $I_2 = 0.75 V_2 - 0.5 [V_1 - I_1]$

$$I_2 = 0.75 V_2 - 0.5 V_1 + 0.5 [\underbrace{0.5 V_1 - 0.25 V_2}_{\text{eqn ③ } I_1}]$$

$$I_2 = 0.75 V_2 - 0.5 V_1 + 0.25 V_1 - 0.125 V_2$$

$$I_2 = -0.25 V_1 + 0.625 V_2 \quad \text{——— ④}$$

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$$Y_{21} = -0.25\,\mho \qquad Y_{22} = 0.625\,\mho$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.625 \end{bmatrix} \mho$$

WKT,
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1/Y_{11} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$$

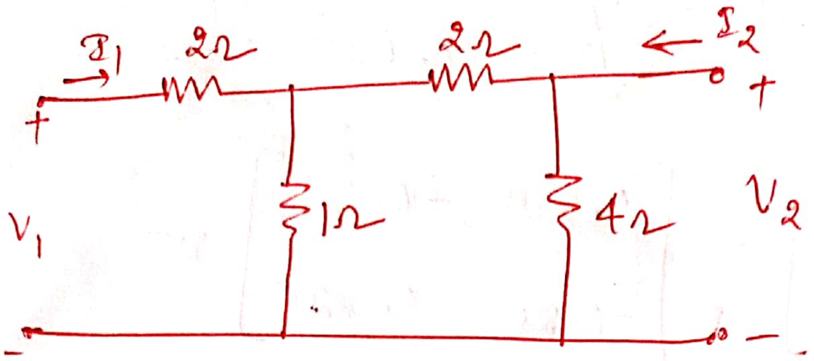
$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta Y = 0.25$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1/0.5 & -\frac{(-0.25)}{0.5} \\ \frac{-0.25}{0.5} & \frac{0.25}{0.5} \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

21) S.T the n/w is reciprocal but not symmetrical by finding h parameters.



Ans :- $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2.66 & 0.33 \\ -0.33 & 0.59 \end{bmatrix}$

Since $h_{12} = -h_{21}$

$$0.33 = -(-0.33)$$

$$0.33 = 0.33$$

$\therefore$ The n/w is reciprocal.

$$\Delta h = \begin{vmatrix} 2.66 & 0.33 \\ -0.33 & 0.59 \end{vmatrix}$$

$$\Delta h = 1.67$$

Since $\Delta h \neq 1$ $\therefore$ the n/w is not Symmetrical.

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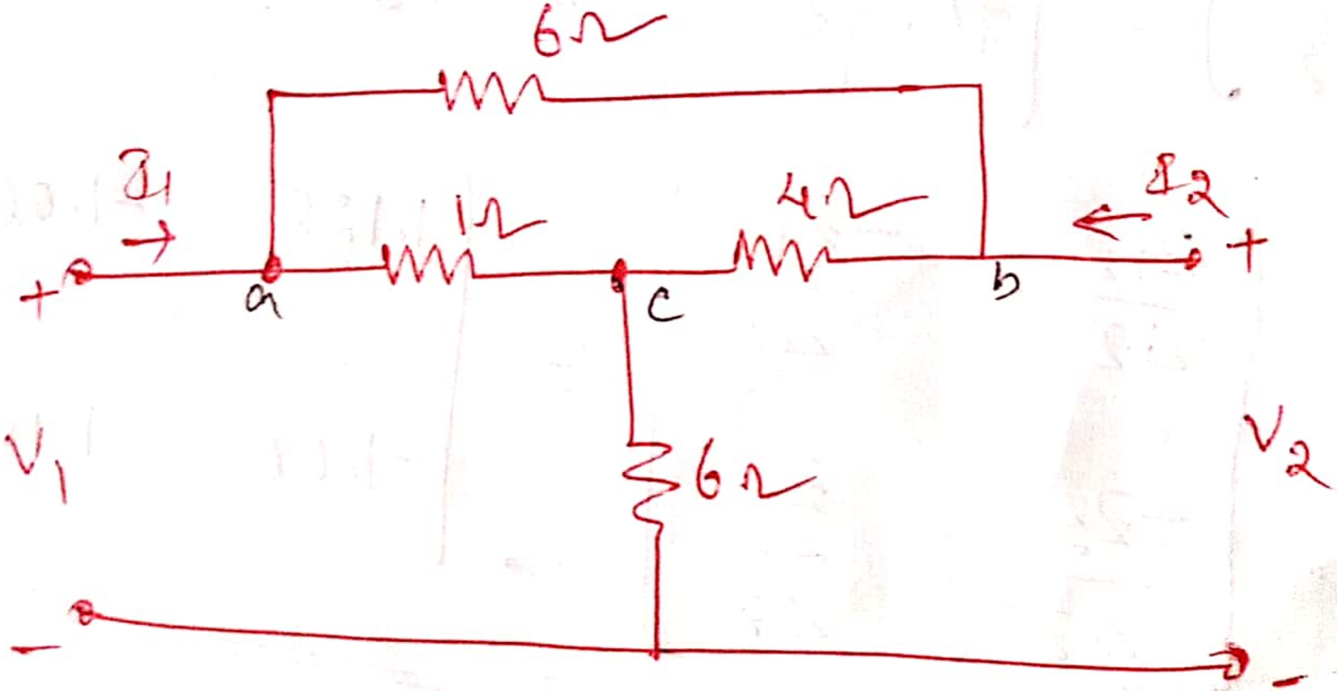
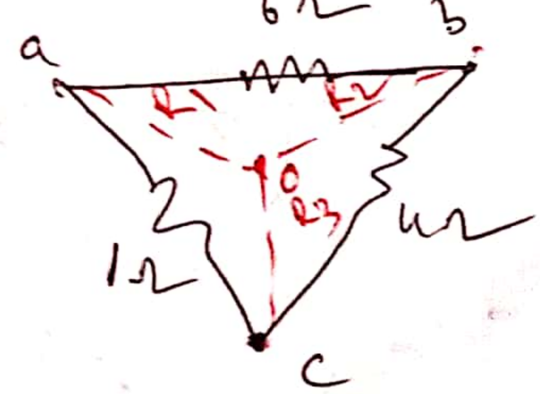
method.

22) Find the Y- parameters

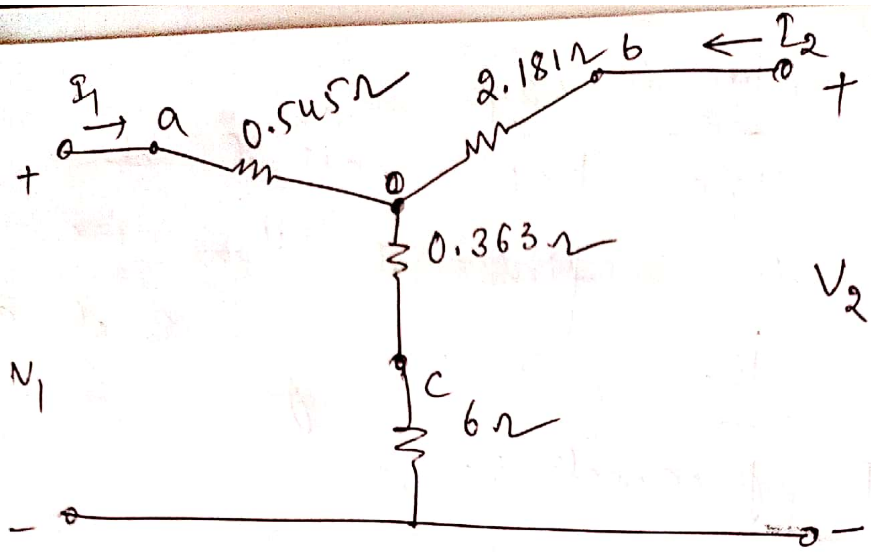


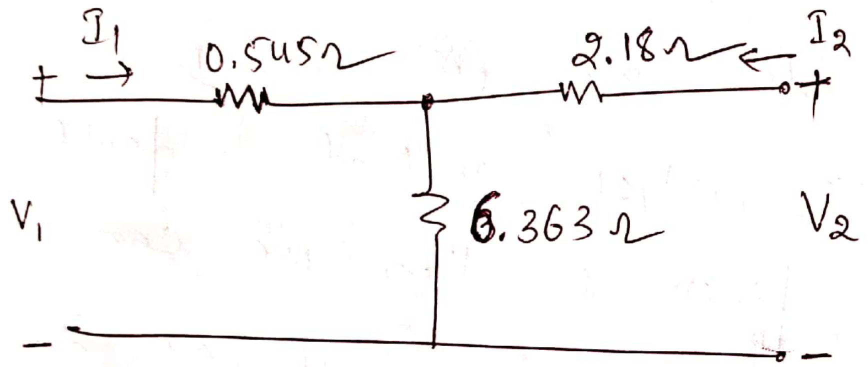


$R_1 = 0.545\,\Omega$

$R_2 = 2.18\,\Omega$

$R_3 = 0.363\,\Omega$





Since T- n/w,

$Z_{11} = 6.908\,\Omega$ $\qquad Z_{12} = Z_{21} = 6.363\,\Omega$

$Z_{22} = 8.54\,\Omega$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 6.908 & 6.363 \\ 6.363 & 8.54 \end{bmatrix} \Omega$$

$$Y = Z^{-1} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix} = \begin{bmatrix} 1.125 & -1.06 \\ -1.06 & 1.15 \end{bmatrix} \mho$$

$\Delta Z = 5.97\,\Omega$

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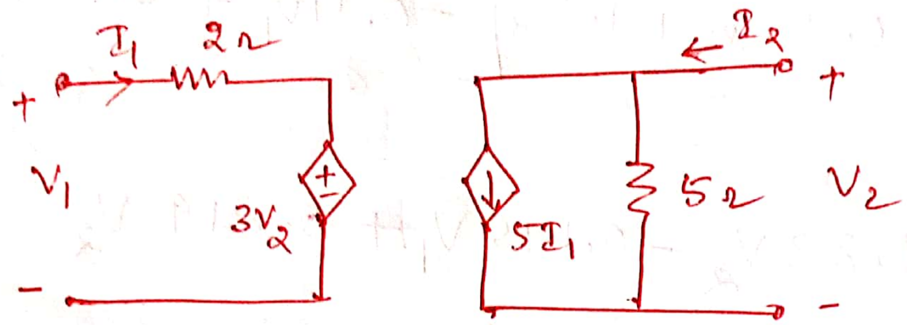
23) Determine the transmission parameters for the n/w shown below

June/July - 2019



WKT Transmission parameters,

$$V_1 = AV_2 - BI_2 \quad \text{——— (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{——— (2)}$$

By KVL (to the LHS side)

$$+V_1 - 2I_1 - 3V_2 = 0$$

or $$V_1 = 2I_1 + 3V_2 \quad \text{——— (3)}$$

By KCL (to the RHS side)

$$I_2 = \frac{V_2}{5} + 5I_1$$

$$5I_1 = I_2 - 0.2V_2$$

$$I_1 = \frac{-0.2}{5}V_2 + \frac{I_2}{5}$$

$$I_1 = -0.04V_2 + 0.2I_2 \quad \text{——— (4)}$$

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Compare (4) & (2)

$$\boxed{C = -0.04} \qquad \boxed{D = -0.2}$$

Substituting (4) in (3)

$$V_1 = 2\left[-0.04\,V_2 + 0.2\,I_2\right] + 3V_2$$

$$V_1 = -0.08\,V_2 + 0.4\,I_2 + 3V_2$$

$$V_1 = 2.92\,V_2 + 0.4\,I_2 \quad \text{——(5)}$$

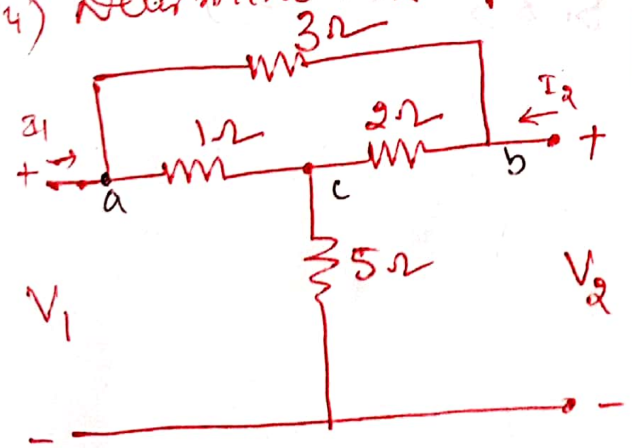
Compare (5) & (1)

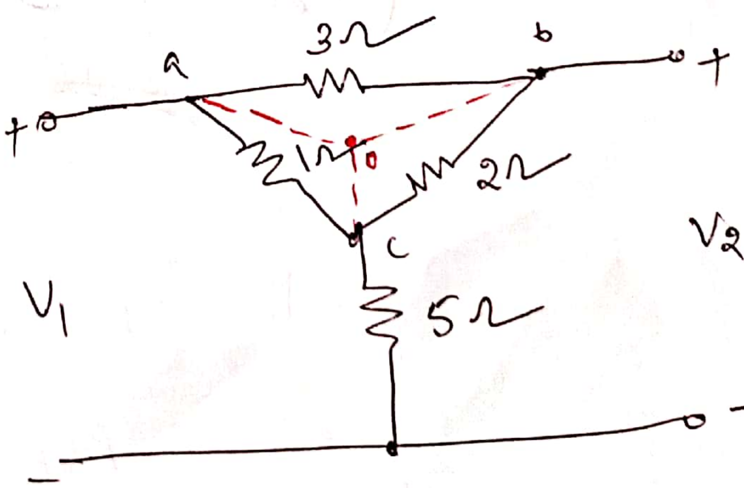
$$\boxed{A = 2.92} \qquad \boxed{B = -0.4}$$

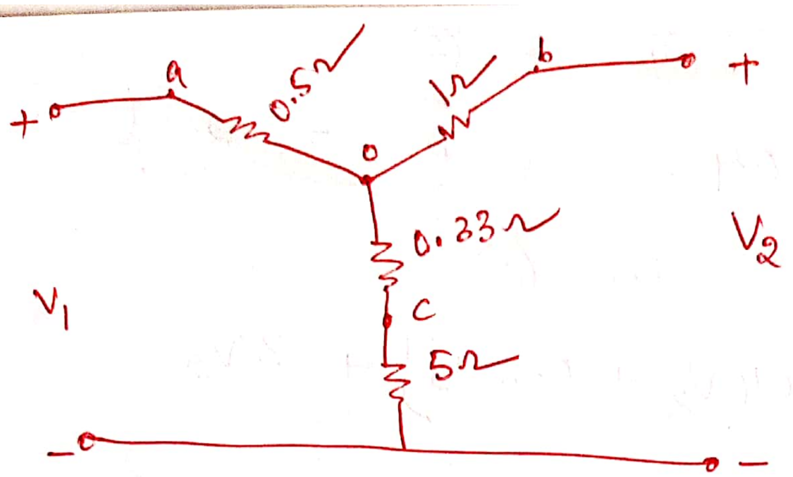
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2.92 & -0.4 \\ -0.04 & -0.2 \end{bmatrix}$$

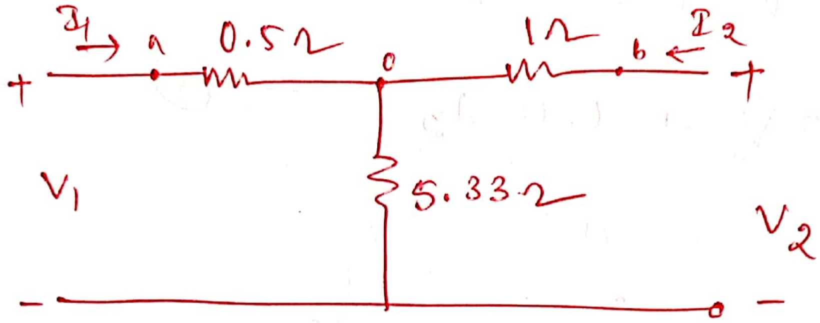
June/July -18

24) Determine Z-parameters for the n/w.









Since T-n/w,

$Z_{11} = 5.83\,\Omega$ $\quad Z_{22} = 6.33\,\Omega$

$Z_{12} = Z_{21} = 5.33\,\Omega$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5.83 & 5.33 \\ 5.33 & 6.33 \end{bmatrix} \Omega$$

CBCS SCHEME

USN | | | | | | | | | | 15EC34

Third Semester B.E. Degree Examination, June/July 2018
Network Analysis

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Determine the equivalent resistance across XY shown in Fig.Q1(a) **(05 Marks)**

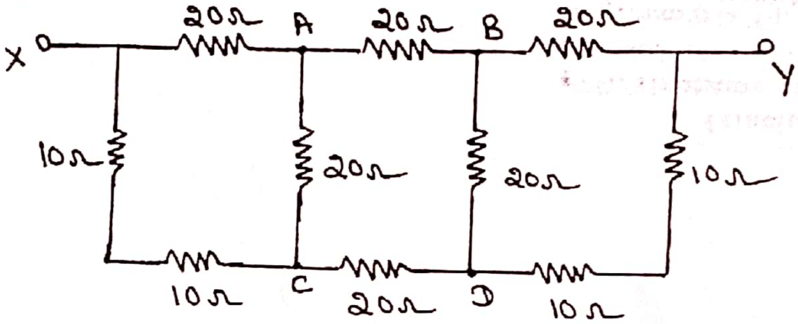


Fig.Q1(a)

b. Calculate the voltage across the 6Ω resistor using source shifting and transformation technique shown in Fig.Q1(b). **(05 Marks)**

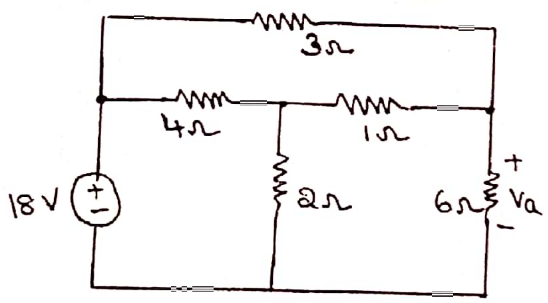


Fig.Q1(b)

c. Determine the power supplied by the dependent source of Fig.Q1(c) shown. **(06 Marks)**

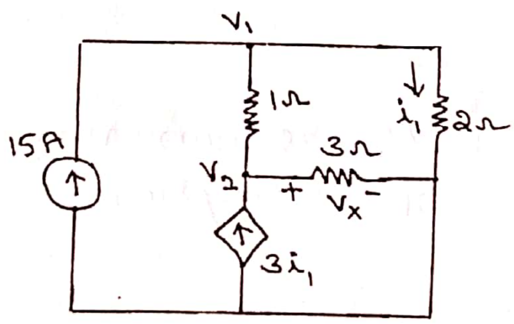


Fig.Q1(c)

OR

4 a. Using Millman's theorem, find I_L through R_L for the network shown in Fig.Q4(a). **(06 Marks)**

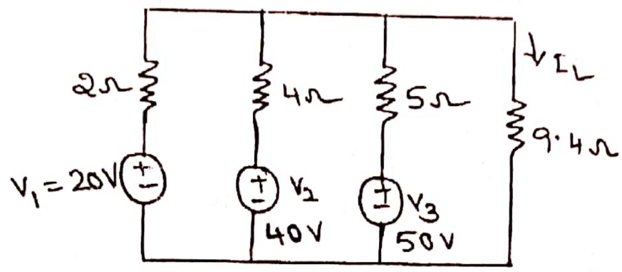


Fig.Q4(a)

b. Verify reciprocity theorem for the circuit shown in Fig.Q4(b). **(06 Marks)**

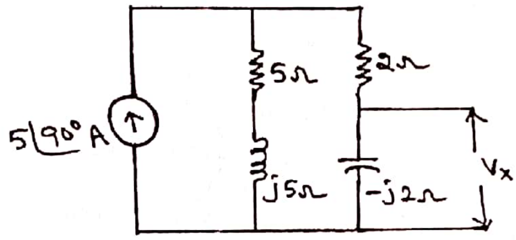


Fig.Q4(b)

c. State and explain maximum power transfer theorem. **(04 Marks)**

Module-3

5 a. In the circuit shown in Fig.Q5(a), the switch K is changed from position 1 to position 2 at t = 0, the steady state has been reached before switching. Find the values of $i, \frac{di}{dt}$ and $\frac{di^2}{dt^2}$ at $t=0$. **(08 Marks)**

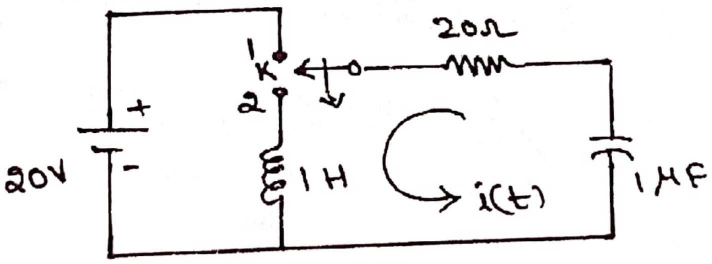


Fig.Q5(a)

b. The switch in the network shown in Fig.Q5(b) is closed at t = 0. Determine the voltage across the capacitor. Use Laplace transform. **(08 Marks)**

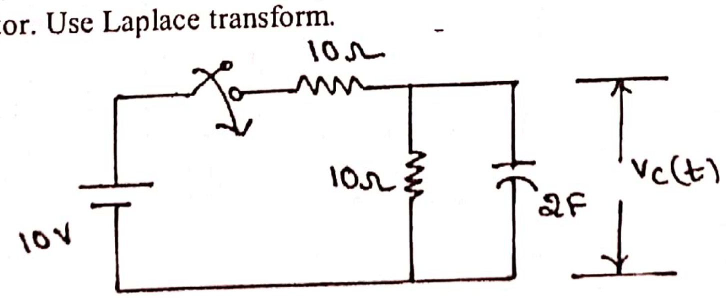


Fig.Q5(b)

OR

6 a. In the network shown in Fig.6(a), the switch K is opened at $t = 0$. At $t = 0^+$, solve for the values of v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ if $I = 2A$, $R = 200\Omega$ and $L = 1H$. (08 Marks)

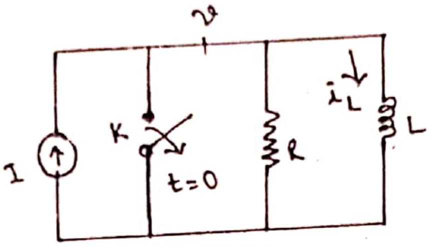


Fig.Q6(a)

b. Determine the Laplace transform of the periodic saw tooth waveform of Fig.Q6(b). Use gate function. (08 Marks)

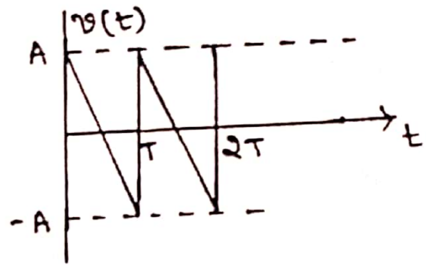


Fig.Q6(b)

Module-4

7 a. Derive for a resonant circuit, the resonant frequency $f_0 = \sqrt{f_1 f_2}$, where f_1 and f_2 are the two half power frequencies. (07 Marks)

b. Find the value of L for which the circuit shown in Fig.Q7(b) is resonant at a frequency of $w = 5000$ rad/sec. (06 Marks)

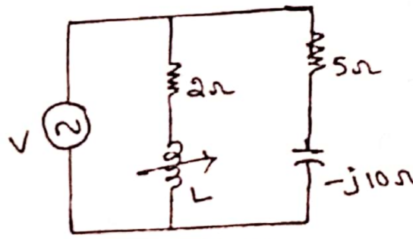


Fig.Q7(b)

c. A series RLC circuit has $R = 10\Omega$, $L = 0.01H$ and $c = 0.01\mu F$ and it is connected across 10mV supply. Calculate : i) f_0 ii) Q_0 iii) B.w. (03 Marks)

OR

8 a. A series RLC circuit has a resistance of 10Ω, an inductance of 0.3H and a capacitance of $100\mu F$. The applied voltage is 230V. Find : i) Resonant frequency ii) Quality factor iii) Lower and upper cut off frequencies iv) Bandwidth v) Current at resonance vi) currents at f_1 and f_2 vii) voltage across inductance at resonance. (08 Marks)

b. Derive an expression for the resonant frequency of a parallel resonant circuit. Also show that the circuit is resonant at all frequencies if $R_L = R_C = \sqrt{\frac{L}{C}}$ where R_L = Resistance in the inductor branch, R_C = resistance in the capacitor branch. (08 Marks)

Module-5

9 a. Find Y parameters and Z parameters for the circuit show in Fig.Q9(a). (08 Marks)

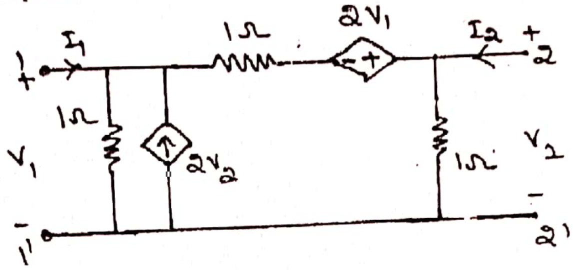


Fig.Q9(a)

b. Express ABCD parameters interms of Y-parameters and h-parameters. (08 Marks)

OR

10 a. Determine z parameters for the network shown in Fig.Q10(a). (08 Marks)

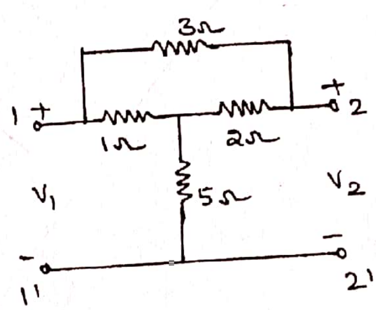


Fig.Q10(a)

b. Express h–parameters interms of Y–parameters. (08 Marks)

* * * * *

CBCS SCHEME

USN

17EC35

Third Semester B.E. Degree Examination, June/July 2019
Network Analysis

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define the following terms with examples:
i) Active elements
ii) Passive elements
iii) Linear and non linear elements
iv) Lumped node
v) Unilateral and bilateral elements. **(10 Marks)**

b. Use the node analysis and find the value of V_x in the circuit shown in below Fig.Q.1(b). Such that the current through the impedance $(2 + J3)\Omega$ is zero.

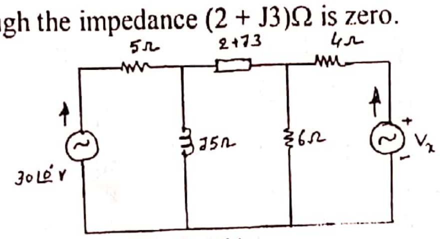


Fig.Q.1(b) **(10 Marks)**

OR

2 a. Derive an expression for i) Δ to Y transformation ii) Y to Δ transformation. **(10 Marks)**

b. Find the voltage across 20Ω resistor in the network shown in Fig.Q.2(b) below by using Mesh analysis method. **(10 Marks)**

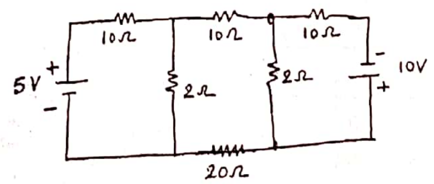


Fig.Q.2(b)

Module-2

3 a. State and prove Millman's theorem with an example. **(10 Marks)**

b. Find the Thevenin's equivalent circuit of Fig.Q.3(b) shown below: **(10 Marks)**

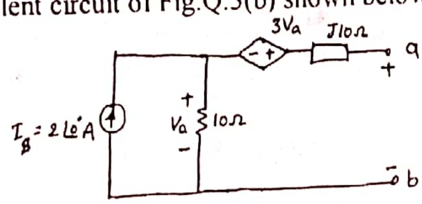


Fig.Q.3(b)

OR

4 a. Prove that the maximum power transferred from source to load when,

i) $R_L = R_o$ ii) $R_L = |Z_o|$ iii) $Z_L = \dot{Z}_o$ **(10 Marks)**

b. Find the value of i_b using Norton's equivalent circuit when R = 667Ω, refer Fig.Q.4(b). **(10 Marks)**

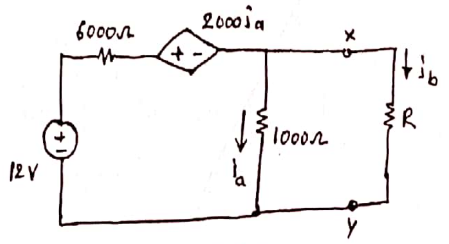


Fig.Q.4(b)

Module-3

5 a. Determine i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$, when the switch is closed at t = 0, from the Fig.Q.5(a) shown below. **(10 Marks)**

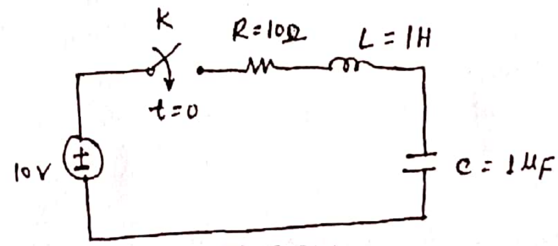


Fig.Q.5(a)

b. Find :

i) $i(0^-)$ and $v(0^+)$

ii) $\frac{di(0^+)}{dt}$ and $\frac{dv(0^+)}{dt}$

iii) $I(\infty)$ and $v(\infty)$

from the circuit shown in Fig.Q.5(b) below. **(10 Marks)**

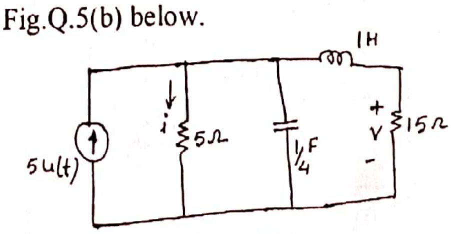


Fig.Q.5(b)

OR

6 a. Deduce the Laplace transform of the following:

i) $\text{Sin}^2 t$ ii) $\text{Cos}^2 t$ iii) Sinwt iv) $\int_0^t i(t).dt$ **(10 Marks)**

b. State and prove Initial and Final value theorems. **(10 Marks)**

Module-4

7 a. Demonstrate the terms: i) Resonance ii) Q-factor iii) Band width iv) Selectivity v) Half power frequency pertaining to a R-L-C series circuit. **(10 Marks)**

b. Prove that the Resonating frequency in a R-L-C series circuit is geometrical mean of half power frequencies i.e. $f_0 = \sqrt{f_1 f_2}$. **(10 Marks)**

OR

8 a. Evaluate w_0, Q, BW and half power frequencies and the output voltage V at W_0, refer Fig.Q.8(a). **(10 Marks)**

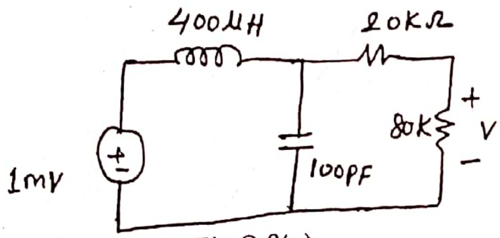


Fig.Q.8(a)

b. Derive an expression for resonance by varying R_L in parallel RLC circuit. **(10 Marks)**

Module-5

9 a. Express Z parameters in terms h parameters and what are hybrid parameters. **(10 Marks)**

b. Determine the transmission parameters for the network shown Fig.Q.9(b) below. **(10 Marks)**

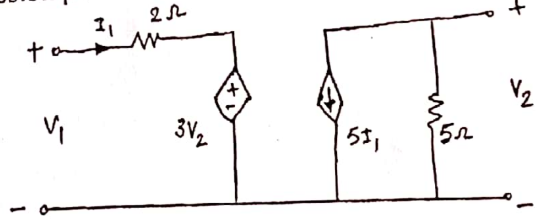


Fig.Q.9(b)

OR

10 a. Obtain the condition of transmission parameters for two networks connected in cascade. **(10 Marks)**

b. Determine the Z-parameters for the circuit shown in Fig.Q.10(b) below. **(10 Marks)**

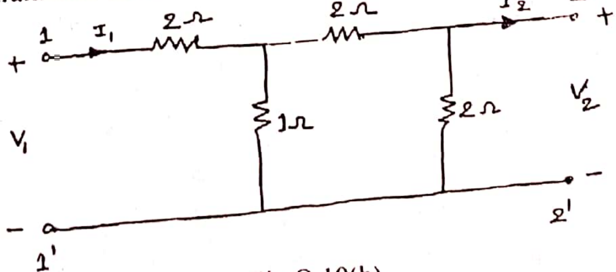


Fig.Q.10(b)

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CBCS SCHEME

USN

17EC35

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Network Analysis

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive the expression for: (i) Δ to Y transformation (ii) Y to Δ transformation **(10 Marks)**

b. Calculate the voltage across the 6Ω resistor in the network of Fig.Q1(b) using source shifting technique.

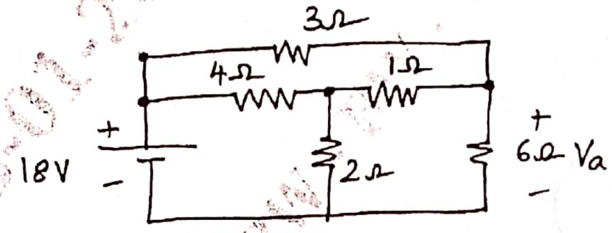

Fig.Q1(b) **(10 Marks)**

OR

2 a. Determine the resistance between the terminals A and B of the network shown in Fig.Q2(a).

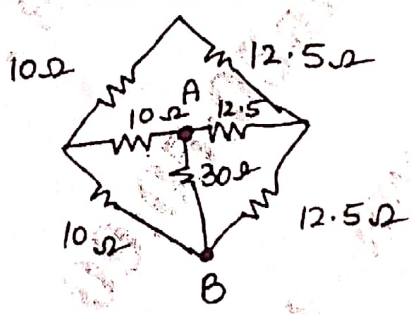

Fig.Q2(a) **(10 Marks)**

b. Find currents in all the branches of the network shown in Fig.Q2(b) using mesh analysis.

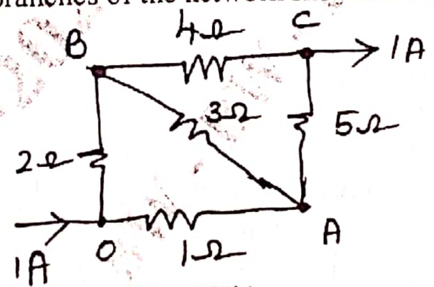

Fig.Q2(b) **(05 Marks)**

c. Find voltages V_1 and V_2 in the network shown in Fig.Q2(c) using node analysis method.

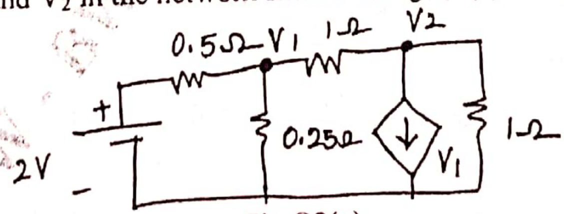

Fig.Q2(c) **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-2

3 a. Obtain Thevenin's equivalent network for Fig.Q3(a).

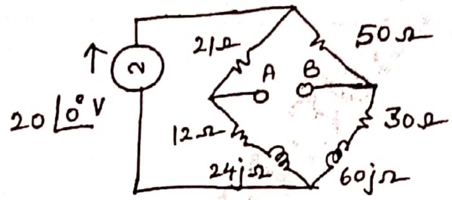

Fig.Q3(a) (08 Marks)

b. State and prove Millman's theorem. (06 Marks)

c. For the circuit shown in Fig.Q3(c), find the voltage V_x and verify reciprocity theorem.

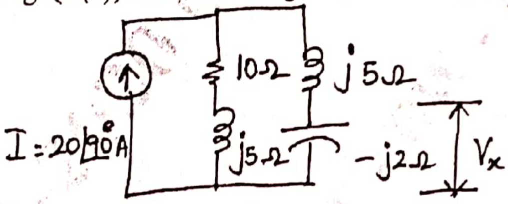

Fig.Q3(c) (06 Marks)

OR

4 a. State and prove maximum power transfer theorem for AC circuits (when R_L and X_L are varying) (10 Marks)

b. Find 'V' in the circuit shown in Fig.Q4(b) using super position theorem.

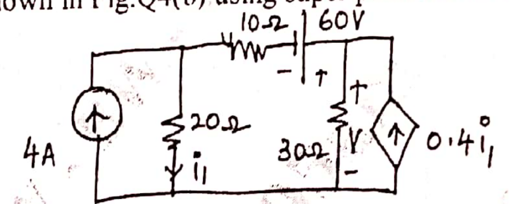

Fig.Q4(b) (10 Marks)

Module-3

5 a. What is the significance of initial conditions? Write a note on initial and final conditions for basic circuit elements. (05 Marks)

b. In the network shown in Fig.Q5(b) switch 'S' is changed from A to B at t = 0 having already established a steady state in position A shown that at $t = 0^+$, $i_1 = i_2 = \dfrac{-V}{R_1 + R_2 + R_3}$ and $i_3 = 0$.

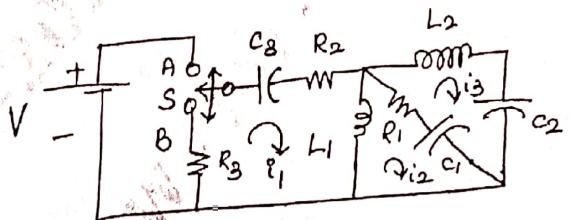

Fig.Q5(b) (10 Marks)

c. In the network of Fig.Q5(c) switch 'S' is closed at t = 0 with zero initial current in the inductor. Find i, $\dfrac{di}{dt}$ and $\dfrac{d^2i}{dt^2}$ at $t = 0^+$ if $R = 10\,\Omega$, $L = 1$ H and $V = 10$ Volts.

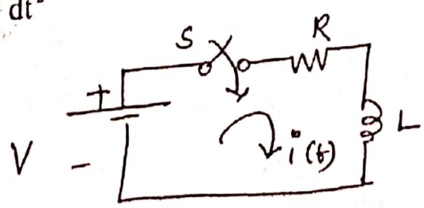

Fig.Q5(c) (05 Marks)

OR

6 a. Obtain Laplace transform of:
(i) Step function
(ii) Ramp function
(iii) Impulse function **(10 Marks)**

b. Find the Laplace transform of the waveform shown in Fig.Q6(b).

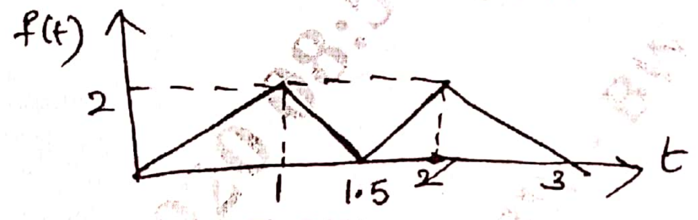

Fig.Q6(b) **(10 Marks)**

Module-4

7 a. Derive the relation between bandwidth and quality factor B.W = f_0/Q. **(10 Marks)**

b. Show that the value of capacitance for max voltage across the capacitor in case of capacitor tuning series resonance is given by $C = \frac{L}{R^2 + X_L^2}$.

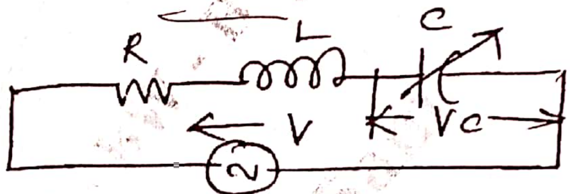

Fig.Q7(b) **(10 Marks)**

OR

8 a. Derive for f_0 for parallel resonance circuit when the resistance of the capacitance is considered.

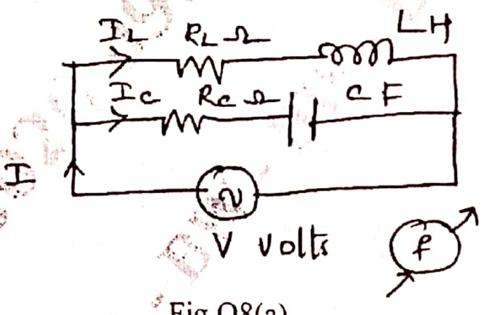

Fig.Q8(a) **(10 Marks)**

b. Find the value of L for which the circuit in Fig.Q8(b) resonates at ω = 5000 rad/sec.

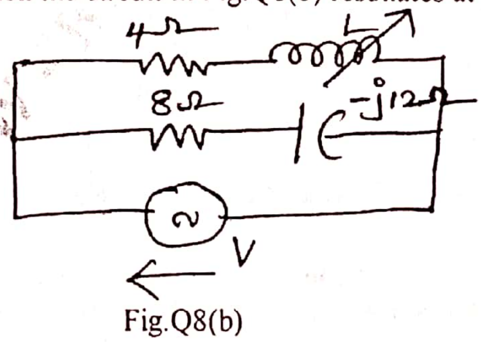

Fig.Q8(b) **(10 Marks)**

Module-5

9 a. Derive the expression of Z parameters in terms of Y parameters. (10 Marks)

b. Determine Y and Z parameters for the network shown in Fig.Q9(b).

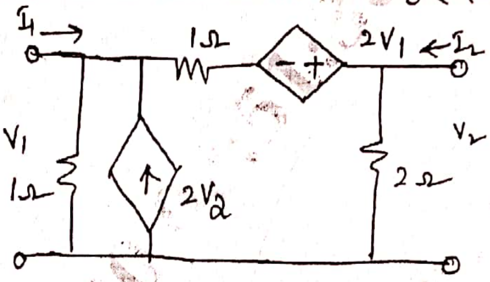

Fig.Q9(b) (10 Marks)

OR

10 a. Derive the expression of h parameters in terms of ABCD parameters. (10 Marks)

b. Find ABCD constants and show that AD – BC = 1 for the network shown in Fig.Q10(b).

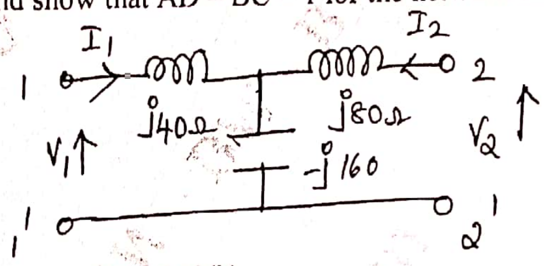

Fig.Q10(b) (10 Marks)

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ADICHUNCHANAGIRI UNIVERSITY

18EC35

Third Semester BE Degree Examination November 2020
(CBCS Scheme)

Time: 3 Hours Max Marks: 100 marks

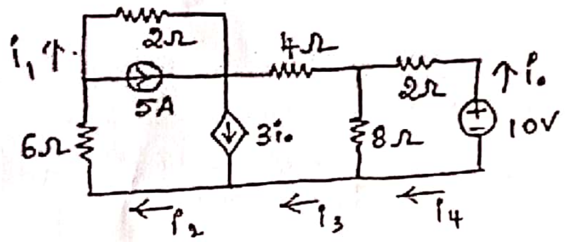
Sub: Network Analysis

Q P Code: 62305

Instructions: 1. Answer **five full** questions.
2. Choose one full question from each module.
3. Your answer should be specific to the questions asked.
4. write the same question numbers as they appear in this question paper.
5. Write Legibly

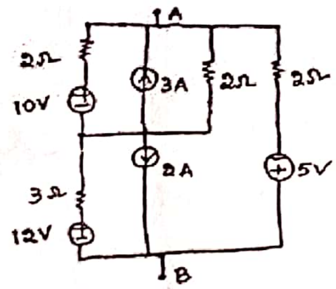
Module – 1

1 a Find the currents i_1, i_2, i_3 and i_4 using mesh analysis for the circuit shown in figure Q1(a). 07 marks



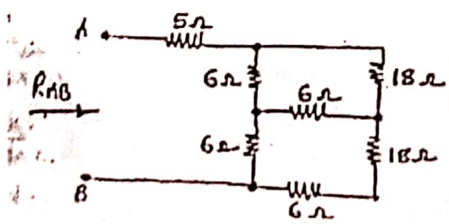
Fig, Q1(a)

b Reduce the network shown in figure Q1(b) to a single voltage source in series with a resistance between terminals A and B. 07 marks



Fig, Q1(b)

c Determine R_{AB} in the network shown in figure Q1(c). 06 marks

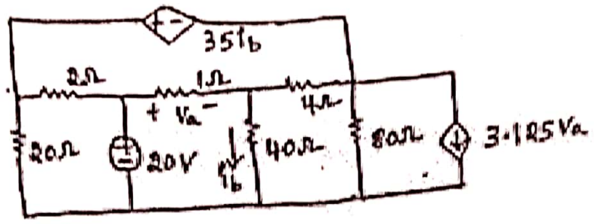


Fig, Q1(c)

PTO

Or

2 a Determine the power supplied by the 20V voltage source to the circuit shown in figure Q2(a) using nodal analysis. 10 marks



Fig, Q2(a)

b Distinguish between the following with suitable examples 10 marks

i) Linear and non-linear elements.
ii) Dependent and independent sources.
iii) Supernode and supermesh.
iv) Ideal and practical current sources.
v) Unilateral and bilateral elements.

Module – 2

3 a State and prove Thevenin's theorem. 10 marks

b Using superposition theorem, obtain the response I for the network shown in figure Q3(b). 10 marks

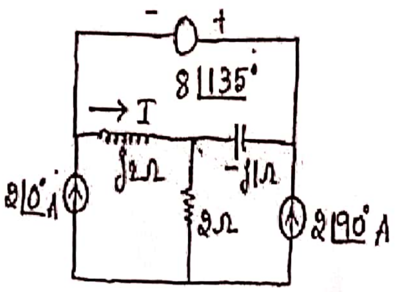


Fig. Q3(b)

Or

4 a State and prove maximum power transfer theorem for an AC circuit with an impedance as the load with variable R_L and fixed load reactance. 10 marks

b. For the circuit shown in figure Q4(b), find Thevenin's equivalent circuit across the terminals ab. 10 marks

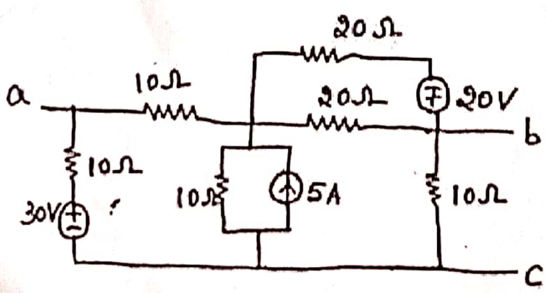


Fig. Q4(b)

Module – 3

a The network shown in figure Q5(a), has two independent node pairs. Switch K is opened at $t=0$, find the following quantities at $t=0^+$. 10 marks

i) V_1 ii) V_2 iii) dV_1/dt iv) dV_2/dt v) di_L/dt

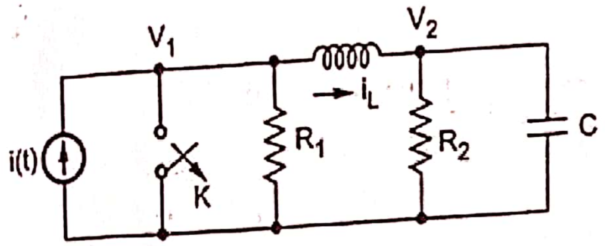


Fig. Q5(a)

b In the network shown in figure Q5(b), K is changed from position 1 to 2 at $t=0$. Solve for i, di/dt and d^2i/dt^2 at $t=0^+$. 10 marks

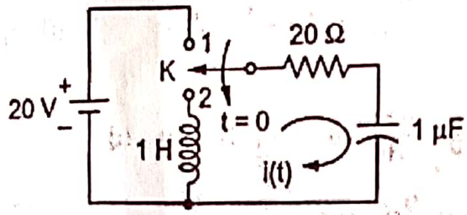


Fig. Q5(b)

Or

6 a Obtain the Laplace transform of saw tooth waveform shown in figure Q6(a). 06 marks

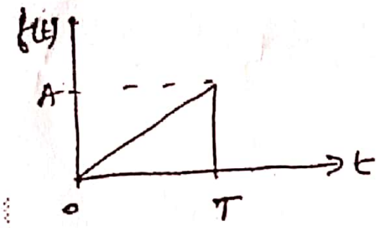


Fig. Q6(a)

b Find the Laplace transform of i) $\delta(t)$ ii) t iii) e^{-at} 06 marks

c Find initial and final value theorem for the function given below. 08 marks

$F(s)=(s^3+7s^2+5)/s(s^3+3s^2+4s+2)$

Module – 4

7 a Two coils one of $R_1=0.51\Omega$, $L_1=32mH$, the other of $R_2=1.3\Omega$ and $L_2=15mH$ and two capacitors of $25\mu F$ and $62\mu F$ are all in series with a resistance of 0.24Ω. Determine the following of this circuit. 10 marks

i) Resonance frequency ii) Q of each coil iii) Q of the circuit
iv) Cut-off frequencies v) Power dissipated of resonance if $E=10V$.

b In a two RL-RC parallel resonant circuit $L=0.4H$ and $C=40\mu F$, obtain resonant frequency for the following values of R_L and R_C. 10 marks

i) $R_L=120\ \Omega$, $R_C=80\ \Omega$ ii) $R_L=R_C=80\Omega$ iii) $R_L=80\ \Omega$, $R_C=0\ \Omega$
iv) $R_L=R_C=100\ \Omega$ v) $R_L=R_C=120\ \Omega$

PTO

Or

8 a A RLC series circuit consists of 50 Ω resistance, 0.2H inductance and 10μF capacitance with an applied voltage of 20V. Determine i) Resonant frequency ii) Q factor iii) Lower and upper frequency limits iv) Bandwidth. 10 marks

b Define the following terms with reference to resonant circuit 04 marks
i) Resonance ii) Q-factor iii) Half-power frequency iv) Selectivity

c Derive the expression for resonant frequency of a parallel resonant circuit with lossless capacitor in parallel with a coil of resistance R and inductance L. 06 marks

Module – 5

9 a Define Y parameters. Determine the Y parameters for the network shown in figure Q9(a). 08 marks

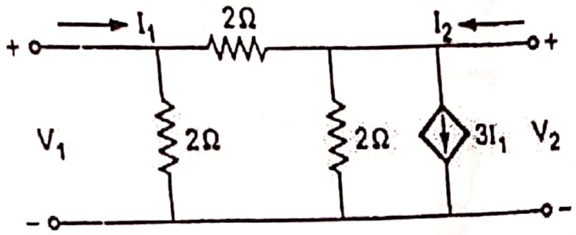


Fig. Q9(a)

b The Z parameters of a two port network are Z_{11}=20Ω, Z_{12}=10Ω, Z_{21}=10Ω and Z_{22}=10Ω. Find its Y and ABCD parameters. 06 marks

c Define h-parameters. Represent h-parameters in terms of ABCD parameters. 06 marks

Or

10 a Define transmission parameters and Z parameters. Express transmission parameters in terms of impedance parameters. 10 marks

b Find the h parameters of the network shown in figure Q10(b). Also draw its equivalent circuit. 10 marks

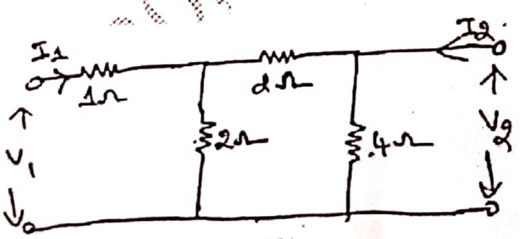


Fig. Q10(b)

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ADICHUNCHANAGIRI UNIVERSITY

18EC35

Third Semester BE Degree Examination January 2020

(CBCS Scheme)

Time: 3 Hours | Max Marks: 100 Marks

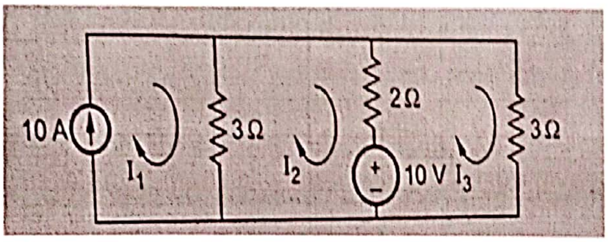
Sub: Network Analysis

Instructions:
1. Answer five full questions
2. Choose one full question from each module
3. Your answer should be specific to the questions asked
4. Write the same question numbers as they appear in this question paper
5. Write Legibly.

Module -1

1 a. Derive expressions for i) Star to Delta conversion **(10 marks)**
ii) Delta to Star conversion

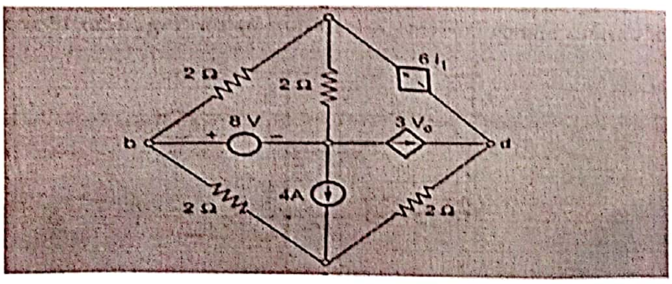
b. Write the mesh equation for the circuit shown below and determine mesh currents using mesh analysis. **(10 marks)**



OR

2 a. Explain the classification of Networks. **(10 marks)**

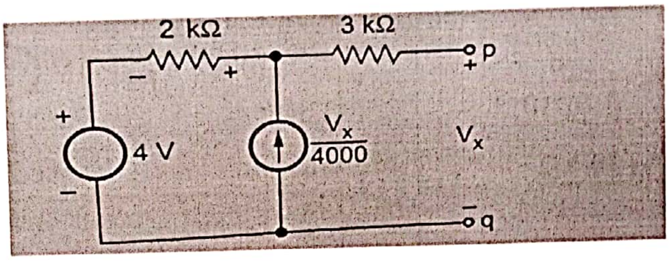
b. For the network shown below, find the node voltages V_d and V_e. **(10 marks)**



Module -2

3 a. State and prove Maximum power transfer theorem for AC circuits. **(10 marks)**

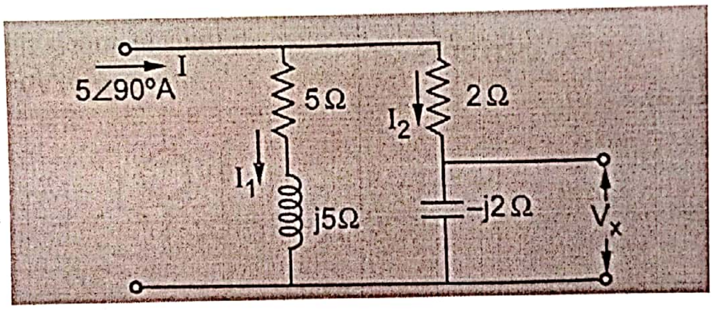
b. Find the Thevenin's equivalent of the network shown below. **(10 marks)**



OR

4 a. State and prove Millman's Theorem. **(10 marks)**

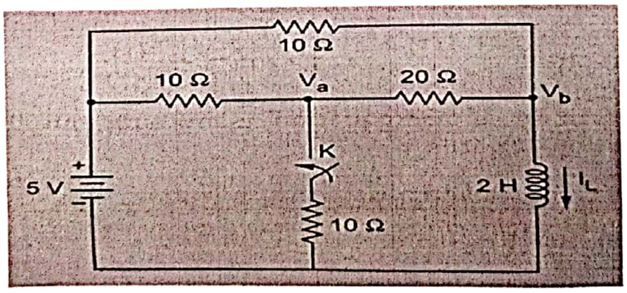
b. Find the voltage V_x and verify the reciprocity theorem for the network shown below. **(10 marks)**



Module -3

5 a. Write a note on initial conditions in basic circuit elements. **(10 marks)**

b. In the network shown below, a steady state is reached with the switch K open. At t =0, the switch is closed. For the element values given, determine the values of $V_a(0^-)$ and $V_a(0^+)$. **(10 marks)**



OR

6 a. State and prove i) Initial value theorem and ii) Final value theorem. **(10 marks)**

b. Find the Laplace transform of the fallowing: i) $\mathrm{Sin}^2 t$ and ii) $\mathrm{Cos}^2 t$ **(10 marks)**

Module -4

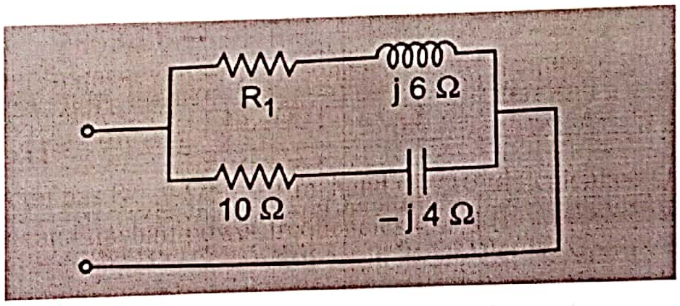
7 a. Show that resonant frequency of series resonance circuit is equal to the geometric mean of two half power frequencies. **(10 marks)**

b. A series RLC circuit has $R = 4\ \Omega$, $L = 1$ mH and $C = 10\ \mu F$, calculate Q-factor, bandwidth, resonant frequency and the half power frequencies f_1 and f_2. **(10 marks)**

OR

8 a. Derive the expression for resonant frequency for parallel circuit containing resistance in both the branches. **(10 marks**

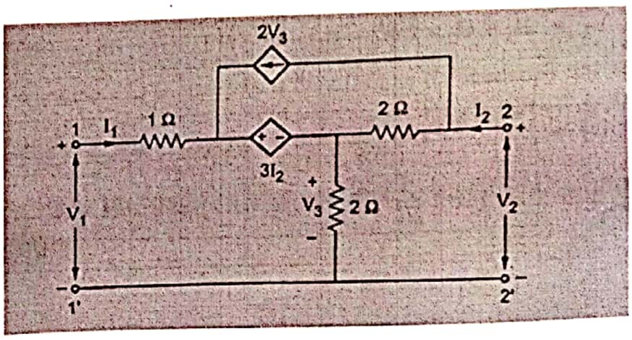
b. Find the value of R_1 such that the circuit given below is resonant. **(10 marks)**



Module -5

9 a. Define Y parameters and derive Y parameters in terms of h parameters. **(10 marks)**

b. Find Z parameters for the circuit shown below. **(10 marks)**



OR

10 a. Define Z parameters and derive Z parameters in terms of y parameters. **(10 marks)**

b. Determine Y parameters for the circuit shown below. **(10 marks)**